CHAMBERS'S EDUCATIONAL COURSE—EDITED BY W. AND R. CHAMBERS.

ARITHMETIC

THEORETICAL AND PRACTICAL.

NEW EDITION.

WILLIAM AND ROBERT CHAMBERS,
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NOTICE.

This Volume, while intended as a sequel to the *Introduction to Arithmetic* in Chambers's Educational Course, forms also an independent treatise, conducting the pupil from the first steps in the science of numbers to that stage where it becomes necessary to adopt the more general symbols of Algebra. Constructed with reference to the *Introduction to Arithmetic* on the one hand, and to the *Elements of Algebra* on the other, a uniformity of system has been observed, which will not only facilitate the progress of the student who follows a complete course, but impart to him a conception of the science more perfect and permanent than could otherwise be attained.

In the arrangement of the work, a more strictly sequential method has been adopted than is to be found in kindred treatises; and while most of the usual rules are retained, others have been introduced which bear more directly upon the business requirements of the present day. In the expression of the rules, simplicity and conciseness have been observed, allowing the pupil to be instructed in details by the *methods of solution* and *explanations* which follow, and which in each case should be thoroughly mastered before proceeding to the exercises, as the prime aim throughout has been to inculcate principles rather than empirical formulæ. In the exercises, everything like puzzle and paradox has been avoided, and such a gradation and variety given, as may

at once train the reasoning faculties of the learner, and accustom him to forms and subjects that are likely to present themselves in active life. Series of miscellaneous exercises have been inserted at certain stages; and where these can be readily solved, the pupil may safely proceed to the next stage; but where any difficulty is experienced, it may be inferred that some of the previous principles have been imperfectly understood, and therefore demand revision.

This volume, which forms the second of the Arithmetical and Commercial series of the Course, has been followed by a treatise on BOOK-KEEPING, and a volume of COMMERCIAL TABLES, containing such mercantile information as may render it an acceptable guide to the Warehouse and Counting-room.

CONTENTS.

ELEMENTS OF ARITHMETIC.

ARITHMETIC is the science of numbers; it treats of their properties, and of the methods of making computations by means of them.

A *unit*, or, as it is sometimes called, *unity*, is *one* of anything; as, *one* foot, *one* man, *one* table, &c.

Number signifies either a unit, as, *one* man; or a collection of units of the same kind, as, *two* men, *five* books, *seven* houses, a *thousand* feet, &c.; *two*, *five*, *seven*, *thousand*, &c., are called *numbers*.

NOTATION AND NUMERATION.

NOTATION is the method of expressing numbers by means of certain signs or figures; thus—1, 2, 3.

NUMERATION is the art of numbering, that is, of reading or expressing numbers in words; thus—1, one; 2, two.

THE FIGURES used to express numbers are the following:

one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	nothing or cipher.
1	2	3	4	5	6	7	8	9	0

The first nine of these figures, when standing separately or singly, thus—1, 2, 3, 9, represent the numbers from *one to nine*. The last, 0, called a nothing or cipher, expresses no number, and has no value in itself; but it is used to affect the value of the other figures when annexed to them, as will presently be explained.

It is by means of these ten figures and their combinations that all numbers are expressed.

Each of these figures, besides the *simple* value which it has when standing alone, as, 1, one, 2, two, &c., has also, when standing in connection with other figures, a *local* value—that is, a value depending on the *place* it occupies in the number.

Thus, 5 by itself means simply *five;* but if it become the *second* figure from the *right*, by a nothing or any other figure being placed after it—thus, 50—the 5 has ten times its former value, and means 5 *tens*, or *fifty;* and if it become the *third* from the right, by another figure being annexed—thus, 500—the 5 is again increased tenfold in value, and means 5 *hundreds;* and so on, the addition of each new figure increasing tenfold the value of those before it.

It is in this way that the nothing or cipher, though it has no value in itself, affects the value of the other figures, by altering their place or rank. Each of the other figures has a similar effect on the rest when annexed to them.

It is to be observed, however, that though the value of a figure depends on the place it occupies, the meaning of the figure always remains the same. Thus, 5 always expresses *five* of something, either *five* single things, or *five* groups of ten, or *five* groups of a hundred, &c.

NUMBERS *Ten to ninety-nine* are expressed by combining *two* figures; thus—10, ten; 20, twenty; 85, eighty-five; 99, ninety-nine.

NUMBERS *One hundred to nine hundred and ninety-nine*, are expressed by combining *three* figures; thus—100, one hundred; 500, five hundred; 867, eight hundred and sixty-seven; 999, nine hundred and ninety-nine.

Thousands are expressed by *four* figures; thus—1000, one thousand; 7320, seven thousand three hundred and twenty.

Tens of thousands are expressed by *five* figures; *hundreds of thousands*, by *six* figures; *millions*, by *seven* figures; and so on. By using more figures, we are enabled to express any number, however great.

It will thus be seen that in Notation, the *rank* or place of a figure in any number is what determines the value it bears.

The *first* figure at the *right* in a line of figures, counting from right to left, has only its *simple* value; a figure in the *second* place has ten times its simple value; a figure in the *third* place has ten times the value it would have in the second place, or a hundred times its simple value, and so on; and, generally, a figure in *any* place has ten times the value it would have in the next *lower* place.

The number 10, according to which the *local* values of the other figures are determined, is called the *base* of the system; and the system of notation itself, the *decimal scale of notation.*

The first or lowest place in a line of figures is called the *units'* place; the second, the *tens'* place; the third, *hundreds';* the fourth, *thousands';* and so on, as shewn in the following NUMERATION TABLE. It is read from *right* to *left*, thus—units, tens, hundreds, &c.; the rank or position in which these stand in regard to each other should be carefully studied and committed to memory.

In expressing large numbers in figures, it is usual, for the sake of distinctness, to point off the figures as far as possible into sets of three, called *periods*, by means of commas, beginning at the *right* hand, and counting towards the *left*. Thus—87,463,292. Each Period of three is named as marked in the Table.

NUMERATION TABLE.*

Quintillions.			Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
Hundreds of Quintil's.	Tens of Quintillions.	Quintillions.	Hundreds of Quadril's.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
3	2	1,	9	8	7,	6	5	4,	3	2	1,	9	8	7,	6	5	4,	3	2	1

The above number, 321,987,654,321,987,654,321, is read three hundred and twenty-one quintillions, nine hundred and eighty-seven quadrillions, six hundred and fifty-four trillions, three hundred and twenty-one billions, nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one.

The periods succeeding those in the Table are *sextillions, septillions, octillions*, &c., but these names are seldom employed. In ordinary affairs, we rarely hear of any sum beyond hundreds of millions.

I. TO EXPRESS NUMBERS IN *Figures*.

Begin at the *left* hand, and put down the required figures one after the other in a line, taking care to put each figure in the *place* or rank necessary to express the number, according to the Numeration Table—that is, millions must be put in the

* This Table is given in the improved form used by the French and Italians. It has been recently introduced into this country, as being much more simple and convenient than the old form used by ourselves. The figures are grouped into periods of three, and are named and read accordingly. In the old form, which is given below, the figures are

millions' place, or seventh from the right hand; thousands in the thousands' place, or fourth from the right; and so on.

In doing this, *nothings* must be put in all those *places* of which none are mentioned in the given number. Thus, if thousands are mentioned, but no hundreds, a nothing must be put in the hundreds' place, to keep the other figures in their proper rank.

After the figures are written down, point them off, as far as possible, into *periods* of three, beginning at the *right*.

It may be useful, at first, for the pupil, in writing small numbers, to mark down as many of the *places* (such as units, tens, hundreds, &c.) in the Numeration Table as are required to express the given number, and then to write the respective figures below the names that express them: thus, write in figures, thirty-six thousand and seventy-three.

tens of thousands,	thousands,	hundreds,	tens,	units.	
3	6,	0	7	3;	or, 36,073.

In writing down larger numbers, such as millions, proceed as follows: Draw three lines $\underline{m}$ $\underline{t}$ $\underline{u}$ to represent the three *periods*, into which the figures forming a million, or millions, are divided; below that marked *m*, write the millions of the given number; below that marked *t*, the thousands; and below that marked *u*, the hundreds, tens, and units.

Thus, suppose the number to be expressed is one hundred and six millions, five thousand and thirty; below the *m*, write 106; below the *t*, 005; and below the *u*, 030; thus—

m	*t*	*u*	
106	005	030 ;	or without the lines, 106,005,030.

It will be observed that there being in the MILLIONS' period no *tens* of millions, in the THOUSANDS' period no *hundreds* or *tens* of thousands, and in the UNITS' period no hundreds or units, nothings have been inserted in all these *places* to keep the other figures in their right position.

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grouped into periods of six, which is an arrangement very inconvenient in practice. It will be observed that the two Tables are the same up to hundreds of millions.

OLD NUMERATION TABLE.

	Billions.						Millions.						Units.					
Trillions.	Hundreds of thousands of	Tens of thousands of	Thousands of	Hundreds of	Tens of	Billions.	Hundreds of thousands of	Tens of thousands of	Thousands of	Hundreds of	Tens of	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
1,	9	8	7	6	5	4,	3	2	1	9	8	7,	6	5	4	3	2	1

After a little practice, the pupil will be able to dispense with the names and lines altogether, and to write any number whatever with great facility.

Exercises in Notation.

Note down in figures the following numbers:

1. Six thousand seven hundred and eight.
2. One million two thousand and sixty.
3. Seven millions four hundred and thirty-two thousand four hundred and seven.
4. Sixty-three millions and fifty thousand.
5. One hundred and twenty-three millions four hundred and fifty-six thousand seven hundred and eighty-nine.
6. Two millions and two.
7. Seven thousand and forty-eight.
8. One hundred and seven thousand six hundred and seventeen.
9. Twenty-four millions thirty-six thousand and five hundred.
10. Seven hundred and eighty millions six hundred thousand and four.
11. Ninety-one millions sixty-three thousand two hundred and twenty-five.
12. Eight hundred and seventy-nine millions and fifteen.

The distances of the principal planets from the sun are as follows:

13. Mercury, thirty-seven millions of miles.
14. Venus, sixty-eight millions of miles.
15. The Earth, ninety-five millions of miles.
16. Mars, one hundred and forty-two millions of miles.
17. Jupiter, four hundred and ninety-five millions of miles.
18. Saturn, nine hundred and six millions of miles.
19. Uranus, one billion eight hundred and twenty-two millions of miles.
20. Neptune, two billions eight hundred and fifty-four millions of miles.

It may give some idea of large numbers, such as millions and billions, to mention that about 25 millions of books, each an inch thick, would make a pile 400 miles high—the distance from London to Edinburgh; and about 15¼ billions of such books would make a pile 240,000 miles high—being the distance from the earth to the moon.

Answers.

1.	6708	6.	2000002	11.	91063225	16.	142000000
2.	1002060	7.	7048	12.	879000015	17.	495000000
3.	7432407	8.	107617	13.	37000000	18.	906000000
4.	63050000	9.	24036500	14.	68000000	19.	1822000000
5.	123456789	10.	780600004	15.	95000000	20.	2854000000

II. To express Numbers in *Words*.

Begin at the *right*, and, going towards the left, divide the numbers, by means of commas, into as many periods of three figures each, as possible; then name the order or rank of each figure of the given number; thus—units, tens, hundreds, thousands, &c.—that is, the first figure at the *right* is units; the next, tens; and so on.

Having in this way ascertained the *rank* of each figure, or its position in the Numeration Table, express the whole sum in words, reading in the usual way from *left* to right.

In reading large numbers, it will be useful, after dividing them into periods, to write above the first period at the *right*, *u* for units; above the second, *t* for thousands; above the third, *m* for millions, &c.; the names of the successive periods being obtained from the *Numeration Table.* Then commence at the *left*, and read as in the following example:

The number 85605832, when divided into *periods*, stands thus—

m t u
85,605,832,

and is read *eighty-five millions, six hundred and five thousands, eight hundred and thirty-two.*

After a little practice, it will become unnecessary to name the order or rank of the figures, or to mark the periods, before reading them.

Exercises in Numeration.

Express the following numbers in words:

1.	45	13.	7354	25.	39000051
2.	89	14.	1296	26.	682379265000
3.	76	15.	608	27.	300031027
4.	67	16.	1007	28.	416927384
5.	83	17.	7105	29.	10000310020
6.	95	18.	83472	30.	84700502060
7.	59	19.	100685	31.	12345678
8.	16	20.	29307	32.	1020304050
9.	432	21.	5410200	33.	60070800917
10.	119	22.	61794325	34.	413000000
11.	191	23.	31416	35.	271094684
12.	746	24.	7854	36.	2718281828459

37. Mercury, diameter in miles, . . 3140
38. Venus, " " . . 7700
39. Earth, " " . . 7912
40. Mars, " " . . 4100
41. Jupiter, " " . . 90000
42. Saturn, " " . . 76791
43. Uranus, " " . . 35307
44. Neptune, " " . . 39793

ROMAN NOTATION.

The Romans made use of the following letters, with their combinations, to express numbers. They are still in use among ourselves for some purposes, such as the headings of chapters, divisions, &c.

I.	=	1	C.	=	100
V.	=	5	D. or IↃ.	=	500
X.	=	10	M. or CIↃ.	=	1000
L.	=	50			

Two or more of the *same* letter placed together, mark two or more of the same number; thus—II. means twice I., or two.

A letter of inferior value placed *before* one of superior value, means that the inferior is to be *deducted* from the superior; thus in IX., the I placed before the X means that I is to be taken from X, and IX. therefore expresses 9.

A letter of inferior value placed *after* one of superior value, means that the inferior is to be *added* to the superior—thus in LX., the X placed after L means that X is to be added to L, and LX. therefore expresses 60.

A line drawn above a letter increases its value a thousand times—as $\overline{\text{X}}$., 10,000; $\overline{\text{D}}$., 500,000.

The number IↃ (= D. or 500) is increased in value ten times for every Ↄ annexed; thus—IↃↃ. means 5000. The number CIↃ (= M. or 1000) is increased in value ten times for every C and Ↄ joined to it; thus—CIↃ., by joining C and Ↄ, becomes CCIↃↃ., or 10,000. The letters IↃ are not now in use.

I.	1	XIV.	14	LXXX.	80
II.	2	XV.	15	XC.	90
III.	3	XVI.	16	C.	100
IV.	4	XVII.	17	CC.	200
V.	5	XVIII.	18	CCC.	300
VI.	6	XIX.	19	CCCC.	400
VII	7	XX.	20	D.	500
VIII.	8	XXI.	21	DC.	600
IX.	9	XXX.	30	DCC.	700
X.	10	XL.	40	DCCC.	800
XI.	11	L.	50	DCCCC.	900
XII.	12	LX.	60	M.	1000
XIII.	13	LXX.	70	MDCCCLIV.	1854

SIMPLE ADDITION.

ADDITION is the method of finding a single number that is equal to two or more other numbers taken together. The single number obtained by the addition, is called the *sum* of the other numbers.

It is only numbers of the same kind or denomination that can be added together; thus, though 3 and 4 make 7, we cannot say that 3 *yards* and 4 *feet* make either 7 *yards* or 7 *feet*.

A *Simple* number is that which is expressed in *one* denomination, as 10 pounds, *or*, 10 shillings. When simple numbers are added—that is, numbers that are *all* pounds, *or*, *all* shillings, &c. —the operation is called SIMPLE ADDITION. This Rule is given below.

A *Compound* number is that which is expressed in several denominations, as 10 pounds, 5 shillings, and 3 pence. When compound numbers are added, the operation is called COMPOUND ADDITION. This Rule is given at page 50.

THE SAME DISTINCTION of *Simple* and *Compound* applies to the subsequent rules of Subtraction, Multiplication, and Division.

For convenience, certain signs are often employed in connection with the rules of arithmetic.

Addition is denoted by the sign +, called *plus*; thus, $7 + 6$ means that 7 and 6 are to be added together.

The sign = (which means *equal to*), when placed between two quantities, denotes that they are equal to one another: thus— $7 + 8 = 15$, means that 7 and 8 added together are equal to 15.

The sign $\therefore$ signifies *therefore*, and $\because$ *because*.

As the pupil cannot be expected to make any progress until he know at once the sum of any two figures, the following table must be accurately committed to memory:

ADDITION TABLE.

The pupil ought to verify this Table by means of Counters.

2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
2 are 4	2 are 5	2 are 6	2 are 7	2 are 8	2 are 9	2 are 10	2 are 11
3 ″ 5	3 ″ 6	3 ″ 7	3 ″ 8	3 ″ 9	3 ″ 10	3 ″ 11	3 ″ 12
4 ″ 6	4 ″ 7	4 ″ 8	4 ″ 9	4 ″ 10	4 ″ 11	4 ″ 12	4 ″ 13
5 ″ 7	5 ″ 8	5 ″ 9	5 ″ 10	5 ″ 11	5 ″ 12	5 ″ 13	5 ″ 14
6 ″ 8	6 ″ 9	6 ″ 10	6 ″ 11	6 ″ 12	6 ″ 13	6 ″ 14	6 ″ 15
7 ″ 9	7 ″ 10	7 ″ 11	7 ″ 12	7 ″ 13	7 ″ 14	7 ″ 15	7 ″ 16
8 ″ 10	8 ″ 11	8 ″ 12	8 ″ 13	8 ″ 14	8 ″ 15	8 ″ 16	8 ″ 17
9 ″ 11	9 ″ 12	9 ″ 13	9 ″ 14	9 ″ 15	9 ″ 16	9 ″ 17	9 ″ 18
10 ″ 12	10 ″ 13	10 ″ 14	10 ″ 15	10 ″ 16	10 ″ 17	10 ″ 18	10 ″ 19

The pupil should be exercised on the table in the following manner: Write the first ten numbers in any order on a *black-board*, and below each write the number 2, and let him name the sum of every two numbers, in succession; proceed in the same way with 3, 4, 5, &c. For example, suppose that 6 is placed below each of the figures, thus:

6	8	3	10	5	2	7	4	1	9
6	6	6	6	6	6	6	6	6	6

the pupil should then begin at the right, and say 6 and 9 are 15, 6 and 1 are 7, 6 and 4 are ten, &c. After he can do this with facility, the teacher should ask him: How many are 16 and 9? 26 and 9? 36 and 9?—calling his attention to the fact, that the right-hand figure is the same in each case: and so on with all the other figures. By pursuing this method, the teacher will be much gratified with the future progress of the pupil. It may be remarked, that by means of the *counters*, the *black-board*, and *vivâ-voce* examination, the pupil's senses of *touch*, *sight*, and *hearing*, 'the three great avenues of knowledge,' are called into exercise.

RULE FOR ADDING.

1. Write the numbers to be added together, distinctly under each other, so that *units* may stand directly under *units*, *tens* under *tens*, &c., and draw a line under the whole.

2. Add together the figures in the column of units, put down the *last* figure of the sum under this column, and carry the other figure or figures, if any, to the next column.

3. Proceed in the same way with the other columns in succession, adding in each case any figures *carried* from the previous column; on adding the last column, put down *all* the figures of the summation.

The various figures that have been written down form the answer or sum.

Example.—Add together 534, 4397, 61, 406, and 7325.

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  534
 4397
   61
  406
 7325
-----
12723
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In this example, the numbers are set down as in the margin, and beginning at the lowest figure of each column, the pupil should proceed thus:

Pupil.—5 and 6 are 11, and 1 are 12, and 7 are 19, and 4 are 23; that is, 2 tens and 3 units. I put down the 3 units, and *carry* the 2 tens to the next column. 2 to 2 are 4, and 6 are 10, and 9 are 19, and 3 are 22 tens; that is, 2 hundreds and 2 tens. I put down the 2 tens, and carry the 2 hundreds. Again, 2 and 3 are 5, and 4 are 9, and 3 are 12, and 5 are 17 hundreds; that is, 1 thousand and 7 hundreds. I put down the 7, and carry 1. Lastly,

1 and 7 are 8, and 4 are 12. I put down the whole sum, 12 thousands, because there are no more columns to be added. *Teacher.* What is the sum of all the columns? *Pupil.* Twelve thousand seven hundred and twenty-three.

After a short time, the pupil should learn to sum the several columns without naming the figures. For instance, in the above example, instead of saying 5 and 6 are 11, and 1 are 12, and 7 are 19, and 4 are 23, he should only be required to name the result at each step; thus—5, 11, 12, 19, 23, &c.

Reason of the Rule.—The rule for adding numbers depends on the following obvious principles:

1. That the whole sum is equal to all its parts taken together.

2. That it is only numbers of the same *local* value that can be added together; hence, units are added to units, tens to tens, &c.

3. *One* is carried for every *ten*, because, from the nature of notation, *ten* units in any one column is equal to *one* only in the column immediately to the left of it.

Proof.—To check the correctness of the addition, count the columns downward, and if the sum is the same as before, the work is *probably* right.

Or, Divide the account into two or more parts, and add the parts separately; then find the amount of these partial sums. If this amount is equal to the sum obtained in the usual way, the work may be presumed to be right.

Example.—	7346	
	1275	8621
	8394	
	2957	
	1826	13177
Total	21798	21798

Exercises.

1.	2.	3.	4.	5.
184	579	695	866	516
392	125	953	596	783
571	791	857	127	521
694	506	191	932	668
963	384	802	702	214
182	179	864	341	817

Answers.

1. 2986 2. 2564 3. 4362 4. 3564 5. 3519

6.	7.	8.	9.	10.
565	650	8521	7235	9167
436	392	3947	5984	8582
398	415	1876	1678	9764
325	686	2345	2959	1852
791	735	9812	3827	3956
452	606	7195	1898	1234

11.	12.	13.	14.	15.
5678	3957	7354	3954	3198
9012	1268	8141	1276	4569
3456	9854	6957	3919	7352
7891	7369	2718	4578	1127
2345	1275	2845	9087	3964
6789	9631	9687	1685	3576

16.	17.	18.	19.	20.
9735	9162	6783	4068	8964
1976	3976	2954	2879	4258
3759	8537	3936	9965	7068
4169	7352	1483	8470	3829
3785	8964	1846	2886	4956
2135	8572	3972	9742	9748

21.	22.	23.	24.	25.
4065	4209	5126	2677	6973
2387	5670	9632	8321	2185
4021	8463	7923	9678	9657
5689	2338	3084	5322	8326
9148	4075	9146	9172	9577
2796	4087	1752	6854	8763

26.	27.	28.	29.	30.
2982	1352	5176	5810	6237
7812	3976	3498	6781	9748
9657	4757	5132	9574	8034
1285	3182	6790	3448	9154
4037	7957	9259	5486	9257
1877	2632	3807	4798	2863

Answers.

6. 2967	11. 35171	16. 25559	21. 28106	26. 27580
7. 3484	12. 33354	17. 42068	22. 29442	27. 23856
8. 33696	13. 33152	18. 20964	23. 41613	28. 33662
9. 23571	14. 24449	19. 37010	24. 42024	29. 35397
10. 29555	15. 23786	20. 38818	25. 45481	30. 45288

31.	32.	33.	34.	35.
3789	7984	3043	2463	6287
9415	8196	8923	4087	4509
9789	9768	9748	5868	6243
6431	8437	2896	4293	7801
9283	1689	5148	8068	9360
7965	9874	2981	3647	4918

36.	37.	38.	39.	40.
9837	7306	5983	2138	9273
7754	5479	3209	3845	2734
6383	8452	9270	4569	4689
3847	3874	3478	3672	4386
2905	9765	7640	5986	7892
9978	3209	5963	2186	7584
4106	2973	9742	8573	6705

41.	42.	43.	44.	45.
3925	1234	7817	1297	59317
1874	5678	7939	9084	98765
3927	9123	2157	3275	39278
9872	4567	9736	7926	41692
3197	8901	5273	1837	54183
2864	2345	1892	2749	27952
5972	6782	3685	8475	61847

46.	47.	48.	49.	50.
91934	473926	397426	825927	392576
89756	593842	593075	259728	834912
19372	795384	329705	298732	782519
49108	682173	574839	987659	235678
21685	373925	328975	283139	901234
39724	814198	895763	173916	579257
58372	765432	654321	876544	395872
27998	127904	718902	328597	782764
54796	752847	537198	123456	397185

51. 3917 + 46 + 31254 + 293 + 8162 + 753 + 82 + 7854.
52. 17542 + 39126 + 3175 + 93 + 814 + 79268 + 123 + 17.
53. 891 + 216 + 7139 + 25 + 318 + 9832 + 72956 + 18 + 9.

Answers.

31. 46672	36. 44810	41. 31631	46. 452745	51. 52361
32. 40948	37. 41058	42. 38630	47. 5379631	52. 140158
33. 32239	38. 45285	43. 38499	48. 5030204	53. 91404
34. 28426	39. 30969	44. 34643	49. 4157698	
35. 39118	40. 43263	45. 383034	50. 5301997	

54. Add together fifty-four thousand seven hundred and sixteen, four thousand and eighty-nine, three hundred and twenty thousand six hundred and four, one hundred and sixty-five, thirty-nine, four millions three thousand two hundred and thirty-seven, nine hundred and sixty-one thousand five hundred and twenty-nine.

55. In 1851, the population of London was 2362236; that of Liverpool, 375955; of Manchester, 303385; of Birmingham, 232841; of Leeds, 172270; of Bristol, 137328; of Sheffield, 135310; of Bradford, 103778; of Glasgow, 329097; of Edinburgh and Leith, 191221. What was the total population of all these places in that year?

56. In 1801, the population of London was 958863; that of Liverpool, 82295; of Manchester, 94876; of Birmingham, 70670; of Leeds, 53162; of Bristol, 61153; of Sheffield, 45755; of Bradford, 13264; of Glasgow, 77058; of Edinburgh and Leith, 81404. What was then the total population of all these places?

57. The Great Exhibition of 1851 was open in May 27 days, and the number of visitors during that time was 734814; in June, 25 days—visitors, 1133114; in July, 27 days—visitors, 1314176; in August, 26 days—visitors, 1023435; in September, 26 days—visitors, 1155240; and in October, 13 days—visitors, 808237. How many days was the Great Exhibition open, and how many visitors were in it?

58. At the end of 1843 there were open 2036 miles of railway in Great Britain; in 1844 there were opened 204 miles; in 1845, 296 miles; in 1846, 606 miles; in 1847, 803 miles; in 1848, 1182 miles; in 1849, 869 miles; and in 1850, 625 miles. How many miles of railway communication were open at the end of 1850?

59. The total receipts for all the railways in Great Britain were as follows: in 1842, £4341781; in 1843, £4842625; in 1844, £5610982; in 1845, £6669224; in 1846, £7689874; in 1847, £8975671; in 1848, £10059006; in 1849, £11013817; and in 1850, £12755235. What were the total receipts of all the railways during these nine years?

60. In the year 1857 the quantity of mineral dug in the United Kingdom was as follows: tin ore, 9783 tons; copper ore, 229455 tons; lead ore, 96820 tons; silver, 15 tons; zinc ore, 9289 tons; sulphur ore, 74679 tons; arsenic, 476 tons; cobalt, 4 tons; nickel, 1 ton; iron ore, 9573281 tons; coals, 65394707 tons; salt, 1462045 tons. How many tons were dug in all?

Answers.

54. 5344379	57. 144 days	59. £71958215
55. 4343421	6169016 visitors	60. 76850555
56. 1538500	58. 6621	

SIMPLE SUBTRACTION.

SUBTRACTION is the method of taking a less number from a greater, to find what remains, or what is the difference between the two numbers.

The number left after subtracting the one from the other, is called the *Difference*, or *Remainder*.

Subtraction is denoted by the sign – called minus; thus, $7 - 5 = 2$, denotes that 5 is to be subtracted from 7, and the remainder is equal to 2.

SUBTRACTION TABLE.

To be verified by means of Counters.

2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
3 = 1	4 = 1	5 = 1	6 = 1	7 = 1	8 = 1	9 = 1	10 = 1
4 " 2	5 " 2	6 " 2	7 " 2	8 " 2	9 " 2	10 " 2	11 " 2
5 " 3	6 " 3	7 " 3	8 " 3	9 " 3	10 " 3	11 " 3	12 " 3
6 " 4	7 " 4	8 " 4	9 " 4	10 " 4	11 " 4	12 " 4	13 " 4
7 " 5	8 " 5	9 " 5	10 " 5	11 " 5	12 " 5	13 " 5	14 " 5
8 " 6	9 " 6	10 " 6	11 " 6	12 " 6	13 " 6	14 " 6	15 " 6
9 " 7	10 " 7	11 " 7	12 " 7	13 " 7	14 " 7	15 " 7	16 " 7
10 " 8	11 " 8	12 " 8	13 " 8	14 " 8	15 " 8	16 " 8	17 " 8
11 " 9	12 " 9	13 " 9	14 " 9	15 " 9	16 " 9	17 " 9	18 " 9

RULE FOR SUBTRACTING.

1. Write the less number under the greater, so that *units* may stand under *units*, *tens* under *tens*, &c., and draw a line under them.

2. Begin at the right hand, and take in succession each figure in the *lower* line, if possible, from that which stands above it in the *upper* line, and set down the remainder.

3. But if any figure in the *lower* line be greater than the figure above it, 10 is to be added to the *upper* figure before subtracting.* As an equivalent for adding the 10, the next *under* figure requires to be considered as 1 *more*. This is called carrying 1 to the under figure.

The several remainders that have been written down form the answer, or difference between the two lines.

* The 10 that is here added is got by taking, or *borrowing*, as it is called, 1 from the next *upper* figure. That figure should, therefore, be counted as one *less*; but it is more convenient in practice, and produces the same result, to make the next *under* figure 1 *more*. The reason that 1 taken from the one figure is counted as 10, in adding it to the other, is, that the figure from which the 1 is taken is of a higher rank than the figure to which it is added; and, consequently, 1 of the former is equal to 10 of the latter.

Example.—Take 1896573 from 3200318.

3200318
1896573
1303745

In this example, the numbers are put down as in the margin, and beginning at the right hand the *pupil* says—3 from 8 and 5 remain, I put down the 5; 7 from 1, I cannot, but 7 from 11 and 4 remain, I put down the 4. *Teacher*. How do you get 11? *Pupil*. I take 1 from the next figure 3; but 1 in any place of figures makes 10 in the next lower place, and this 10 added to 1 makes 11. As an equivalent for borrowing 1 from the upper figure 3, I *carry* 1 to the next under figure 5, and say, 6 from 3, I cannot, but 6 from 13 and 7 remain, then carrying 1 to 6, 7 from 10 and 3 remain, 10 from 10 and 0 remains, 9 from 2 I cannot, but 9 from 12 and 3 remain, 2 from 3 and 1 remains. *Teacher*. What is the whole difference? *Pupil*. One million three hundred and three thousand seven hundred and forty-five.

Proof.—Add the remainder to the lower line of figures, and the result will be equal to the upper line, if the work is right; for it is evident that the sum taken away, added to that which is left, will make up the whole. From this it appears that *Subtraction* is the converse of *Addition*.

51309	
26847	
24462	Remainder.
51309	Proof.

Reason of the Rule.—The reason of the rule may be gathered from the example, combined with the observations made under the Reason of the Rule of Addition. The example, broken up and arranged, may render the operation more obvious.

$$\begin{array}{lcl}
3200318 & = & 3000000 + 1200000 + 100000 + 10000 + 1300 + 110 + 8 \\
1896573 & = & 2000000 + 900000 + 100000 + 7000 + 600 + 70 + 3 \\
\hline
\text{Difference} & = & 1000000 + 300000 + 0 + 3000 + 700 + 40 + 5 = 1303745
\end{array}$$

Exercises.

1. 719384	2. 594768	3. 938726	4. 839269	5. 947385
206123	123456	124311	627135	413052

6. 543912647	7. 835164295	8. 594168231	9. 615932718
275183176	187106837	427641897	309168271
10. 184391715	11. 315712937	12. 791936752	13. 683175398
139276189	182619308	139271825	596483712
14. 309215730	15. 500384106	16. 216004132	17. 300026154
186493271	123725918	108217184	17938417

Answers.

1. 513261	6. 268729471	10. 45115526	14. 122722459
2. 471312	7. 648057458	11. 133093629	15. 376658188
3. 814415	8. 166526334	12. 652664927	16. 107786948
4. 212134	9. 306764447	13. 86691686	17. 282087737
5. 534333			

18. 803658687	19. 526809259	20. 980138420	21. 505061023
618932751	309215417	961083597	123785527

22. What is the difference between 381725 and 683508519?

23. 7126 + 943 − (920 + 793).

24. 3768 + 517 − (38 + 76 + 305); 10000 − (382 + 7196 + 275).

25. 76 − 29; 31 − 17; 803 − 126; 700 − (61 + 75).

26. George Heriot's Hospital was founded in the year 1628; how old was it in 1858?

27. By how much is 71032500 greater than 8261092?

28. From the sum of 283, 6024, 3961, 35, and 72436, subtract the sum of 3192, 576, 2953, and 12345.

29. At the end of the year 1857, the number of miles of railway open for traffic was in England 6773, and in Scotland 1230 miles; in 1850 there were open for traffic in England and Scotland together, 6621 miles. How many miles were opened for traffic during the seven intervening years?

30. According to the report of the Post-office for 1857, there were delivered in the United Kingdom during the year, 504421000 letters; of these, 410003000 were delivered in England and Wales, 42806000 in Ireland, and the remainder in Scotland. Find how many were delivered in Scotland.

31. The total value of gold imported into Great Britain during the seven years beginning with 1851, and ending with 1857, was £130876000; the value imported in 1851 was £8654000; in 1852, was £15194000; in 1853, was £22435000; in 1854, was £22077000; in 1855, was £19875000; in 1856, was £21275000. Find the value imported in 1857.

32. The solid content of the Sun is 399839629687 cubic miles; that of the Earth, 260775; of Mercury, 10195; of Venus, 223521; of Mars, 48723; of Jupiter, 343125828; of Saturn, 245089877; of Uranus, 19727774. By how much does the content of the Sun exceed that of all those planets?

33. The Money-order Office in connection with the Post-office was instituted in 1838; the following are in round numbers the sums transmitted through it up to 1855—namely, in 1839, £313000; in 1841, £3127000; in 1843, £5113000; in 1845, £6413000; in 1847, £7903000; in 1849, £8153000; in 1851, £8880000; in 1853, £9916000; in 1855, £11009000. Find the increase in each two years. Ans. £2814000, £1986000, £1300000, £1490000, £250000, £727000, £1036000, £1093000.

Answers.

18. 184725936	23. 6356	28. 63673
19. 217593842	24. 3866, 2147	29. 1382
20. 19054823	25. 47, 14, 677, 564	30. 51612000
21. 381275496	26. 230	31. £21366000
22. 683126794	27. 62771408	32. 399231142994

SIMPLE MULTIPLICATION.

MULTIPLICATION is a short method of ascertaining what a number will amount to, when repeated a given number of times; as, for example, what 6 repeated 4 times will amount to. The number to be repeated is said to be multiplied by the number which indicates how *often* it is to be repeated.

The number to be multiplied is called the *Multiplicand;* the number that multiplies it, the *Multiplier;* and the result of multiplying the two numbers together, the *Product.* The multiplicand and multiplier are also called *Factors.*

It is usually most convenient in multiplying two numbers together, to multiply the larger number by the smaller.

Multiplication is denoted by the sign $\times$; thus, $8 \times 3 = 24$, signifies, that when 8 is multiplied by 3, the product is 24.

The process of multiplication is carried on by means of the following Multiplication Table, which shews the numbers obtained by multiplying together any two numbers up to 12.

The table should be carefully committed to memory, as a knowledge of it is of great value in arithmetic, and saves much trouble in after-life.

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times	8 times	9 times	10 times	11 times	12 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 〃 4	2 〃 6	2 〃 8	2 〃 10	2 〃 12	2 〃 14	2 〃 16	2 〃 18	2 〃 20	2 〃 22	2 〃 24
3 〃 6	3 〃 9	3 〃 12	3 〃 15	3 〃 18	3 〃 21	3 〃 24	3 〃 27	3 〃 30	3 〃 33	3 〃 36
4 〃 8	4 〃 12	4 〃 16	4 〃 20	4 〃 24	4 〃 28	4 〃 32	4 〃 36	4 〃 40	4 〃 44	4 〃 48
5 〃 10	5 〃 15	5 〃 20	5 〃 25	5 〃 30	5 〃 35	5 〃 40	5 〃 45	5 〃 50	5 〃 55	5 〃 60
6 〃 12	6 〃 18	6 〃 24	6 〃 30	6 〃 36	6 〃 42	6 〃 48	6 〃 54	6 〃 60	6 〃 66	6 〃 72
7 〃 14	7 〃 21	7 〃 28	7 〃 35	7 〃 42	7 〃 49	7 〃 56	7 〃 63	7 〃 70	7 〃 77	7 〃 84
8 〃 16	8 〃 24	8 〃 32	8 〃 40	8 〃 48	8 〃 56	8 〃 64	8 〃 72	8 〃 80	8 〃 88	8 〃 96
9 〃 18	9 〃 27	9 〃 36	9 〃 45	9 〃 54	9 〃 63	9 〃 72	9 〃 81	9 〃 90	9 〃 99	9 〃 108
10 〃 20	10 〃 30	10 〃 40	10 〃 50	10 〃 60	10 〃 70	10 〃 80	10 〃 90	10 〃 100	10 〃 110	10 〃 120
11 〃 22	11 〃 33	11 〃 44	11 〃 55	11 〃 66	11 〃 77	11 〃 88	11 〃 99	11 〃 110	11 〃 121	11 〃 132
12 〃 24	12 〃 36	12 〃 48	12 〃 60	12 〃 72	12 〃 84	12 〃 96	12 〃 108	12 〃 120	12 〃 132	12 〃 144

The pupil may be drilled on the table in the following way: What is the product of 6 and 7? or, to save time, *Teacher.* 6 times 7? *Pupil.* 42. *Teacher.* 7 times 6? *Pupil.* 42; &c. After the pupil knows the product of any two numbers in the table, and has solved several questions in which the multiplier consists of a single figure, he may be exercised thus:

1	2	3	4	5	6	7	8	9	7	5	2
5	11	8	1	6	9	2	7	12	3	10	4

Write any two series as in the margin, the lower containing all the first twelve numbers; cause the pupil to multiply the lower line by 2, and to add the upper line to the product: thus, 2 times 4 are 8, and 2 make 10; 2 times 10 are 20, and 5 make 25; 2 times 3 are 6, and 7 make 13, &c.; and so on with each line, 3, 4, 5, 6, &c. After he has gone over the table in this way, let him only tell the results; instead of 2 times 4 are 8, and 2 make 10, &c., he should simply say, 8, 10—20, 25—6, 13—24, 33, &c. A reason for adding the upper line is to accustom the pupil to the *carrying*. At this stage several questions of the following kind ought to be proposed: —*Teacher*. If I give 3 nuts to each of 4 boys, how many nuts do I give away? If I pay 7d. for one pound of sugar, how much will I pay for 8 pounds? If 12 boys have 5 marbles each, how many have they altogether? &c.

MULTIPLICATION TABLE EXTENDED.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

When any number in the *side* row at the left, multiplies any number in the *top* row, the product is found in the square *beneath* the number in the top row, and *opposite* the number in the side row. Thus—2 times 2 are 4; 5 times 6 are 30; 20 times 20 are 400.

THIS TABLE is also used for DIVISION. When any number in the *side* row at the left, *divides* a number in any of the squares opposite to it in the same line, the answer is found in the *top* row directly above the number divided. Thus—12 is contained in 168, 14 times.

RULES FOR MULTIPLICATION.

I. When the Multiplier does not exceed 12.

Rule.—1. Write the multiplier below the right-hand figure of the multiplicand, and draw a line under them.

2. Begin at the *right*, and multiply each successive figure of the multiplicand by the multiplier; marking the last figure of each product below the figure multiplied, and carrying the other figure or figures to the product of the next figure, when it is multiplied in its turn.*

When all the figures have been multiplied, the result is the answer or product required.

Example.—Multiply 30874 by 6.

30874	*Multiplicand.*
6	*Multiplier.*
185244	*Product.*

Here, beginning at the right, we say, 6 times 4 are 24; putting down 4, we carry 2 to the next figure: 6 times 7 are 42, and 2 make 44; putting down 4, carry 4: 6 times 8 are 48, and 4 make 52; putting down 2, carry 5: 6 times 0 are 0, and 5 make 5; putting down 5, nothing is carried: 6 times 3 are 18, both of which figures are written down, as there are no more to multiply.

Reason of the Rule.—As multiplication is merely a short method of performing the addition of several equal numbers, it is to addition that we must refer for the principles on which this operation depends. For example, let it be required to multiply 397 by 3; this means that 397 is to be taken 3 times. Let therefore 397 be put down 3 times as in the margin. Now, by the multiplication table, we can sum each column at once; thus, 3 times 7 are 21, we put down 1 and carry 2; 3 times 9 are 27, and 2 are 29, we put down 9 and carry 2; 3 times 3 are 9, and 2 are 11, we put down the whole number. This operation is more shortly written as in the right-hand margin.

397
397
397
1191

397
3
1191

Exercises.

1. Multiply 1852963074 by 2, 3, 4, 5, 6, 7, 8, 9, 12.
2. " 6837014925 " 2, 3, 4, 5, 6, 7, 8, 9, 12.

Answers.

1.	11117778444	2	41022089550
3705926148	12970741518	13674029850	47859104475
5558889222	14823704592	20511044775	54696119400
7411852296	16676667666	27348059700	61533134325
9264815370	22235556888	34185074625	82044179100

* In Multiplication, the last figure is marked down, and the other figures *carried* at each stage of the process, for the same reasons as are explained in Simple Addition, page 10.

3. Multiply 3051847296 by 2, 3, 4, 5, 6, 7, 8, 9, 12.
4. " 8263940517 " 2, 3, 4, 5, 6, 7, 8, 9, 12.
5. " 9360471852 " 2, 3, 4, 5, 6, 7, 8, 9, 12.

II. When the Multiplier exceeds 12.

Rule.—1. Write the multiplier below the multiplicand, placing units under units, tens under tens, and so on, and draw a line under them.

2. Multiply each successive figure of the multiplicand by the *units* of the multiplier, as in Rule I.; next multiply by the *tens;* and so on with the other figures of the multiplier.

Each new line of products is written below the previous one, but a *place* further to the *left;* so that each line may commence exactly below the figure in the multiplier which produces it. When a nothing occurs in the multiplier, pass on to the next figure.

3. Add up all the lines of products, and their sum is the product required.

Examples.—Multiply 5463 by 34; and 76843 by 4063.

(1.)

```
   5463
     34
  21852
 16389
 185742 Ans.
```

In example 1, the number is multiplied first by the 4, the product of which being written down, we next multiply by the 3, and write its product below the other, but one place further to the left. The products are then added together.

(2.)

```
     76843
      4063
    230529
   461058
 307372
 312213109 Ans.
```

Example 3.—Multiply 39207 by 8037.

```
           39207
            8037
          274449 =    7 times the multiplicand.
         1176210 =   30   "   "
       313656000 = 8000   "   "
Product. 315106659 = 8037   "   "
```

The numbers being put down as in the margin, and the operations performed, the pupil may be required to go over the account in the following way: *Pupil.* Here I am required

Answers.

3.	4.	5.
6103694592	16527881034	18720943704
9155541888	24791821551	28081415556
12207389184	33055762068	37441887408
15259236480	41319702585	46802359260
18311083776	49583643102	56162831112
21362931072	57847583619	65523302964
24414778368	66111524136	74883774816
27466625664	74375464653	84244246668
36622167552	99167286204	112325662224

to multiply 39207 by 8037. I first multiply by 7; this gives 274449 = 7 times the multiplicand. Next, to multiply by 30, I put down a cipher, and multiply by 3; this gives 1176210 = 30 times the multiplicand. Again, to multiply by 8000, I put down three ciphers, and multiply by 8; this gives 313656000 = 8000 times the multiplicand. Now, it is plain that, if I add 8000 times the multiplicand, 30 times the multiplicand, and 7 times the multiplicand together, the sum will give 8037 times the multiplicand; that is, 315106659, which is the product required.

THE REASON for writing each successive line of products a place further to the left, as in the preceding examples, is, that the first line being the product of the units of the multiplier, the second, of the tens, the third, of the hundreds, and so on, each new product is a place higher in order than the previous one, and must accordingly be written one place further to the left.

To multiply 436 by 324 according to the rule, is the same as to multiply separately by 4 units, 2 tens, and 3 hundreds; thus—

(1.)
```
   436
   324
  ----
  1744
  872
 1308
------
141264
```

(2.)
$$
\begin{aligned}
436 \times 4 &= 1744 \\
\times 20 &= 8720 \\
\times 300 &= 130800 \\
\hline
&\ 141264
\end{aligned}
$$

Here the result of multiplying is the same in both cases. In No. 2, the figures have their proper position given to them, according to the rank of the multiplier, by nothings being annexed; and in No. 1, by putting each line of figures a place further to the left, which serves the same purpose as the nothings.

METHODS OF PROOF.—I. Make the multiplier the *multiplicand*, and the multiplicand the *multiplier*, and if the product thus obtained is the same as the former product, the account is right.

```
  32      47
  47      32
 ---     ---
 224      94
1280    1410
----    ----
1504    1504
```

For example, when 32 is multiplied by 47, the product is 1504; and when 47 is multiplied by 32, the product is 1504; therefore the account is correct.

II. Add together all the figures, except the 9's, in the *multiplicand*, take the *nines* out of the sum, and set down the remainder opposite the multiplicand.

Example.
```
    73195   7
     7904   2
   ------
   292780
  658755
 512365
---------
578533280   5
```

Do the same thing with the *multiplier* and the *product*. Then multiply the remainder opposite the multiplicand by the remainder opposite the multiplier; take the *nines* out of the result, and if the new remainder is the same as that opposite the product, the account is *probably* correct.

Here the multiplicand amounts to 16, leaving 7 over.
〃 the multiplier 〃 11, 〃 2 〃
〃 the product 〃 41, 〃 5 〃
Then 7×2 makes 14, leaving 5 over, the same as the remainder opposite the product.

When factors are multiplied together, the result is the same in whatever order the multiplications are performed. Take any two factors, as 5 and 3, then the product of 5 by 3 is equal to the product of 3 by 5.

$$\begin{array}{rlc} \text{For} & 5 = 1 + 1 + 1 + 1 + 1 & * \quad * \quad * \quad * \quad * \\ & 5 = 1 + 1 + 1 + 1 + 1 & * \quad * \quad * \quad * \quad * \\ & 5 = 1 + 1 + 1 + 1 + 1 & * \quad * \quad * \quad * \quad * \end{array}$$

Adding the columns vertically, we obtain $5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$; that is, 5 taken 3 times = 3 taken 5 times.

The same thing is obvious from the arrangement of the stars on the right of the page: reading the stars *horizontally*, we have 5 taken 3 times; and reading the same stars *vertically*, we have 3 taken 5 times. Similarly, $3 \times 4 \times 5 = 4 \times 5 \times 3 = 5 \times 3 \times 4$.

When there are more factors than two, the result is called the *continued product* of the factors. Thus 60 is the continued product of 3, 4, and 5.

When all the factors are the same number, the continued product is then called a *power* of that number. Thus, $3 \times 3 \times 3 \times 3 = 81$; 81 is the fourth power of 3. Instead of repeating the equal factors, it is usual to write it only once, with a small figure at its top, called the *index*, to indicate the number of times that it is employed as factor: thus $3 \times 3 \times 3 \times 3$ is indicated by 3^4; the *index* 4 shewing how often the number enters as a factor.

Hence it appears that when a number is to be multiplied, by a whole number which is the product of two other numbers, instead of multiplying by that number, we may first multiply by one of the factors, and then this product by the other factor. Thus, $29 \times 6 \times 6$, $29 \times 9 \times 4$, or, $29 \times 12 \times 3 = 29 \times 36$.

Exercises.

1.	Multiply	18530729	by	21, 34, 42.
2.	"	43915806	"	37, 26, 19.
3.	"	70268315	"	43, 51, 84.
4.	"	92573684	"	78, 89, 35.
5.	"	39753984	"	47, 96, 28.
6.	"	48637251	"	73, 45, 67.
7.	"	83974695	"	89, 91, 23.
8.	"	18370298	"	45, 57, 98.

Answers.

1.	2.	3.	4.
389145309	1624884822	3021537545	7220747352
630044786	1141810956	3583684065	8239057876
778290618	834400314	5902538460	3240078940

5.	6.	7.	8.
1868437248	3550519323	7473747855	826663410
3816382464	2188676295	7641697245	1047106986
1113111552	3258695817	1931417985	1800289204

9. Multiply 95721886 by 89, 79, 89.
10. " 84397857 " 605, 320, 75.
11. " 89260489 " 406, 470, 38.
12. " 17935982 " 35, 207, 98.
13. " 31694708 " 78, 43, 59.
14. " 89357064 " 65, 91, 82.
15. " 39712584 " 123, 456, 789.
16. " 80379218 " 372, 958, 461.
17. " 92873905 " 837, 325, 138.
18. " 61938796 " 504, 982, 837.
19. " 54917283 " 271, 659, 504.
20. " 93650389 " 948, 326, 271.
21. " 41705826 " 516, 486, 305.
22. " 18472593 " 785, 493, 827.
23. " 85149260 " 6521, 61594.
24. " 52816937 " 3298, 73261.
25. " 29583604 " 19695, 4938.
26. " 96250371 " 86372, 8476.
27. " 63927048 " 9328, 60745.
28. " 30694715 " 8975, 32164.
29. " 123456789 " 123456789.

Ans. 15241578750190521.

Answers.

9.	10.	11.	12.
3733134054	51060703485	15939758534	627759370
7561989494	27007314240	18452429830	3712748274
8519203354	6329839275	1491898582	1757726236

13.	14.	15.	16.
2472187224	5808209160	4884647832	29901069096
1362872444	8131492824	18108938804	77003290844
1869987772	7327279248	31333228776	37054819498

17.	18.	19.	20.
77735456485	31217153184	14882583693	88780568772
30184019125	60823897672	36190489497	30530026814
12816598890	51842772252	27678310632	25379255419

21.	22.	23.	24.
21520206216	14500985505	555258324460	174190258226
20269031436	9106988349	5244683520440	3869421621557
12720276930	15276834411		

25.	26.	27.	28.
582649080780	8313337044012	596311503744	275485067125
146083836552	815818144596	3883248530760	987264813260

30. Two factors are 30957 and 839; what is their product? Ans. 25972923.

31. In a desk there were 6 drawers, each drawer was divided into 8 compartments, and in each compartment were 87 pounds; how many pounds did the desk contain? . Ans. 4176 pounds.

32. The equatorial diameter of the earth is 7926 miles, and the diameter of the sun is 112 times as great; find the diameter of the sun, ... Ans. 887712 miles.

WHEN THE MULTIPLIER IS THE PRODUCT OF TWO NUMBERS, neither of which exceeds 12; the given quantity may be multiplied by *one* of the numbers or factors producing the multiplier, and the product by the other; the last product is the answer.

This method is useful chiefly as an exercise for the scholar. In practice, the method stated in the Rule will usually be found more convenient.

Example.—Multiply 4794 by 27.

```
  4794
     9
------
 43146
     3
------
129438 Ans.
```

Here the multiplier 27 is the product of the two numbers 9 and 3; we may therefore first multiply the given quantity by 9, and then the product by 3.

Exercises.—Multiply the following numbers:

				Answers.	
1. 18530729	by	21	42	389145309	778290618
2. 43915806	"	36	25	1580969016	1097895150
3. 70268315	"	32	84	2248586080	5902538460

III. WHEN THE MULTIPLIER IS A NUMBER HAVING *nothings* ANNEXED TO IT.

RULE.—Write down the multiplier in such a way that the nothings shall extend beyond the multiplicand, then put down the nothings as part of the answer, and multiply by the other figure or figures of the multiplier, as in Rules I. and II.

Examples.—Multiply 7312 by 70; 47683 by 70600; 87300 by 760.

```
   1.                2.                3.
  7312             47683             87300
    70             70600               760
------          ----------         --------
511840 Answer.    28609800          5238000
                 333781             611100
                ----------         --------
                3366419800 Answer. 66348000 Answer.
```

Exercises.

					Answers.
1.	Multiply	3179408	by	20	63588160
2.	„	„	„	4000	12717632000
3.	„	„	„	300	953822400
4.	„	„	„	90000	286146720000
5.	„	„	„	1200	3815289600
6.	„	87039	„	6090	530067510
7.	„	930675	„	8700	8096872500
8.	„	7856973	„	590600	4640328253800
9.	„	19076573	„	80090	1527842731570

WHEN THE MULTIPLIER IS 10, 100, 1000, or 1 with any other number of nothings annexed, the multiplication is accomplished merely by annexing as many nothings to the multiplicand as are contained in the multiplier; thus—

386 is multiplied by 10, by annexing 1 nothing = 3860
736 „ „ 1000, „ „ 3 nothings = 736000

IV. WHEN THE MULTIPLIER IS A FRACTION—AS $\frac{1}{2}$, A HALF, OR $\frac{3}{4}$, THREE-FOURTHS. *See Note.**

RULE.—Multiply the given number by the *upper* figure of the fraction, and *divide* the product by the *under* figure.

Example.—Multiply 16 by $\frac{3}{4}$.

$$\begin{array}{r} 16 \\ 3 \\ \hline 4)\overline{48} \\ \hline 12 \end{array}$$ *Answer.*

To multiply a number by a fraction such as $\frac{1}{2}$, a half, $\frac{3}{4}$, three-fourths, &c., is to find what is the *half* or *three-fourths* of the number: thus—to multiply 16 by $\frac{3}{4}$, is the same thing as to find how much is $\frac{3}{4}$ of 16.

In this example, 16 is multiplied by 3, the upper figure of the fraction, and then divided by 4, the under figure.

When the upper figure of a fraction is 1, such as $\frac{1}{3}$, it is unnecessary to multiply by the 1, as it will obviously make no difference on the number; all that is required is to divide the given number by the *under* figure; thus—to multiply 24 by $\frac{1}{3}$, we divide 24 by 3, and the answer is 8—that is, 8 is one-third of 24.

---

* Rules IV. and V., strictly speaking, belong to FRACTIONS; but as they are of frequent use in ordinary calculations, they are placed here for convenience. The teaching of them requires to be deferred till the Rule for DIVISION has been learned, page 28, as a knowledge of that rule is necessary to their working. They may be taught most conveniently along with DIVISION, Rule IV., page 34.

For explanation of FRACTIONS, see page 80.

V. When the Multiplier is an integer or *whole* number, with a Fraction annexed—as $7\frac{5}{6}$.

Rule.—Multiply first by the fraction, as in Rule IV., then by the integer, and add both the products for the answer.

Example.—Multiply 4387 by $7\frac{5}{6}$.

```
   4387
     7⅚
6)21935
   3655⅚
  30709
  34364⅚  Answer.
```

Here $3655\frac{5}{6}$, the product of multiplying by $\frac{5}{6}$, and 30709, the product of multiplying by 7, are added together.

Exercises.

1. Multiply	5376	by	$5\frac{1}{2}$
2. "	7493	"	$6\frac{1}{4}$
3. "	8375	"	$7\frac{2}{3}$
4. "	17654	"	$8\frac{4}{5}$
5. Multiply	89705	by	$9\frac{7}{8}$
6. "	73476	"	$15\frac{3}{7}$
7. "	89596	"	$85\frac{7}{12}$
8. "	73685	"	$376\frac{8}{9}$

Miscellaneous Exercises.

1. The circumference of the earth is about 25000 miles, and light would travel round it eight times in a second of time; what is the velocity of light per second? . . Ans. 200000 miles.

2. The distance of the moon from the earth is about 240000 miles, and the distance of the sun is 400 times as great; what is the distance of the sun from the earth? . Ans. 96000000 miles.

3. The surface of the earth is 197336595 square miles, and the surface of the sun is 12544 times as great; find the surface of the sun in square miles, ... Ans. 2475390247680.

4. The number of schools connected with the national system of education in Ireland in 1853 was 5123, and the average number of children on the rolls of each was 109; how many children were attending these schools? . . . . . . Ans. 558407.

5. There are 31558148 seconds in a sidereal year, and the earth moves in its orbit about 18 miles per second; what is the circumference of the earth's orbit? . . . . Ans. 568046664.

6. A cubic foot of water, when expanded into steam, under a pressure of 15 lbs. per square inch, becomes 1669 cubic feet of steam; a ton of water is $35\frac{21}{25}$ cubic feet; how many cubic feet of steam under a pressure of 15 lbs. per square inch would be produced from a ton of water? . . . . Ans. $59816\frac{24}{25}$.

Answers.

1. 29568
2. $46831\frac{1}{4}$
3. $64208\frac{1}{3}$
4. $155355\frac{1}{5}$
5. $885836\frac{7}{8}$
6. $1133629\frac{5}{7}$
7. $7667924\frac{4}{12}$
8. $27771057\frac{7}{9}$

DIVISION

OF SIMPLE NUMBERS.

Division is that operation by which we find how often one number called the *Divisor* is contained in another called the *Dividend*, or how many times the divisor can be taken away from the dividend.

The result of the process is called the *Quotient*, which literally signifies, how many times? If anything is left after the process is finished, it is called the *Remainder*.

When there is no remainder, then the dividend contains the divisor an exact number of times, and the quotient in this case, when multiplied by the divisor, will give a product which is equal to the dividend. If, therefore, we divide the dividend by the quotient, the result will be the divisor; hence division may be defined as that process by which we divide a given number into any proposed number of equal parts.

Division is denoted by the sign $\div$; thus, $75 \div 25$, signifies that 75 is to be divided by 25.

Division is also indicated by placing the divisor under the dividend, with a line between them; thus, $\frac{9}{3}$, denotes that 9 is to be divided by 3.

It is obvious that the number of times that the dividend contains the divisor, may be found by repeatedly subtracting the divisor from the dividend, until the remainder is less than the divisor. Division therefore bears the same relation to Subtraction that Multiplication does to Addition; and as Subtraction is the converse of Addition, so Division is the converse of Multiplication.

When the divisor does not exceed 12, and the dividend does not exceed 12 times the divisor, the quotient and remainder are easily found by the Multiplication Table. Thus, if it was required to divide 65 by 9, we know at once that the quotient is 7, and the remainder 2. A Division table is therefore unnecessary; but the pupil should be well drilled in the following way: How many twos are there in 4? or, to save time—*Teacher*. Twos in 4? *Pupil*. 2 times. *Teacher*. Twos in 19? *Pupil*. 9 times and 1; and so on with all numbers under 20; threes in all numbers under 30; fours in all under 40, &c.

RULES FOR DIVISION.

I. When the Divisor does not exceed 12.

Rule.—1. Write down the dividend, and draw a curved line on the left side of it, also a straight line below it: then write the divisor on the left of the curved line.

2. Find how often the divisor is contained in the *first* figure of the dividend, or—if the divisor is larger than it—in the first *two* figures, and write the quotient below.

3. If there is no remainder, divide the next figure* of the dividend in the same way. But if there is a remainder, annex to it, mentally, the next figure of the dividend; then find how often the divisor is contained in this sum,* and write down the quotient, as before; and so on, till all the figures of the dividend have been divided, when the division is completed.

* If this figure or sum be less than the divisor, place a *nothing* in the quotient to express this, then annex another figure, and proceed with the division.

4. If there is a remainder after the division is finished, it is marked down as part of the answer, with the divisor written below it, forming a fraction.

Example.—Divide 73016 by 9.

$$9\overline{)73016}$$
$$8112\tfrac{8}{9}$$

The numbers being written as in the margin. *Pupil.* Nines in 73? 8 times and 1; I put down the 8, and place the next figure 0 to the right of the remainder 1, which makes 10; therefore I say nines in 10? 1 and 1; nines in 11? 1 and 2; nines in 26? 2 times and 8. Hence the quotient is 8112, with 8 of a remainder.

The process of Division here described, is termed *Short* Division, when part of the process is carried on in the mind, and the result only written down. Short division is employed when the divisor does not exceed 12.

In numbers above 12, it is necessary, for convenience, to write down at length the various steps of the process; and when this is done, it is termed *Long* Division. (See Rule II., page 29.) The *principle* is the same in both cases, the sole difference being, that in the one, the operation is only partly written down, whilst in the other, all the figures of the process are written.

Example of both methods.—Divide 7958 by 6.

Short Division.

$$6\overline{)7958}$$
$$1326\tfrac{2}{6} \textit{ Quot.}$$

Long Division.

$$\begin{array}{l}
6)7958(1326\tfrac{2}{6} \textit{ Quot.} \\
\underline{6} \\
19 \\
\underline{18} \\
\ \ 15 \\
\ \ \underline{12} \\
\ \ \ \ 38 \\
\ \ \ \ \underline{36} \\
\ \ \ \ \ \ 2 = \tfrac{2}{6}
\end{array}$$

Here we find that there is *one* 6 in 7, the first figure of the dividend, and 1 over; we therefore write 1 in the quotient, and (in *long* division) multiplying the divisor, 6, by 1, the quotient, subtract the product, 6, from 7 of the dividend. To the remainder, 1, we bring down 9, the next figure of the dividend, making 19. As there are 3 times 6 in 19, we place 3 in the quotient, and multiplying the divisor, 6, by 3, subtract 18 from 19, which leaves 1. To this 1 we

bring down 5, making 15; and as there are 2 times 6 in 15, we place 2 in the quotient, and multiplying 6 by 2, subtract 12 from 15, leaving 3. To this 3 we bring down 8, making 38, in which there are 6 sixes; therefore, placing 6 in the quotient, we multiply 6 by 6, and subtract 36 from 38, leaving 2 over. Here the account terminates, it being found that there are 1326 sixes in 7958, with a remainder of 2, below which the divisor is written, thus—$\frac{2}{6}$, and the fraction is annexed to the quotient as part of the answer.

Exercises.

1.	Divide	75176640	by	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
2.	"	225529920	"	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
3.	"	81829440	"	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
4.	"	157006080	"	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

II. When the Divisor exceeds 12.

Rule.—1. Write down the dividend, draw a curved line on each side of it, and place the divisor on the left.

2. Count off as many figures from the left of the dividend as make a number greater than the divisor: find how often the divisor is contained in this number, and place the result at the *right* of the dividend, as the first figure of the quotient.

It is to be observed, that 9 is the highest number to be placed in the quotient at any one time.

3. Multiply the divisor by the quotient, and subtract the product from the number pointed off; then *bring down* and annex to the remainder, if any, the next figure of the dividend.* Divide this sum as before, placing the quotient as the next

* If, after bringing down a figure to any of the remainders, during the process, the sum is *less* than the divisor, place a nothing in the quotient to express this, then bring down another figure to the remainder, and proceed with the division.

Answers.

1.	2.	3.	4.
37588320	112764960	40914720	78503040
25058880	75176640	27276480	52335360
18794160	56382480	20457360	39251520
15035328	45105984	16365888	31401216
12529440	37588320	13638240	26167680
10739520	32218560	11689920	22429440
9397080	28191240	10228680	19625760
8352960	25058880	9092160	17445120
7517664	22552992	8182944	15700608
6834240	20502720	7439040	14273280
6264720	18794160	6819120	13083840

figure of the answer; and so on, till all the figures of the dividend have been brought down and divided, when the operation is completed.

4. **If there is a remainder after the division is finished, it is marked down as part of the answer, with the divisor written below it, forming a fraction.**

It will be found useful to place *dots* below the figures in the dividend, on bringing them down in succession to the remainders, to prevent any mistakes as to what have been brought down.

WHEN THE DIVISOR IS LARGE, it is often difficult for the pupil to tell how often the divisor is contained in any partial dividend.

Let him therefore consider how many times the *first* figure in the divisor is contained in the first, or two first figures of the partial dividend, and let him take this as a *trial* quotient figure; if, on multiplying the divisor by it, the product be greater than the partial dividend, the trial figure must be diminished until he find the product equal to or less than the number under which it stands. If any remainder be greater than, or equal to, the divisor, the quotient figure must be increased.

When the second figure of the divisor is 8 or 9, one is to be added to the first figure of the divisor for a trial divisor; in this case the pupil must watch that the remainder be not greater than the divisor.

Example.—Divide 58379 by 76.

76)58379(768$\frac{11}{76}$	768$\frac{11}{76}$	
532..	76	
517	4608	
456	53760	
619	11	Remainder.
608	58379	Proof.
11		

Let the divisor be placed on the left of the dividend, as in the margin. *Pupil.* Since 583 is the least number of figures which make a number greater than 76, I count it off, and say sevens in 58? 8 times; but on multiplying my divisor by 8, I find the product to be greater than 583, therefore I take it a time less, namely, 7; I place 7 on the right of the dividend as the first figure of the quotient; I then multiply 76 by 7, and the product, 532, I put under 583, and subtract, and find the remainder to be 51. I now bring down the next figure 7 to the right of 51, this makes 517. Again, I say sevens in 51? 7 times, which is too great, therefore I take it 6 times; multiplying and subtracting, I find the remainder to be 61. Next figure 9; sevens in 61? 8 times; multiplying and subtracting, I find the remainder to be 11; and as there are no more figures to bring down, the work is finished.

PROOF.—Multiply the quotient by the divisor, and add to the product any remainder. The sum will be the same as the dividend, if the working has been correct.

In the above example, multiply the quotient 768 by the divisor 76, and to the product add the remainder 11; the sum 58379 being equal to the dividend, shews that the operation is correct.

THE REASON for the rule of Division will appear from the following example:

Divide 94608 by 73.

1.	2.
73)94608(1000	73)94608(1296
73000	73..
21608(200	216
14600	146
7008(90	700
6570	657
438(6	438
438	438
Total, 1296 *times.*	

Here it will be seen, on comparing No. 1 with No. 2, that the process is virtually the same as if we were first to ascertain how many *thousand* times 73 is contained in the dividend; how many *hundred* of times in the remainder; how many times *ten* in the next remainder; and how many times *one* in the last remainder; and adding all these, we have the total number of times. In this example, we have 73 contained in the dividend and the successive remainders, 1000, 200, 90, and 6 times—in all, 1296 times. It will be seen, that the nothings annexed to the quotients, &c., in No. 1, may, in practice, be left out, as the other figures have the same value without them, by being placed as in No. 2, according to their rank.

Exercises.

1.	Divide	79512587	by	13, 23, 31.
2.	"	89659053	"	34, 41, 73.
3.	"	19271873	"	51, 43, 83.
4.	"	85296307	"	61, 71, 85.
5.	"	41824680	"	62, 52, 74.
6.	"	36925814	"	82, 53, 65.
7.	"	70369257	"	91, 17, 19.
8.	"	84169273	"	29, 37, 38.
9.	"	12345678	"	68, 79, 87.
10.	"	90273189	"	97, 432, 513.
11.	"	87625432	"	199, 843, 726.
12.	"	17927618	"	925, 379, 476.

Answers.

1.	2.	3.	4.
$6116352\frac{11}{13}$	$2637030\frac{33}{34}$	$377879\frac{44}{51}$	$1398300\frac{7}{61}$
3457069	$2186806\frac{7}{41}$	$448183\frac{4}{43}$	$1201356\frac{31}{71}$
$2564922\frac{5}{31}$	$1228206\frac{15}{73}$	$232191\frac{20}{83}$	$1003485\frac{82}{85}$
5.	**6.**	**7.**	**8.**
$674591\frac{38}{62}$	$450314\frac{66}{82}$	$773288\frac{49}{91}$	$2902388\frac{21}{29}$
$804320\frac{40}{52}$	$696713\frac{25}{53}$	$4139368\frac{1}{17}$	$2274845\frac{8}{37}$
$565198\frac{28}{74}$	$568089\frac{29}{65}$	$3703645\frac{2}{19}$	$2214980\frac{33}{38}$
9.	**10.**	**11.**	**12.**
$181554\frac{6}{68}$	$930651\frac{42}{97}$	$440328\frac{160}{199}$	$19381\frac{193}{925}$
$156274\frac{32}{79}$	$208965\frac{309}{432}$	$103944\frac{640}{843}$	$47302\frac{160}{379}$
$141904\frac{30}{87}$	$175971\frac{66}{513}$	$120696\frac{136}{726}$	$37663\frac{30}{476}$

Exercises.

13. Divide 419352716 by 123, 456, 789
14. " 900416824 " 54321, 98765.
15. " 519387549 " 17297, 2731.
16. " 183926157 " 37246, 8799.
17. A product is 554270292192, and one of the factors is 7584. What is the other factor? . . . Ans. 73084163.
18. I distributed 1080 marbles among a number of boys, and gave each boy 6 marbles. How many boys were there? Ans. 180.

When the Divisor is the product of two numbers, neither of which exceeds 12, *short* Division may be employed.

Rule.—Resolve the divisor into its two factors; divide the dividend by one of them, and the resulting quotient by the other.

If there be remainders, the true remainder is found by multiplying the last remainder by the first divisor, and then adding the first remainder to the product.

Example.—Divide 37255 by 63.

```
9)37255
7) 4139⁴
Quotient 591 22/63
```

Here the *divisor* 63 is resolvable into the two factors, 9 and 7; I therefore divide first by 9, and then the resulting quotient, 4139, by 7: hence the quotient required is 591. Again, since the second remainder is 2, I therefore multiply it by 9, the first divisor, and to the product 18 I add 4, the first remainder; and the sum 22 is the true remainder.

The first remainder 4 is obviously so many units; the second dividend being so many nines, hence the second remainder 2 is to be regarded as 2 nines: ∴ the true remainder is equal to $9 \times 2 + 4 = 22$.

Exercises.

1. Divide 38297546 by 14, 15, 16.
2. " 12345678 " 18, 20, 21.
3. " 90127539 " 22, 24, 25.
4. " 84736008 " 27, 30, 28.

Answers.

13.	14.	15.	16.
$3409371\frac{83}{123}$	$16575\frac{46249}{54321}$	$30027\frac{10530}{17297}$	$4938\frac{5409}{37246}$
$919633\frac{68}{456}$	$9116\frac{75084}{98765}$	$190182\frac{507}{2731}$	$20903\frac{60}{8799}$
$531499\frac{5}{789}$			

1.	2.	3.	4.
2735539	685871	$4096706\frac{7}{22}$	$3138370\frac{18}{27}$
$2553169\frac{11}{15}$	$617283\frac{18}{20}$	$3755314\frac{3}{24}$	$2824533\frac{18}{30}$
$2393596\frac{10}{16}$	$587889\frac{9}{21}$	$3605101\frac{14}{25}$	3026286

Exercises.

5.	Divide	16254973	by	32, 36, 42.
6.	"	89653741	"	33, 49, 35.
7.	"	29538279	"	40, 56, 64.
8.	"	18374625	"	81, 84, 63.
9.	"	97325008	"	44, 45, 60.
10.	"	23456789	"	54, 72, 70.
11.	"	34567890	"	96, 108, 121.
12.	"	27931529	"	121, 64, 81.
13.	"	84739265	"	49, 144, 81.
14.	"	69718257	"	144, 64, 121.
15.	"	30984752	"	81, 56, 144.
16.	"	91938276	"	36, 25, 144.

III. WHEN THE DIVISOR IS A NUMBER HAVING *nothings* ANNEXED TO IT.

RULE.—1. Point off the nothings from the divisor, and also point off an equal number of figures from the *right* of the dividend; then divide by the remaining figures of the divisor.

If these do not exceed 12, the question is wrought by *short* division. If above 12, *long* division is employed.

2. To any remainders after the division is completed, annex the figures pointed off from the dividend, and mark the whole in the quotient, with the divisor written below it, to form a fraction.

Example.—Divide 384127 by 6000.

$$6,000)\underline{384,127}$$
$$64\tfrac{127}{6000}$$

As there are three nothings in the divisor, we point off 3 figures from the right of the dividend, and divide by the remaining figure, 6.

Answers.

5.	6.	7.	8
$507967\frac{29}{32}$	$2716780\frac{1}{33}$	$738456\frac{39}{40}$	$226847\frac{18}{81}$
$451527\frac{1}{36}$	$1829668\frac{9}{49}$	$527469\frac{15}{56}$	$218745\frac{45}{84}$
$387023\frac{7}{42}$	$2561535\frac{16}{35}$	$461535\frac{39}{64}$	$291660\frac{45}{63}$

9.	10.	11.	12.
2211932	$434384\frac{53}{54}$	$360082\frac{18}{96}$	$230839\frac{10}{121}$
$2162777\frac{43}{45}$	$325788\frac{53}{72}$	$320073\frac{6}{108}$	$436430\frac{9}{64}$
$1622083\frac{28}{60}$	$335096\frac{69}{70}$	$285685\frac{5}{121}$	$344833\frac{56}{81}$

13.	14.	15.	16.
$1729372\frac{37}{49}$	$484154\frac{81}{144}$	$382527\frac{65}{81}$	2553841
$588467\frac{17}{144}$	$1089347\frac{49}{64}$	$553299\frac{8}{56}$	$3677531\frac{1}{25}$
$1046163\frac{62}{81}$	$576183\frac{114}{121}$	$215171\frac{128}{144}$	$638460\frac{36}{144}$

Exercises.

1. Divide 21793628574	by	230,	Answer	$94754906\frac{194}{230}$
2. " "	"	78000,	"	$279405\frac{38574}{78000}$
3. " "	"	3290000,	"	$6624\frac{668574}{3290000}$

When the Divisor is 10, 100, 1000, or 1 with any other number of nothings annexed, the division is accomplished merely by pointing off as many figures from the right of the dividend as there are nothings in the divisor.

The remaining figures are the quotient, and the figures pointed off, with the divisor written below them, form a fraction to be annexed to the quotient; for example—

To divide 1748 by 10, point off 1 figure, and the quotient is $174\frac{8}{10}$

" 2376 " 100, " 2 figures, " " $23\frac{76}{100}$

IV. When the Divisor contains a Fraction—as $7\frac{3}{4}$.*

A number with a fraction annexed, such as $7\frac{3}{4}$, is called a *mixed number*. Before we can divide by such a number, it must be converted into a simple one; that is, into a number having one denomination.

Rule.—Multiply the *whole* number or integer of the divisor, by the *lower* figure of the fraction, and add the *upper* figure to the product: multiply the dividend also by the lower figure of the fraction: then divide the one sum by the other.

Example.—Divide 5736 by $6\frac{3}{7}$.

$6\frac{3}{7}$) 5736
7 7
45)40152(892
360
415
405
102
90
12 = $\frac{12}{45}$

Here the divisor, 6, is multiplied by 7, the lower figure of the fraction, and 3, the upper figure, is added to the product, making 45; that is, $6\frac{3}{7}$ are the same as 45 *sevenths*.

In order, however, to preserve the proportion between the numbers, the sum to be divided must also be multiplied by 7, reducing it in like manner to sevenths. The two numbers being now both reduced to sevenths, the division takes place: the answer is 892, and the remainder is 12, which is expressed as $\frac{12}{45}$.

* This Rule, strictly speaking, belongs to Fractions, but being of frequent use in ordinary calculations, it is placed here for convenience. Rules IV. and V. of Simple Multiplication, pages 25, 26, may be taught along with this Rule.

Exercises.

1. Divide	417638	by	$4\frac{2}{3}$,	Answer	$89493\frac{12}{14}$
2. ″	386543	″	$8\frac{4}{5}$,	″	$43925\frac{15}{44}$
3. ″	7695478	″	$12\frac{5}{9}$,	″	$612914\frac{20}{113}$
4. ″	673685	″	$14\frac{3}{4}$,	″	$45673\frac{33}{59}$
5. ″	7103647	″	$9\frac{5}{8}$,	″	$738041\frac{19}{77}$
6. ″	8301765	″	$57\frac{3}{7}$,	″	$144558\frac{39}{402}$

Miscellaneous Exercises.

1. In 1857 the total quantity of sheep, lamb, and alpaca wool imported into Britain was 129749898 lbs.; how many lbs. were imported on an average daily, allowing 365 days to the year? Ans. $355479\frac{63}{365}$ lbs.

2. The population of London in 1851 was 2362236, on a surface of 122 square miles; what was the average population per square mile? . . . . Ans. $19362\frac{72}{122}$.

3. In 1851 the south of Scotland contained a population of 1813562 on a surface of 9000 square miles; what was the average population per square mile? . . . . . Ans. $201\frac{4562}{9000}$.

4. The surface of the earth is about 197140100 square miles, and the surface of Great Britain and Ireland is 122550 square miles; how often is the surface of the United Kingdom of Great Britain and Ireland contained in the whole earth? . Ans. $1608\frac{79700}{122550}$.

5. The surface of Great Britain is 90038 square miles, and in 1851 it contained a population of 20959477; what was then the average population per square mile? . . . Ans. $232\frac{70661}{90038}$.

6. In 1853 the population of the Russian empire was 65237437, and the number of inhabitants to each square geographical mile was 174; find the number of square geographical miles in the Russian empire, ... Ans. $374927\frac{139}{174}$.

Exercises on
Simple Addition, Subtraction, Multiplication, and Division.

1. Multiply the sum of six thousand seven hundred and two, fifteen thousand nine hundred and twenty-six, and two millions seven hundred and thirty-nine thousand three hundred and eighty-nine, by six thousand seven hundred and forty-five, and divide the product by one thousand seven hundred and twenty-nine, ... Ans. $10774901\frac{836}{1729}$.

2. Add together 1894, 37592, 1006, 283597, 8463925, 173964, 2157184, and 327; from the sum subtract 3578961, multiply the remainder by 78359, and divide the product by 9385, Ans. $62958788\frac{8173}{9385}$.

3. Divide the sum of 237965843, 3921726, 58379, and 83796254, by the difference between 71689 and 140942,

Ans. $4703\frac{45343}{69253}$.

4. Multiply 675392619 by 8735, and divide the product by 54321,

Ans. $108605410\frac{50355}{54321}$.

5. Perform the operations indicated below:

$$\frac{718 + 3102 + 61584 - 17916}{5927}$$

Ans. $8\frac{72}{5927}$.

6. Perform the operations indicated below:

$$\frac{312570 \times 598 + 76125 \times 47 + 318 \div 3 - 79583}{6139 \times 15}$$

Ans. $2067\frac{80845}{92085}$.

7. Perform the operations indicated below:

$$\frac{(7 + 75) \times 43 + (4698 + 315) \div 9}{(73 + 14 - 2) - (16 \div 2 + 4 \times 7)}$$

Ans. $83\frac{16}{49}$.

8. Divide the continued product of 2, 3, 4, 5, 6, 7, 8, and 9 by the continued product 12, 13, and 14,

Ans. $166\frac{336}{2184}$.

9. Divide $781 \times 437 \times 596$ by $71 \times 24 \times 89$,

Ans. $1255\frac{62132}{151656}$.

ALL CALCULATIONS IN ARITHMETIC are made by means of one or other of the four fundamental rules, Addition, Subtraction, Multiplication, and Division, or by combining them in various ways, according to the Rules given in the remaining portion of this work.

COMPOUND NUMBERS.

Compound Numbers are those which consist of two or more kinds or denominations.

When we speak of a *pound*, a simple number is expressed, it is one kind of thing; but when we speak of a sum consisting of *pounds*, *shillings*, and *pence*, then we refer to various denominations or kinds; in other words, a sum *compounded*, of various kinds of money, and hence termed a compound number.

All questions which refer to sums of money, and to calculations of weights, measures, &c., which consist of various kinds or denominations, are placed under the head of *Compound Numbers*.

Calculations in *simple* numbers are the same in every country, and must continue without change throughout all time; for example, that 2 and 2 are equal to 4, is a universal truth, which all mankind cannot alter. The rules of Simple Addition, Subtraction, Multiplication, and Division, are therefore in use in every country in the world.

With calculations in *compound* numbers, the case is entirely different, as almost every country has its own standards of money, weights, and measures, and the arithmetical rules for working differ accordingly. These calculations, however, could be rendered as easy as those in simple numbers, if the standards of money, weights, and measures, were constructed on the same decimal principle of advancing by tens.

In Great Britain and Ireland, the standards of money, weights, and measures are mixed and various, as may be learned from the following tables.

It is necessary that the pupil commit these tables to memory, before proceeding with the calculations in compound numbers.

ARITHMETICAL TABLES.

STERLING MONEY.

			Marked.
		1 farthing.	$\frac{1}{4}$
2 Farthings	=	1 half-penny.	$\frac{1}{2}$
4 Farthings	=	1 penny.	*d.*
12 Pence	=	1 shilling.	*s.*
20 Shillings, or 240 pence, or 960 farthings	=	1 pound.	£

				d.		*f.*
		s.		1	=	4
£		1	=	12	=	48
1	=	20	=	240	=	960

STERLING MONEY—*continued.*

Silver Coins.		s.	d.	Gold Coins.		s.	d.
Threepenny-piece .	=	0	3	Half-sovereign .	=	10	0
Groat . . .	=	0	4	Sovereign or pound	=	20	0
Sixpence . .	=	0	6	Guinea, a coin now disused, but still often spoken of, consisted of 21*s*.			
Shilling . .	=	1	0				
Florin . . .	=	2	0				
Half-crown .	=	2	6				
Crown . . .	=	5	0				

In the gold coinage, 22 parts (commonly called *carats*) = 11 oz. of fine gold, and 2 parts (*carats*) = 1 oz. of copper are melted together, and 40 troy pounds of this mixture are coined into 1869 sovereigns. The weight of a sovereign is 123¼ grains.

In the silver coinage, 11 oz. 2 dwt. of fine silver are melted with an alloy of 18 dwt. of copper, and 66 shillings are coined from a troy pound of this mixture. The weight of a shilling is 87¼ grains.

In the copper coinage, 24 pence are made from an avoirdupois pound of copper.

All accounts are kept and reckoned by *pounds*, *shillings*, *pence*, and *farthings*. L. or £ is the initial letter of the Latin word *libra*, a pound, and is used to denote pounds; *s*., from the Latin word *solidus*, stands for shillings; and *d*., from *denarius*, for pence: £ *s*. *d*. are therefore respectively placed over columns of pounds, shillings, and pence. The mark for a half-penny is ½; for a farthing, or fourth of a penny, ¼; and for three farthings, ¾.

The old Scottish pound was equal to 1*s*. 8*d*. sterling: hence £100 Scots was = £8, 6*s*. 8*d*. of our present money.

NOTE.—For an account of a new system of reckoning money decimally, that has been proposed as a substitute for the present mode of reckoning, see Appendix.

TROY WEIGHT.

			Marked.
		1 grain.	*gr.*
24 Grains	=	1 pennyweight.	*dwt.*
20 Dwts., or 480 grains .	=	1 ounce.	*oz.*
12 Ounces, or 5760 " . .	=	1 pound.	*lb.*

7000 Troy grains = 1 lb. Avoirdupois.
100 Troy ounces = 109⅘ Ounces Avoirdupois.

lb.		oz.		dwt.		gr.
				1	=	24
		1	=	20	=	480
1	=	12	=	240	=	5760

Gold, silver, and precious stones are weighed by troy weight. In determining the purity of gold, the gold is supposed to be divided into 24 carats, and if pure, is said to be 24 carats fine; if there be 23 carats of pure gold, and 1 of alloy, it is said to be 23 carats fine; and so on.

APOTHECARIES' WEIGHT.

			Marked.
		1 grain.	*gr.*
20 Grains	=	1 scruple.	*sc.* or ℈
3 Scruples	=	1 dram.	*dr.* " ʒ
8 Drams	=	1 ounce.	*oz.* " ℥
12 Ounces	=	1 pound.	*lb.*

The ounce and pound are the same as in the troy weight. This weight is used only in preparing medicines.

AVOIRDUPOIS WEIGHT.

			Marked.		grs. troy.
		1 dram.	*dr.*		
16 Drams	=	1 ounce.	*oz.*	=	$437\frac{1}{2}$
16 Ounces	=	1 pound.	*lb.*	=	7000
28 Pounds	=	1 quarter.	*qr.*		
4 Quarters, or 112 lbs.	=	1 hundredweight.	*cwt.*		
20 Hundredwts., or 2240 lbs.	=	1 ton.	*T.*		

A Stone is 14 pounds: 8 Stones, 1 cwt. A Cental, 100 pounds.

100 Ounces Avoirdupois = $91\frac{7}{48}$ Ounces Troy.

ton.		cwt.		qr.		lb.		oz.		dr.
								1	=	16
						1	=	16	=	256
				1	=	28	=	448	=	7168
		1	=	4	=	112	=	1792	=	28672
1	=	20	=	80	=	2240	=	35840	=	573440

All ordinary articles are weighed by this weight, which is known as the imperial standard, and is in universal use in this country.

A standard imperial pound avoirdupois is equal to the weight of 27·727 cubic inches of distilled water (or the tenth of a gallon) at the temperature of 62° Fahrenheit, and when the barometer stands at 30 inches.

LINEAL MEASURE, OR MEASURE OF LENGTH.

			Marked.
		1 inch.	*in.*
12 Inches	=	1 foot.	*ft.*
3 Feet, or 36 inches	=	1 yard.	*yd.*
$5\frac{1}{2}$ Yards	=	1 perch or pole.	*per.*
40 Poles, or 220 yards	=	1 furlong.	*fur.*
8 Furlongs, or 1760 yards, or 5280 feet	=	1 mile.	
3 Miles	=	1 league.	

A Fathom is 6 feet: a Hand, 4 inches: a Military pace, $2\frac{1}{2}$ feet.

mi.		fur.		ch.		po.		yd.		ft.		in.
										1	=	12
								1	=	3	=	36
						1	=	$5\frac{1}{2}$	=	$16\frac{1}{2}$	=	198
				1	=	4	=	22	=	66	=	792
		1	=	10	=	40	=	220	=	660	=	7920
1	=	8	=	80	=	320	=	1760	=	5280	=	63360

For smaller lengths than an inch, eighths and sixteenths of an inch, also tenths, are used.

This is the ordinary measure for dimensions of all kinds. Distances are measured by miles: depth, by fathoms: the height of horses, by hands. The chain used for measuring land = 66 feet, and consists of 100 links, each of them 7·92 inches.

The yard is the imperial standard measure of length. A pendulum vibrating seconds of mean time in the latitude of London, in a vacuum, at the sea-level, and at 62° Fahrenheit, is equal to 39·1393 inches; and a yard is to the length of the seconds pendulum, in the proportion of 36 to 39·1393 inches.

CLOTH MEASURE.

			Marked.
$2\frac{1}{4}$ Inches	=	1 nail.	*nl.*
4 Nails, or 9 inches	=	1 quarter.	*qr.*
4 Quarters, or 36 "	=	1 yard.	*yd.*
5 Quarters, or 45 "	=	1 English ell.	

The yard in this measure is of the same length as that in Lineal Measure.

YARN MEASURE.

COTTON.			LINEN.		
54 Inches	=	1 thread.	90 Inches	=	1 thread.
80 Threads	=	1 skein.	120 Threads	=	1 cut.
7 Skeins	=	1 hank.	2 Cuts	=	1 hear.
18 Hanks	=	1 spindle.	6 Hears	=	1 hasp.
			4 Hasps	=	1 spindle.

SQUARE OR SUPERFICIAL MEASURE.

144 Square inches	=	1 square foot.
9 " feet	=	1 " yard.
$30\frac{1}{4}$ " yards	=	1 " perch.
40 " perches	=	1 rood.
4 Roods	=	1 acre.
640 Acres	=	1 square mile.

ac.		ro.		sq. po.		sq. yd.		sq. ft.		sq. in.
								1	=	144
						1	=	9	=	1296
				1	=	$30\frac{1}{4}$	=	$272\frac{1}{4}$	=	39204
		1	=	40	=	1210	=	10890	=	1568160
1	=	4	=	160	=	4840	=	43560	=	6272640

36 square yards are 1 rood of building: 100 square feet, 1 square of flooring: $272\frac{1}{4}$ square feet, 1 rood of bricklayer's work. The chain with which land is measured is 22 yards long, and 1 square chain = 10,000 square links, contains 22 × 22 = 484 square yards: 10 square chains = 1 acre.

Superficial measure refers to breadth as well as length. For instance, a *superficial foot*, usually called a square foot, is either a square of a foot in length and a foot in breadth—in other words, a foot each way —or it is any dimension in which the length multiplied by the breadth will form a foot, or 12 inches, each way: thus, 12 times 12 inches =

144 inches, is a square foot; or 6 times 24 inches = 144, is also a square foot. Sometimes the term *square feet* is confounded with that of *feet square*, which is quite a different thing. A piece of cloth said to measure six square feet, consists of six squares of a foot each; but a piece said to measure *six feet square* would be six feet along each side, and comprise thirty-six squares of a foot each.

It will now be understood, that the *square* of any number is that number multiplied by itself; thus—the square of 3 is 3 times 3, or nine; the square of 4 is 16; and so on. On this principle is formed the table of square measure, for the measurement of breadth and length; it is commonly used for measuring land, walls, &c.

The lower denominations, inches, feet, and yards, are used in the measurement of surfaces of small extent; poles, roods, and acres, in the measurement of land. In measuring land, the chain is generally employed; hence the square chain and square link may be very properly introduced as subdivisions of an acre. By an easy calculation, it will appear that 100,000 square links are equal to an acre.

CUBIC OR SOLID MEASURE.

1728 Cubic inches	=	1 cubic foot.
27 " feet, or 46,656 inches . .	=	1 " yard.
40 " "	=	1 ton shipping.
5 " "	=	1 barrel bulk.

40 Cubic feet of rough timber, or 50 of hewn timber, is a load.

Solid measure is computed by multiplying the length by the breadth, and the product by the thickness. This measure is used in calculating the solid contents of masses of earth, &c.; in measuring the holds of vessels, to ascertain the tonnage; and in all cases where length, breadth, and thickness are reckoned; hence, a solid inch, solid foot, and solid yard, are sometimes used for cubic inch, cubic foot, and cubic yard respectively.

LIQUID MEASURE OF CAPACITY.

			Marked.
4 Gills	=	1 pint.	*pt.*
2 Pints	=	1 quart.	*qt.*
4 Quarts, or 8 pints, or 32 gills .	=	1 gallon.	*gal.*

A hogshead (hhd.) contains 63 gallons. A pipe is 2 hogsheads, and 2 pipes form a tun. But in trade, these measurements are not rigidly adhered to, as casks differ in capacity. 1 gill = 5 oz. avoirdupois of water, or about 8⅔ cubic inches. All liquids are measured by this table.

The imperial gallon is the standard measure of capacity, both for liquids and dry goods; it contains 277·274 cubic inches, being equal to a volume of distilled water weighing 10 lb. avoirdupois, at 62° Fahrenheit, and the barometer being at 30 inches of mercury.

APOTHECARIES use the following liquid measure—

			Marked.
1 Fluid minim	=	0·0045 cubic inches.	♏
60 Fluid minims . . .	=	1 dram.	ʒ
8 Drams	=	1 ounce.	℥
16 Ounces	=	1 pint.	O

The pint is the same as the ordinary pint measure.

GRAIN MEASURE, OR DRY MEASURE OF CAPACITY.

			Marked.
2 Gallons	=	1 peck.	*pk.*
4 Pecks, or 8 gallons	=	1 bushel.	*bu.*
8 Bushels	=	1 quarter.	*qr.*

5 Bushels are a Sack: 5 Quarters, a Load.

The gallon is the same as in Liquid Measure. By this table, grain, seeds, flour, &c., are measured.

The *imperial bushel* contains 2218·192 cubic inches, and weighs 80 lb. avoirdupois of distilled water.

TABLE OF TIME.

60 Seconds	=	1 minute.
60 Minutes	=	1 hour.
24 Hours	=	1 day.
7 Days	=	1 week.
52 Weeks 1 day, or 365 days	=	1 ordinary year.
365 Days, 5 hours, 48 minutes, 49 seconds	=	1 solar year.
365 Days, 6 hours	=	1 Julian year.
366 Days	=	1 leap-year.

wk.		day.		hr.		min.		sec.
						1	=	60
				1	=	60	=	3600
		1	=	24	=	1440	=	86400
1	=	7	=	168	=	10080	=	604800

The year is also divided into 12 Calendar Months; namely—

January,	31 days.	May,	31 days.	September,	30 days.
February,	28 "	June,	30 "	October,	31 "
March,	31 "	July,	31 "	November,	30 "
April,	30 "	August,	31 "	December,	31 "

The number of days in each month may be easily remembered from the following well-known lines—

Thirty days have September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
Which hath but twenty-eight days clear,
And twenty-nine in each leap-year.

As the true solar year is nearly 6 hours more than 365 days, every fourth year, termed leap-year, is reckoned as consisting of 366 days, in order to make allowance for the excess; the additional day being given to February.

To ascertain if a given year is leap-year, divide it by 4, and if there is no remainder, it is leap-year;* if there be a remainder, the number over indicates how many years it is *after* leap-year. Thus, 1852 is leap-year, because divisible by 4 without a remainder; and 1854 is two years after leap-year, because there is a remainder of 2, after dividing it.

* The years 1700, 1800, and 1900 are exceptions, but 2000 will be a leap-year.

GEOGRAPHICAL OR NAUTICAL MEASURE.

1	Geographical mile		=	$1\frac{3}{20}$ imperial mile, or 6076 feet.
3	〃	miles	=	1 league.
60	〃	miles	=	1 degree. marked *deg.* or [°].
360	〃	degs. or about 24,855½ imp. miles	=	Circumference of the earth.

The degree is divided into 60 minutes (marked ′), and the minute into 60 seconds (marked ″). This measure is used for geographical purposes, and in reckoning distances at sea. At the equator, a degree of longitude is 69½ imperial miles.

MISCELLANEOUS.

Aum of hock wine	=	30 gals.	Load of bricks	=	500 bricks.
Bag of coffee	=	1¼ to 1½ cwt.	〃 tiles	=	1000 tiles.
〃 hops	=	about 2½ 〃	〃 straw	=	11 cwt. 64 lbs.
〃 rice (E. I.)	=	about 1½ 〃	〃 old hay	=	18 〃
Bale of coffee (Mocha)	=	2 to 2½ 〃	〃 new hay	=	19 〃 32 lbs.
〃 cotton	=	200 to 500 lbs.	Loaf (quartern)	=	4 lbs.
〃 flax (Russia)	=	5 to 6 cwt.	Mat of flax (Dutch)	=	126 〃
Barrel of beef	=	200 lbs.	Pipe of Port wine	=	115 gals.
〃 beer	=	36 gals.	〃 Madeira	=	92 〃
〃 butter	=	224 lbs.	Pocket of hops	=	1½ to 2 cwt.
〃 flour	=	196 〃	Puncheon of brandy	=	100 to 110 gals.
〃 gunpowder	=	100 〃	〃 rum	=	90 to 100 〃
〃 herring	=	500 her.	〃 whisky	=	about 120 〃
〃 soap (soft)	=	256 lbs.	Quintal of fish	=	112 lbs.
〃 tar	=	26¼ gals.	Quire of paper *	=	24 sheets.
Box of raisins	=	56 lbs.	Ream of paper	=	20 quires, or 480 sheets.
〃 salmon	=	120 to 130 〃			
Butt of sherry	=	108 gals.	Robin of coffee	=	1 to 1½ cwt.
Case of mace	=	about 1½ cwt.	Roll of parchment	=	60 skins.
Cask of clover	=	7 to 9 〃	Sack of clover	=	2 to 3½ cwt.
〃 raisins	=	1 to 2½ 〃	〃 flour	=	280 lbs.
〃 rice (Amer.)	=	6 〃	Score	=	20 articles.
Chaldron of coal	=	25½ 〃	Seron of almonds	=	1½ to 2 cwt.
Chest of soap	=	3½ 〃	Sheet of paper folded—		
〃 tea, congou	=	about 84 lbs.	into 2 leaves,	is termed	folio size.
〃 〃 hyson	=	60 to 80 〃	〃 4 〃		4to, or quarto.
Dozen	=	12 articles.	〃 8 〃		8vo, or octavo.
Drum of figs	=	24 lbs.	〃 12 〃		12mo, or duodecimo.
Firkin of beef	=	100 〃	〃 16 〃		16mo.
〃 butter	=	56 〃	〃 18 〃		18mo.
Fodder of lead	=	19½ cwt.	〃 24 〃		24mo.
Frail of figs	=	32 to 56 lbs.	〃 48 〃		48mo.
Gross	=	12 doz.	Tierce of coffee	=	5 to 7 cwt.
Hogshead of beer	=	54 gals.	〃 sugar	=	7 to 9 〃
〃 brandy	=	60 〃	Truss of straw	=	36 lbs.
〃 rum	=	45 to 50 〃	〃 old hay	=	56 〃
〃 sugar	=	13 to 16 cwt.	〃 new hay	=	60 〃
Keel of coal	=	21 tons.	Tub of butter	=	84 lbs.

* In Scotland, a quire consists of 24 sheets of folio paper, but of 48 sheets of quarto and octavo size.

QUARTERLY TERMS FOR LEASES, ETC.

ENGLAND AND IRELAND.			SCOTLAND.		
Lady-day	March	25	Candlemas	February	2
Midsummer	June	24	Whitsunday	May	15
Michaelmas	September	29	Lammas	August	1
Christmas	December	25	Martinmas	November	11

When a Scottish Term falls on Sunday, the Monday following is considered Term-day.

REDUCTION

OF COMPOUND NUMBERS.

Reduction is that process by which numbers that are expressed in one or more denominations, are reduced or converted into numbers of other denominations having the same value.

The calculations are made by means of the rules of Multiplication and Division, in connection with the preceding Arithmetical Tables.

I. To convert a given quantity from a higher to a *lower* Denomination. [Multiply the quantity.]

Rule.—1. Consider how many of the next *lower* denomination make *one* of the highest,* and by that number *multiply* the highest denomination, adding to the product any of the lower denomination in the given sum.

* For instance, if the highest denomination were pounds, find how many shillings make *one* pound.

2. If the reduction is to be carried further, convert this product into the next lower denomination, in a similar way; and so on with each denomination, till the whole has been reduced to the lowest given term.

Example 1.—Reduce £31, 17*s.* 2¾*d.* to farthings.

	£	*s.*	*d.*
	31	17	2¾
Multiply by	20		
	620		
Add	17		
	637	Shillings.	
Multiply by	12		
	7644		
Add	2		
	7646	Pence.	
Multiply by	4		
	30584		
Add	3		
	30587	Farthings.	

In this example the highest denomination is pounds, therefore, to change these to shillings, we multiply by 20, and add 17 to the product; the given sum now consists of 637*s.* 2¾*d.* We then change the shillings to pence by multiplying by 12, and adding the 2*d.*; the sum is now changed to 7646*d.* and ¾*d.*: changing the pence to farthings by multiplying by 4, and adding 3 farthings, we have the given sum, £31, 17*s.* 2¾*d.*, changed to 30587 farthings, the name required. The example may be abridged, and the pupil required to go over it in the following manner:

£ s. d.
31 17 $2\frac{3}{4}$
20
637 Shillings.
12
7646 Pence.
4
30587 Farthings.

Pupil. Here I am required to reduce £31, 17*s.* $2\frac{3}{4}$*d.* to farthings. First, to change pounds to shillings. Since 20*s.* make £1, I must have 20 times as many shillings as pounds; therefore, I multiply by 20, and *take in* the 17*s.*, because it is only numbers of the same name that can be added together: this gives 637*s.* Next, to change shillings to pence. Since 12 pence make 1 shilling, I must have 12 times as many pence as shillings; therefore, I multiply by 12, and take in 2*d.*, because it is only numbers of the same name that can be added together: this gives 7646*d.* Lastly, to change pence to farthings. Since 4 farthings make 1 penny, I must have 4 times as many farthings as pence; hence, I multiply by 4, and take in the 3 farthings, because, &c.: this gives 30587 farthings, the denomination required.

II. To convert a given quantity from a lower to a *higher* Denomination. [Divide the quantity.]

Rule.—1. Consider how many of the given denomination make *one* of the next higher,* and by that number *divide* the given sum, which is thus converted to the denomination immediately above it.

* For instance, if the given denomination were farthings, find how many farthings make *one* penny.

2. If the conversion is to be carried further, divide the new denomination in a similar way; and so on, dividing each denomination in succession, till the sum has been converted to its highest required denomination.

The remainders at each stage of the division are of the same denomination as the number from which they arise, and must be marked as such in the answer.

All questions of Reduction of money, weights, measures, &c., whether from a higher to a lower, or from a lower to a higher denomination, are wrought by these two rules of Reduction.

Example 2.—Convert 17127 farthings to pounds.

Farthings.
4)17127
12)4281$\frac{3}{4}$
2,0)35,6 " 9
£17, 16*s.* $9\frac{3}{4}$*d.*

In this example—*Pupil.* I divide by 4, because 4 farthings make a penny, the quotient, 4281, is pence, and 3 farthings over; I divide 4281*d.* by 12, to change them to shillings, the quotient is 356*s.*, and 9*d.* over. Lastly, I divide 356*s.* by 20, to change them to pounds, the quotient is £17, and 16*s.* over; therefore, 17127 farthings is equal in value to £17, 16*s.* $9\frac{3}{4}$*d.*

Exercises.

1. Reduce the following compound numbers to farthings:

   £17, 13s. 4½d.; £63, 13s. 11¾d.; £302, 5s. 9¼d.; 16s. 8¾d.; £2763, 14s. 8½d.; £4587, 19s. 11¾d.; £209, 14s. 9d.; £5129, 4s. 5¼d.; 19s. 11¾d.; £670, 11s. 7¾d.; £27, 13s. 4½d.

2. Reduce the following compound numbers to halfpence:

   £34, 14s. 7d.; £207, 4s. 3½d.; £7624, 13s. 8½d.; £391, 12s. 8d.; 17s. 5½d.; £706, 15s. 11d.; £310, 10s. 0½d.; £206, 15s.; £3726, 15s. 4½d.

3. Convert the following simple numbers to pounds:

   7314 farthings; 59633 farthings; 71058 farthings; 3827 halfpence; 76419 halfpence; 710 halfpence; 38295 pence; 9163 pence; 12345 pence; 7987 farthings; 39276 halfpence; 49168 pence.

4. Convert £3417 to guineas, ... Ans. 3254 g. 6s.
5. " 3916 guineas to crowns, . . Ans. 16447 cr. 1s.
6. " £732, 15s. 8d. to farthings, Ans. 703472 farthings.
7. " 17923 halfpence to sovereigns, Ans. £37, 6s. 9½d.
8. " 9468 guineas to pounds, . . Ans. £9941, 8s.
9. " 3159687 pence to guineas, . Ans. 12538 g. 9s. 3d.
10. " 2345678 farthings to sovereigns, Ans. £2443, 8s. 3½d.

Example 3.—Reduce 3 lb. 4 oz. 17 dwt. 21 gr. to grains.

	12	20	24
lb.	oz.	dwt.	gr.
3	4	17	21
12			
40 oz.			
20			
817 dwt.			
24			
3269			
16360			
19629 grains.			

This is an example in Troy weight, and the proper multipliers are obtained from the Table, page 38. The pupil is recommended to write the multipliers over their respective names, as in the example. We multiply and take in, as in the former examples. In multiplying by 24, when we multiply by the *units*, we take in the unit 1 grain; and when we multiply by the *tens*, we take in the 2 tens.

Answers.

1.	959	339262	£7 19*s.* 5½*d.*
16962	643759	149041	159 4 1½
61151	26562	99240	1 9 7
290197	2.	1788849	159 11 3
803	16670	3.	38 3 7
2653186	99463	£ *s.* *d.*	51 8 9
4404479	3659849	7 12 4½	8 6 4¾
201348	187984	62 2 4¼	81 16 6
4924053	419	74 0 4½	204 17 4

Example 4.—Convert 39257 Troy grains to pounds.

```
          gr.
      ( 6)39257
24 = {  ------
      ( 4)6542  5
        ------
   2,0)163,5  17
      --------
     12)81  15
      --------
   lb.  6  9   15   17
```

In this example the grains are divided by 24, which is = 6 × 4, and the quotient is 1635 dwt., and 17 grains over; these dwt. are divided by 20, and the quotient is 81 oz. and 15 dwt.; these oz. are divided by 12, and the final result is 6 lb. 9 oz. 15 dwt. 17 gr., the answer required.

Exercises.

11. Convert 23 lb. 7 oz. 3 dwt. 12 gr. to grains Troy, Ans. 135924 gr.
12. " 9 oz. 17 dwt. to grains, . . Ans. 4728 gr.
13. " 2885 grains to ounces, Ans. 4 oz. 19 dwt. 9 gr.
14. " 172859 grains to pounds, Ans. 30 lb. 2 dwt. 11 gr.
15. " 17 tons 13 cwt. of sugar to pounds, Ans. 39536 lb.
16. " 39 cwt. 2 qr. 15 lb. to pounds, . Ans. 4439 lb.
17. " 26 cwt. 3 qr. 27 lb. to pounds, . Ans. 3023 lb.
18. " 76123 oz. to cwt. Ans. 42 cwt. 1 qr. 25 lb. 11 oz.
19. " 54726 lb. to tons, Ans. 24 tons 8 cwt. 2 qr. 14 lb.
20. " 17 lb. 12 oz. 13 dr. to drams, . Ans. 4557 dr.
21. " 35 cwt. 2 qr. 11 lb. to pounds, . Ans. 3987 lb.
22. " 3 tons to drams, . . . Ans. 1720320 dr.
23. " 69125 lb. to tons, Ans. 30 tons 17 cwt. 21 lb.
24. " 391726 lb. to cwt. . Ans. 3497 cwt. 2 qr. 6 lb,
25. " 123456 oz. to tons, Ans. 3 tons 8 cwt. 3 qr. 16 lb.
26. " 9 cwt. 1 qr. 23 lb. to pounds, . Ans. 1059 lb.
27. " 2345678 drams to tons, Ans. 4 tons 1 cwt. 3 qr. 6 lb. 12 oz. 14 dr.
28. " 13 yd. 2 qr. to nails, . . . Ans. 216 nails.
29. " 23 miles 2 fur. 219 yd. to feet, Ans. 123417 feet.
30. " 78235 square poles to acres, Ans. 488 ac. 3 ro. 35 po.
31. " 234 ac. 3 ro. 27 po. to poles, . Ans. 37587 po.
32. " 2174 nails to yards, . Ans. 135 yd. 3 qr. 2 nl.
33. " 2174 ounces of gold to pounds, Ans. 181 lb. 2 oz.
34. " 56 gallons 3 pints to gills, . . Ans. 1804 gills.
35. " 208 qr. 3 bush. of wheat to pecks, Ans. 6668 pk.
36. " 73926 inches to miles, Ans. 1 m. 1 fur. 73 yd. 1 ft. 6 in.
37. " 7125 yards to English ells, . Ans. 5700 Eng. ells.
38. " 39127 Flemish ells to yards, Ans. 29345 yd. 1 qr.
39. " 17 ac. 2 ro. to poles, . . . Ans. 2800 po.
40. " 39127 poles to acres, . Ans. 244 ac. 2 ro. 7 po.
41. How many seconds in 365 days 5 hours 48 min. 51 sec.? Ans. 31556931 sec.
42. How many solid yards in 391256 solid inches? Ans. 8 yd. 10 ft. 728 in.

43. How many days in 391256 seconds?
Ans. 4 days 12 hr. 40 min. 56 sec.
44. How many yards in 3912 English ells? . Ans. 4890 yd.
45. How many sixpences in £1234, 17*s.* 6*d.*? Ans. 49395 sixp.
46. How many pounds in 78543 fourpences? Ans. £1309, 1*s.*
47. How many pounds in 35726 dwt. of silver?
Ans. 148 lb. 10 oz. 6 dwt.
48. How many pounds in 26 tons 3 cwt. 1 qr. of soap?
Ans. 58604 lb.
49. How many yards in 78325 nails of cloth?
Ans. 4895 yd. 1 qr. 1 nl.
50. How many poles in 76 ac. 1 ro. 39 po. of land?
Ans. 12239 po.
51. How many years, of 365 days each, would it take a body, moving at the rate of 60 miles an hour, to pass from the earth to the sun, the mean distance being 95 millions of miles?
Ans. 180 yr. 272 d. 5 hr. 20 min.
52. The total revenue of the French government in 1858 was 1773919114 francs, estimating the franc at 9¾*d.*; find the revenue in pounds sterling, ... Ans. £72065464, 0*s.* 1½*d.*
53. The velocity of the earth in its orbit is 101173 feet per second; find its velocity in miles per second,
Ans. 19 miles 1 fur. 64 yd. 1 ft.
54. A cubic foot of gold weighs 19259 ounces avoirdupois; find its weight in hundredweights, &c.,
Ans. 10 cwt. 2 qr. 27 lb. 11 oz.
55. The length of a sidereal year is 365 days 6 hours 9 minutes and 12 seconds; find its length in seconds,
Ans. 31558152 seconds.
56. The total quantity of coffee imported into the United Kingdom in the year 1857 was 58912619 lb.; find its weight in tons, ... Ans. 26300 tons 5 cwt. 2 qr. 3 lb.
57. In the year 1857 there was paid to the king of Denmark £1125206 for abolition of the Sound dues. The value of gold imported from New South Wales the same year was £3, 15*s.* 10*d.* per ounce troy; how many pounds troy of gold would be required to pay the king of Denmark? . . Ans. 24729 lb. 9$\frac{6}{91}$ oz.
58. A pound avoirdupois is 7000 troy grains; find the weight in tons of the gold in the answer to the last question,
Ans. 9 tons 1 cwt. 2 qr. 21 lb. 1⅓ oz. nearly.

Application of Reduction to the calculation of prices, when the value of one article is given.

Rule.—Reduce the price to the lowest denomination it contains, then multiply the price by the number of articles, or the number of articles by the price; the product will be the answer in the same denomination to which the price of one was reduced, which may be changed to a higher denomination, by Rule II.

Example 1.—What is the value of 345 feet of mahogany, at 1*s*. 7½*d*. per foot?

```
Halfpence.        s.  d.
   345            1   7½
    39           12
  ----           --
  3105           19
 10350            2
 -----           --
2)13455          39
 -------
12)6727½
 -------
 2,0)56.0 " 7
    --------
   £28, 0s. 7½d.
```

Pupil. Here the lowest denomination in the price is halfpence, therefore I change 1*s*. 7½*d*. to halfpence; this gives 39 halfpence. Now, it is obvious that 345 feet, at a halfpenny per foot, will be 345 halfpence, and that the price at 39 halfpence will be 39 times as great; therefore, I multiply by 39, and the answer is 13455 halfpence; that is, £28, 0*s*. 7½*d*.

Example 2.—What is the value of 28 yards of broad cloth, at £1, 3*s*. 4*d*. per yard?

```
      £  s.  d.
      1  3   4
     20
     --
     23
     12
    ---
    280
     28
   ----
   2240
   5600
   ----
 12)7840
   -----
 2,0)65 3 " 4
   ----------
   £32, 13s. 4d.
```

Pupil. Here the lowest denomination is pence; therefore, I change £1, 3*s*. 4*d*. to pence: this gives 280 pence. Now, it is plain that the price of 28 yards will be 28 times as much; therefore, I multiply by 28, and the answer is 7840 pence, or £32, 13*s*. 4*d*.

Note.—It is always more convenient to multiply the greater number by the less.

Exercises.

59. What is the value of 327 tons of guano at £7, 3*s*. 4*d*., £6, 7*s*. 11¾*d*., and £5, 19*s*. 7½*d*. per ton?
Ans. £2343, 10*s*., £2092, 9*s*. 2¼*d*., £1955, 17*s*. 4½*d*.

60. What cost 739 gallons of ale at 3*s*. 4½*d*., 2*s*. 11*d*., and 2*s*. 1¾*d*. per gallon?
Ans. £124, 14*s*. 1½*d*., £107, 15*s*. 5*d*., £79, 5*s*. 9¼*d*.

61. What is the price of 29 oxen at £10, 17*s*., £8, 19*s*. 3*d*., and £9, 6*s*. 8*d*. each? . Ans. £314, 13*s*., £259, 18*s*. 3*d*., £270, 13*s*. 4*d*.

62. What is the price of 536 yards of linen at 2*s*. 7½*d*., 3*s*. 6*d*., and 4*s*. 9¾*d*. per yard? . Ans. £70, 7*s*., £93, 16*s*., £128, 19*s*. 6*d*.

63. What will a man's wages amount to in 313 days at 1*s*. 10*d*., 2*s*. 3½*d*., and 4*s*. 9¾*d*. per day?
Ans. £28, 13*s*. 10*d*., £35, 17*s*. 3½*d*., £75, 6*s*. 3¾*d*.

64. What is the price of 3456 lb. of tea at 4*s*. 6*d*., 5*s*. 7¼*d*., and 6*s*. 3½*d*. per lb.? . . Ans. £777, 12*s*., £968, 8*s*., £1087, 4*s*.

65. What cost 659 cwt. of raisins at £4, £4, 11*s*. 5*d*., and £3, 19*s*. per cwt.? . . Ans. £2636, £3012, 3*s*. 7*d*., £2603, 1*s*.

COMPOUND ADDITION.

Rule.—1. Write the numbers under one another, so that those of the same denomination may stand in the same column, and draw a line under them.

2. Add together the numbers in the lowest denomination, and find (by Reduction) how many of the next higher denomination are contained in the sum; mark any remainder below the column added, and carry the quotient to the next denomination.

3. Repeat this process until all the columns are added. The highest denomination is added as in Simple Addition. The last sum, with the several remainders, will be the answer required.

The Methods of Proof are the same as in Simple Addition. The Reasons for the Rules of *Compound* Addition, Subtraction, Multiplication, and Division, are obviously the same as for the Rules of *Simple* Addition, &c.

Example.—Add together £31, 17*s.* $2\frac{3}{4}$*d.*, £307, 6*s.* $9\frac{1}{2}$*d.*, £29, 19*s.* $4\frac{1}{4}$*d.*, £78, 1*s.* $11\frac{1}{2}$*d.*, £691, 13*s.* $11\frac{3}{4}$*d.*, and £12, 18*s.* $9\frac{3}{4}$*d.*

£	*s.*	*d.*
31	17	$2\frac{3}{4}$
307	6	$9\frac{1}{2}$
29	19	$4\frac{1}{4}$
78	1	$11\frac{1}{2}$
691	13	$11\frac{3}{4}$
12	18	$9\frac{3}{4}$
£1151	18	$1\frac{1}{2}$

In this example, the numbers are arranged as in the margin. The lowest being farthings, the pupil begins at this column, and adds—*Pupil.* 3, 6, 8, 9, 11, 14; fours in 14? 3 times and 2; I put down 2 farthings, or a halfpenny, and carry the 3. *Teacher.* Why do you divide the sum by 4? *Pupil.* Because 4 farthings make a penny. I carry 3. 3, 12, 13, 14, 18, 27, 29, 39, 49; twelves in 49? 4 times and 1; I put down 1, and carry 4. 4, 12, 15, 16, 25, 31, 38; I put down 8, and carry 3. 3, 4, 5, 6, 7; twos in 7? 3 times and 1. *Teacher.* This is the short method of dividing by what number? *Pupil.* 20. *Teacher.* Why? &c. *Pupil.* 3, 5, 6, 14, 23, 30, 31; 1 and carry 3. 3, 4, 13, 20, 22, 25; 5 and carry 2. 2, 8, 11. Therefore the whole sum is £1151, 18*s.* $1\frac{1}{2}$*d.*

Note.—In adding the shillings, first add the column of units; put down the right-hand figure, and carry the remaining figure or figures to the tens; divide their sum by 2; if 1 remain, put it down, and carry the quotient to the pounds.

Exercises.

1.			2.			3.			4.		
£27	13	4½	£83	15	11½	£25	14	6	£36	15	7¼
39	6	5¼	24	15	9¼	97	13	11¾	98	14	10
23	17	9	61	18	2¾	25	18	10½	36	19	11½
64	1	11¾	97	14	7½	31	2	7½	42	13	8¾
27	19	10	27	6	10¼	69	13	4¾	70	14	5
91	13	9½	35	13	6½	48	16	2	49	17	9¼

5.			6.			7.			8.		
£72	5	7½	£25	13	11¼	£26	15	7¼	£47	16	8½
27	16	9¼	96	17	8	98	19	0	99	15	11¼
83	17	2¼	69	14	10¼	56	14	11¾	47	10	5¾
38	19	6½	38	5	9¾	92	12	8¾	53	14	9
59	14	8¾	57	12	11¾	75	17	7¼	81	15	6½
95	13	10½	86	19	10½	97	0	11¾	59	18	3¾

9.			10.			11.			12.		
£39	18	5½	£46	19	6¾	£73	12	5½	£65	12	9½
17	13	10¾	28	14	11	26	19	7	26	13	11
68	15	9	79	16	10¼	37	18	4¾	73	4	7
86	19	5½	97	10	6¾	11	1	1¼	29	3	8
33	11	1¼	44	12	2½	96	15	8¾	91	17	5
74	15	9½	85	16	10¾	27	12	11½	63	15	7½

13.			14.			15.			16.		
£727	13	4¾	£672	4	5¾	£634	14	9½	£789	19	11¾
965	14	2	267	13	9½	346	12	11¾	210	0	1¼
837	17	11½	672	15	11¼	683	13	7¼	567	17	3¾
586	19	10¼	935	14	5½	267	7	8½	763	13	8½
398	15	9½	593	18	10¾	695	19	9¼	649	17	6¼
613	12	11¾	359	11	1¼	999	7	5¾	783	12	8½
395	9	7½	111	11	11¼	842	16	10½	594	10	10½
593	19	4¾	369	14	7	795	5	5¾	279	5	3¾
675	12	11½	583	9	10½	689	17	8½	659	8	5

Answers.

1.	£274	18	2		7.	£448	0	10¾		12.	£ 350	8	0	
2.	331	4	11¾		8.	390	11	8¾		13.	5795	16	1½	
3.	298	19	6½		9.	321	14	5½		14.	4566	15	0¾	
4.	335	15	9¾		10.	383	11	0		15.	5955	16	4¾	
5.	378	7	3¾		11.	274	0	2¾		16.	5298	5	11¼	
6.	375	5	1½											

17.

£	s.	d.
£785	13	1
392	10	10¾
583	9	7½
694	5	5½
257	3	9¾
395	17	11¼
287	12	6¾
375	19	8½
987	8	5¾

18.

£	s.	d.
£697	12	6¾
385	17	9½
583	19	5¾
385	10	4½
716	17	11¼
167	14	7¼
973	3	10½
286	19	9¾
682	2	5¼

19.

£	s.	d.
£832	2	10½
756	17	9¼
647	11	3¼
392	13	11¾
238	19	4½
375	15	11¼
932	10	7¼
657	6	3½
746	9	10½

20.

£	s.	d.
£695	10	11½
596	7	5
872	19	10¾
847	8	3
593	13	10½
392	5	9¼
795	19	11¾
258	8	6½
683	16	7¾

21.

	4	28
cwt.	qr.	lb.
76	3	17
23	2	27
11	1	21
9	1	15
15	2	8
14	3	19
79	1	26
65	0	13

22.

	4	28
cwt.	qr.	lb.
437	3	11
743	1	13
695	2	26
217	2	19
416	3	27
932	2	5
584	1	18
123	3	13

23.

	20	4
tons.	cwt.	qr.
68	19	1
34	3	3
72	15	2
97	11	1
9	17	0
35	7	3
64	9	1
53	16	2

24.

	16	16
lb.	oz.	dr.
13	13	6
17	12	15
26	7	11
14	9	7
11	12	3
9	14	9
18	1	1
13	11	10

25.

	12	20	24
lb.	oz.	dwt.	gr.
17	5	19	23
23	11	16	21
96	5	7	22
15	9	3	7
4	3	18	19
17	10	15	14
25	4	12	3

26.

	4	40
ac.	ro.	po.
76	3	39
72	1	17
93	2	26
37	1	15
25	2	7
87	3	19
76	3	35

27.

	4	40
ac.	ro.	po.
367	1	27
123	1	38
792	3	25
614	3	13
795	2	19
318	2	34
573	1	16

28.

	8	220	3	12
m.	fur.	yd.	ft.	in.
7	2	197	2	11
63	6	213	1	4
27	2	39	0	10
38	7	219	1	9
47	3	76	2	8
51	5	23	1	7
39	1	127	2	5

29. A collector of taxes drew in January, £757, 14*s.* 3¾*d.*; in February, £839, 14*s.* 11¾*d.*; in March, £269, 17*s.* 5½*d.*; in April, £392, 12*s.* 11½*d.*; in May, £6732, 7*s.* 5½*d.*; in June, £735, 13*s.* 10¼*d.*; in July, £76, 19*s.* 3¾*d.*; in August, £325, 6*s.* 1¼*d.*; in September, £37, 18*s.* 7¼*d.*; in October, £269, 5*s.* 11*d.*; in November, £3752, 16*s.* 11¾*d.*; in December, £67, 13*s.* 10½*d.* How much did he collect throughout the year? . . Ans. £14258, 1s. 9¾*d.*

Answers.

17. £4760, 1*s.* 6¾*d.*
18. £4879, 18*s.* 10½*d.*
19. £5580, 7*s.* 11¾*d.*
20. £5236, 11*s.* 4*d.*
21. 296 cwt. 2 qr. 1 lb.
22. 4152 cwt. 1 qr. 20 lb.
23. 437 tons 0 cwt. 1 qr.
24. 126 lb. 2 oz. 14 dr.
25. 201 lb. 3 oz. 14 dwt. 13 gr.
26. 470 ac. 2 ro. 38 po.
27. 3586 ac. 1 ro. 12 po.
28. 275 m. 6 fur. 18 yd. 1 ft. 1 in.

30. A servant went to market, and spent on beef £1, 6s. 8½d.; on veal, 17s. 2¾d.; on sugar, £5, 8s. 7¾d.; on tea, £8, 15s. 9¼d.; and on various other articles, £10, 19s. 11½d. How much was spent altogether? . . . . . . Ans. £27, 2s. 10¼d.

31. A farmer sold at one time, 20 qr. 3 bush. 2 pecks; at another, 32 qr. 2 bush. 3 pecks; at a third, 9 qr. 7 bush. 1 peck; at a fourth, 37 qr. 2 pecks; at a fifth, 29 qr. 3 bush.; at a sixth, 13 qr. 4 bush. 3 pecks. How much did he sell altogether?
Ans. 142 qr. 5 bush. 3 pecks.

32. There is a farm consisting of ten fields, the first of which measures 7 acres 1 rood 28 poles; the second, 10 acres 1 rood 39 poles; the third, 5 acres 3 roods 21 poles; the fourth, 1 acre 1 rood 17 poles; the fifth, 11 acres 3 roods 6 poles; the sixth, 15 acres 2 roods 13 poles; the seventh, 9 acres 3 roods 25 poles; the eighth, 8 acres 1 rood 12 poles; the ninth, 13 acres 1 rood 27 poles; the tenth, 4 acres 2 roods 19 poles. What was the extent of the farm? . . . . Ans. 88 acres 3 roods 2 poles.

33. Paid for ground on which to build a house, £360; the mason's bill was £379, 12s. 11d.; the carpenter's, £489, 19s. 5¾d.; the slater's, £76, 4s. 9½d.; the plasterer's, £155, 14s. 3d.; the glazier's, £56, 14s. 2d. At what price must I sell it to gain £102?
Ans. £1570, 5s. 7¼d.

34. I owe to my grocer, £20, 3s. 4½d.; to my baker, £23, 14s. 7½d.; to my shoemaker, £3, 19s. 2¾d.; to my butcher, £18, 14s. 3¼d.; for house-rent, £28; for servants' wages, £10, 7s. 6d.; for taxes, £7, 13s. 10½d. What is the amount of my debt?
Ans. £107, 12s. 10½d.

35. A hop-merchant buys 5 bags of hops, of which the first weighed 2 cwt. 3 qr. 13 lb.; the second, 2 cwt. 3 qr. 11 lb.; the third, 2 cwt. 3 qr. 5 lb.; the fourth, 2 cwt. 3 qr. 12 lb.; the fifth, 2 cwt. 3 qr. 17 lb. I wish to know the weight of the whole,
Ans. 14 cwt. 1 qr. 2 lb.

36. A silversmith sold a gentleman silver-plate as follows: Dishes, 7 lb. 6 oz. 7 dwt.; plates, 5 lb. 9 oz. 12 dwt.; spoons, 2 lb. 6 oz. 13 dwt.; a tea-service weighing 4 lb. 7 oz. 4 dwt. What was the total weight? . . . Ans. 20 lb. 5 oz. 16 dwt.

37. What is the weight of 5 hhds. sugar? No. 1 weighing 15 cwt. 3 qr. 13 lb.; No. 2, 13 cwt. 2 qr. 7 lb.; No. 3, 17 cwt. 1 qr. 18 lb.; No. 4, 14 cwt. 3 qr. 15 lb.; and No. 5, 16 cwt. 1 qr. 13 lb. . . . . Ans. 78 cwt. 0 qr. 10 lb.

38. In the year 1857, the total value of the sovereigns coined was £4495748, 4s. 10d.; the half-sovereigns, £364111, 17s. 4d.; the florins, £167112; the shillings, £128106; the sixpences, £55886; the fourpences, £69, 6s.; the threepences, £22084, 2s.; the twopences in silver, £39, 12s.; the silver pence, £33; the copper pence, £3136; the halfpence, £2464; and the farthings, £1120. What was the total value of all the coins issued in 1857?
Ans. £5239360, 2s. 2d.

39. The income of the United Kingdom of Great Britain and Ireland in the year ending January 5, 1855, was—customs, £20777714, 8*s.* 11*d.*; excise, £16129843, 9*s.* 8*d.*; stamps, £7078004, 10*s.*; taxes (land and assessed), £3040548, 4*s.* 3*d.*; property-tax, £7456025, 2*s.* 8*d.*; post-office, £1288283, 17*s.* 4*d.*; crown-lands, £271571, 16*s.* 8*d.*; duties on pensions and salaries, £2348, 11*s.* 7*d.*; hereditary revenues of the crown, £8256, 17*s.*; surplus fees of regulated public offices, £88567, 4*s.* 3*d.*; produce of the sale of old stores, &c., £386095, 17*s.* 10*d.*; imprest and other moneys, £149922, 18*s.* 1*d.*; money received from the East India Company, £60000. The same year the expenditure exceeded the income by the sum of £3209059, 4*s.* 5*d.* Find the income and expenditure for the year ending January 5, 1855,

Ans. Income, £56737132, 18*s.* 3*d.*; Expenditure, £59946192, 2*s.* 8*d.*

40. The average price of wheat, in the week ending January 16, 1858, was £2, 8*s.* 8*d.* per imperial quarter; of barley, £1, 17*s.*; of oats, £1, 2*s.* 1*d.*; of rye, £1, 13*s.* 7*d.*; of beans, £1, 19*s.* 3*d.*; of pease, £1, 19*s.* 11*d.* What was then the price of one quarter of each grain? . . . . Ans. £11, 0*s.* 6*d.*

41. The sums expended on the *National Collections* in the year 1857 were—British Museum (establishment), £50347, 12*s.* 9*d.*; British Museum (buildings), £38814, 2*s.* 7*d.*; British Museum (purchases), £17425, 5*s.*; National Gallery, £29469, 14*s.* 2*d.*; Scientific Works and Experiments, £3672, 10*s.* 7*d.*; Royal Geographical Society, £500; British Historical Portrait-gallery, £1240, 9*s.* 8*d.*; Science and Art Department, £66011, 12*s.* 11*d.*; Museum of Practical Geology (establishment), £6092, 18*s.* 10*d.*; Royal Society, £1000; South Kensington Museum (building, &c.), £39586. Find the total sum expended during that year on the above National Collections, ... Ans. £254160, 6*s.* 6*d.*

42. The total amount of money advanced as loans to West India proprietors by the Exchequer Bill Commissioners has been £948150, at 4 per cent. The earliest loan was in 1833, and the latest in 1844. On the loans up to 1854 there had been paid of interest by Jamaica, £96691, 6*s.* 3*d.*; by Barbadoes, £155301, 13*s.* 3*d.*; by St Vincent, £17339, 2*s.* 10*d.*; by St Lucia, £4814, 17*s.* 3*d.*; and by Dominica, £4595, 6*s.* 10*d.* How much interest was then paid? . . . . . . Ans. £278742, 6*s.* 5*d.*

43. In the year 1854 the grants for public education in Great Britain were apportioned thus: Church of England Schools, £209871, 3*s.* 7*d.*; British and Foreign Schools, £31681, 4*s.* 8*d.*; Wesleyan Schools, £14049, 8*s.* 10*d.*; Roman Catholic Schools (Great Britain), £10907, 12*s.* 9*d.*; Workhouse Schools, £9882, 12*s.* 7*d.*; Scotland, Established Church Schools, £19193, 13*s.* 5*d.*; Free Church Schools, £21895, 9*s.* 1*d.*; Episcopal Church Schools, £1366, 2*s.* 4*d.*; and the expense of administrating the funds was £7589, 0*s.* 3*d.* How much was expended that year on public education? . . . . £326437, 7*s.* 6*d.*

COMPOUND SUBTRACTION.

RULE.—1. Place the less number under the greater, so that the numbers of the same denomination may stand directly under each other, and draw a line under them.

2. Begin at the right hand, and subtract in succession the numbers in the lower line from those immediately above them in the upper line, when it can be done, and set down the remainders under the numbers from which they arise.

3. But when the under number of any denomination is greater than the upper, add to the *upper* number the value of *one* of the next higher denomination, and then go on with the subtraction.

As an equivalent, however, for this, the *under* figure of the next higher denomination must be considered as having had 1 added to it before it is subtracted from the figure above, on the same principle as in Simple Subtraction.

The subtracting of the highest denomination is performed as in Simple Subtraction. The several remainders form the answer.

THE METHOD OF PROOF is the same as in Simple Subtraction.

Example.—What is the difference between £791, 18*s.* 7¾*d.*, and £1830, 5*s.* 9¼*d.*?

£	*s.*	*d.*
1830	5	9¼
791	18	7¾
£1038	7	1½

The numbers are arranged as in the margin, and the *pupil* says, 3 from 1, I cannot, but adding 1*d.* or 4 farthings, making 5, 3 from 5, and 2 remains, I put down 2 farthings, or a halfpenny. Carrying 1 to 7, 8 from 9, and 1; 18 from 5, I cannot, but adding a pound = 20*s.*, 18 from 25, and 7, I put down 7; 2 from 10, and 8; 10 from 13, and 3; 8 from 8, and 0; 0 from 1, and 1. Therefore the answer is £1038, 7*s.* 1½*d.*

Exercises.

1.			2.			3.			4.		
£638	17	5½	£729	12	11¾	£631	2	4½	£123	15	9¼
429	2	1¼	127	5	6½	236	11	8¾	68	18	5½

5.			6.			7.			8.		
£931	12	0¾	£179	16	5	£673	1	1¼	£712	10	3½
395	18	10¼	86	17	9¼	129	15	10¾	179	5	8¾

Answers.

1.	£209	15	4¼	4.	£ 54	17	3¾	7.	£543	5	2½
2.	602	7	5¼	5.	535	13	2½	8.	533	4	6¾
3.	894	11	0¾	6.	92	18	7¾				

9.			10.			11.			12.		
£847	11	5½	£796	3	2¾	£471	10	4¼	£306	14	10½
359	14	5¾	127	9	7¼	238	15	9¾	129	7	11¼

13.			14.			15.			16.		
£200	14	11	£700	10	0	£560	17	0	£800	0	0
123	2	5½	231	14	6¼	327	12	10½	399	15	4¾

17.			18.			19.			20.		
£783	1	3¾	£207	0	5	£691	0	0	£573	0	0¼
762	15	9	129	0	11¼	432	0	2¾	126	7	4½

21.

Borrowed, .				£739 14 3½
Paid at different times.	£123	12	2¾	
	79	5	4½	
	23	14	10	
	327	12	5¾	
Paid in all, .				£554 4 11
Balance, .				£

22.

Lent, . .				£700 0 0
Received at different times.	£179	13	9	
	46	15	10½	
	259	8	8¾	
	99	19	11	
Received in all,				£
Balance . .				£

23.				24.			25.			26.		
	12	20	24		20	4		4	28		4	4
lb.	oz.	dwt.	gr.	tons.	cwt.	qr.	cwt.	qr.	lb.	yd.	qr.	nails.
321	4	14	13	760	3	0	26	1	13	256	0	1
127	7	10	23	649	12	1	14	2	27	137	1	2

27.				28.			29.			30.		
	1760	3	12		4	40		4	2		8	
m.	yd.	feet.	in.	ac.	ro.	po.	gal.	qt.	pt.	qr.	bush.	pk.
69	39	1	2	102	0	15	191	1	0	326	2	1
27	1123	2	10	93	1	27	69	2	1	179	6	3

Answers.

9.	£487	16	11¾	14.	£468	15	5¾	19.	£258	19	9¼
10.	668	13	7½	15.	233	4	1½	20.	446	12	7¾
11.	232	14	6½	16.	400	4	7¼	21.	185	9	4½
12.	177	6	11¼	17.	20	5	6¾	22.	114	6	8¾
13.	77	12	5½	18.	77	19	5¾				

23. 193 lb. 9 oz. 3 dwt. 14 gr.
24. 110 tons 10 cwt. 3 qr.
25. 11 cwt. 2 qr. 14 lb.
26. 118 yd. 2 qr. 3 nails.
27. 41 m. 675 yd. 1 ft. 4 in.
28. 8 acres 2 roods 28 poles.
29. 121 gal. 2 quarts 1 pint.
30. 146 qr. 3 bush. 2 pecks.

31.				32.			33.			34.		
	24	60	60		4	28		4	40		8	4
d.	hr.	min.	sec.	cwt.	qr.	lb.	ac.	ro.	po.	qr.	bush.	pk.
231	13	11	27	569	1	13	327	1	23	262	5	3
126	7	15	39	172	1	20	219	2	32	176	7	1

35. Subtract a halfpenny from a hundred pounds,
Ans. £99, 19*s.* 11¾*d.*

36. A servant's wages are 12 guineas a year, and she has already received £4, 15*s.* 2¼*d.* How much remains due?
Ans. £7, 16*s.* 9¾*d.*

37. A youth has served 5 years 1 month 3 weeks 5 days of his 7 years' apprenticeship. How much longer has he yet to serve?
Ans. 1 year 10 months 2 days.

38. From a cask containing 36 gallons 1 quart, there was drawn 19 gallons 2 quarts 1 pint. How much liquor remains in the cask? . . . . Ans. 16 gallons 2 quarts 1 pint.

39. In the year 1855 the total cost of the metropolitan police was £405566, 12*s.* 3*d.* The amount paid by the various parishes was £263671, 17*s.*; the Treasury paid £100473, 13*s.* 4*d.*; and £35744, 19*s.* 7*d.* was derived from various special sources, such as attending at public offices, at theatres, and on private individuals, &c. The balance was drawn from the balance left at the end of 1854. What was that balance? . . . Ans. £5676, 2*s.* 4*d.*

40. The latitude of Paris Observatory is 48° 50′ 13″ N., and its longitude 0° 0′ 0″ (taking Paris as the first meridian); the latitude of Edinburgh Observatory is 55° 57′ 20″ N., and its longitude, 5° 31′ 7″ W. Find the difference of latitude and the difference of longitude of Paris and Edinburgh,
Ans. Diff. lat. 7° 7′ 7″; diff. long. 5° 31′ 7″.

41. The latitude of London (Gr. Observatory) is 51° 28′ 38″ N., and its longitude, 0° 0′ 0″ (taking Greenwich as the first meridian); the latitude of New York is 40° 42′ 45″ N., and its longitude, 74° 0′ 3″ W. How far is New York south and west from London? . Ans. 10° 45′ 53″ south, and 74° 0′ 3″ west.

42. The average price of a quarter of wheat on the week ending November 24, 1857, was £2, 11*s.* 8*d.*, and the average price of the same on the week ending October 23, 1858, was £2, 2*s.* 4*d.* How much had a quarter of wheat fallen during the intervening eleven months? . . . . Ans. 9*s.* 4*d.*

43. The gross public income of the United Kingdom of Great Britain and Ireland for the year ending the 31st December 1857, was £70390343, 7*s.* 10*d.*, and the expenditure during the same period was £70354245, 19*s.* 6*d.* How much did the income exceed the expenditure during that year? Ans. £36097, 8*s.* 4*d.*

Answers.

31. 105 days 5 h. 55 m. 48 sec. 33. 107 acres 2 roods 31 poles.
32. 396 cwt. 3 qr. 21 lb. 34. 85 qr. 6 bush. 2 pecks.

COMPOUND MULTIPLICATION.

I. When the Multiplier does not exceed 12.

Rule.—1. Write the multiplier below the lowest denomination of the multiplicand, and draw a line under them.

2. Multiply the lowest denomination by the multiplier; find how many of the next higher denomination is contained in the product, write down any remainder, and carry the quotient to the product of the next denomination.

3. Multiply in the same way the next higher denomination, and add the number *carried* from the last denomination, writing down any remainder, and carrying to the denomination above it, as before; and so on till all the denominations have been multiplied in succession: the highest denomination is multiplied as in Simple Multiplication.

The last product, and the several remainders, form the answer required.

Example.—Multiply £3, 17*s.* $4\frac{3}{4}$*d.* by 9.

£	*s.*	*d.*
3	17	$4\frac{3}{4}$
		9
£34	16	$6\frac{3}{4}$

In this example, the numbers are set down as in the margin, and beginning at the farthings, the *Pupil* says, 9 times 3 are 27; 27 farthings are $6\frac{3}{4}$*d.*, I put down $\frac{3}{4}$*d.*, and carry 6*d.*: 9 times 4 are 36, and 6 make 42; 42 pence are 3*s.* 6*d.*, I put down 6*d.*, and carry 3*s.*: 9 times 7 are 63, and 3 make 66, I put down 6, and carry 6: 9 times 1 are 9, and 6 are 15: twos in 15, 7 times and 1, I put down 1, and carry 7. *Teacher.* This is the short method of dividing by what number? *Pupil.* 20. *Teacher.* Why do you divide by 20? *Pupil.* To change the shillings to pounds. 9 times 3 are 27, and 7 are 34. Therefore, the whole product is £34, 16*s.* $6\frac{3}{4}$*d.*

Exercises.

1. Multiply £47 12 $7\frac{1}{2}$ by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
2. " 95 17 $2\frac{3}{4}$ " 7, 9, 5, 10, 2, 6, 11, 3, 12, 4, 8.

Answers.

1.

£ 95	5	3	£381	1	0
142	17	$10\frac{1}{2}$	428	13	$7\frac{1}{2}$
190	10	6	476	6	3
238	3	$1\frac{1}{2}$	523	18	$10\frac{1}{2}$
285	15	9	571	11	6
333	8	$4\frac{1}{2}$			

2.

£671	0	$7\frac{1}{4}$	£1054	9	$6\frac{1}{4}$
862	15	$0\frac{3}{4}$	287	11	$8\frac{1}{4}$
479	6	$1\frac{3}{4}$	1150	6	9
958	12	$3\frac{1}{2}$	883	8	11
191	14	$5\frac{1}{2}$	766	17	10
575	3	$4\frac{1}{2}$			

II. When the Multiplier is the product of two numbers, neither of which exceeds 12.

Rule.—Resolve the multiplier into its two factors; multiply the multiplicand by one of these, and then the resulting product by the other. This last product will be the answer required.

Example.—Multiply £1, 13*s.* 6¼*d.* by 63.

£	*s.*	*d.*
1	13	6¼
		9
£15	1	8¼
		7
£105	11	9¾

Pupil. In this example 63 = 9 × 7; therefore, I first multiply by 9; this gives £15, 1*s.* 8¼*d.* I next multiply by 7; this gives £105, 11*s.* 9¾*d.*, the product required.

Exercises.

3.	Multiply	£34	10	4¾	by	14, 15, 16.
4.	"	17	5	8	"	18, 20, 21.
5.	"	69	17	11½	"	27, 36, 42.
6.	"	96	13	9¼	"	64, 81, 100.
7.	"	25	8	4¼	"	44, 45, 60.
8.	"	73	19	11¾	"	33, 49, 56.
9.	"	67	1	3½	"	84, 108, 132.
10.	"	81	14	10¾	"	121, 25, 144.

III. When the Multiplier does not exceed 156, but cannot be resolved into two factors, neither of them greater than 12.

Rule.—Take two factors whose product is nearly equal to the given multiplier, and multiply by them as in Rule II. *Add* to this last product the product of the multiplicand multiplied by the *difference* between the given multiplier and the product of the two factors, when the product is less than the multiplier; but *subtract* when it is greater.

Answers.

3.			4.			5.			6.		
£483	5	6½	£311	2	0	£1887	4	10½	£6188	1	4
517	15	11¼	345	13	4	2516	6	6	7831	15	5¼
552	6	4	362	19	0	2935	14	3	9668	17	1

7.			8.			9.			10.		
£1118	7	7	£2441	19	3¾	£5633	8	6	£9891	2	4¾
1143	15	11¼	3625	18	11¾	7242	19	6	2043	12	4¾
1525	1	3	4143	18	10	8852	10	6	11771	5	0

Example 1.—Multiply £3, 15*s*. 8¾*d*. by 29.

	£	*s*.	*d*.		
	3	15	8¾	× 1	
			4		
	15	2	11	= 4	times the multiplicand.
			7		
	106	0	5	= 28	" " "
Add	3	15	8¾	= 1	" " "
Answer	£109	16	1¾	= 29	" " "

Since $29 = 4 \times 7 + 1$, multiply by 4 and by 7; and to the last product *add* £3, 15*s*. 8¾*d*., multiplied by 1.

Example 2.—Multiply £1, 14*s*. 5¼*d*. by 98.

	£	*s*.	*d*.		
	1	14	5¼	× 2	
			10		
	17	4	4½	= 10	times the multiplicand.
			10		
	172	3	9	= 100	" " "
Subtract	3	8	10½	= 2	" " "
Answer	£168	14	10½	= 98	" " "

In this example, since $98 = 10 \times 10 - 2$, multiply by 10 and by 10, which gives the value of 100, and from this last product *subtract* £1, 14*s*. 5¼*d*., multiplied by 2.

Exercises.

		£				
11.	Multiply	7	14	3½	by	19, 17, 23.
12.	"	29	13	7¾	"	26, 31, 39.
13.	"	127	6	9¼	"	48, 47, 52.
14.	"	0	7	11¾	"	67, 71, 78.
15.	"	97	3	0½	"	87, 91, 102.
16.	"	35	19	8¾	"	113, 127, 154.
17.	"	28	14	11¼	"	131, 73, 95.
18.	"	39	17	10¾	"	69, 59, 41.

Answers.

11.	12.	13.	14.
£146 11 6½	£771 14 9½	£5475 11 1¾	£26 14 7¼
131 2 11½	920 3 0¼	5984 18 2¾	28 6 6¼
177 8 8½	1157 12 2¼	6621 12 1	31 2 4½

15.	16.	17.	18.
£8452 4 7½	£4066 9 4¾	£3765 16 9¾	£2752 14 9¾
8840 16 9½	4570 5 7¼	2098 10 5¼	2353 15 10¼
9909 10 8	5541 18 3½	2730 19 0¾	1635 13 8¾

IV. When the Multiplier exceeds 156.

Rule.—1. Multiply the multiplicand by 10; the product by 10; and so on—multiplying *twice* by 10 if there are three figures in the multiplier, *thrice* if there are four figures, &c.

2. Then multiply the *multiplicand* by the units of the multiplier; the line below by the tens; and so on, placing the products as shewn in the following example; then add all the products together for the answer.

Example.—Multiply £1, 4*s.* $7\frac{3}{4}$*d.* by 345.

	£	s.	d.		£	s.	d.		
Multiplicand, .	1	4	$7\frac{3}{4}$	×5=	6	3	$2\frac{3}{4}$	= 5	times the multipl.
			10						
do. × 10 =	12	6	$5\frac{1}{2}$	×4=	49	5	10	= 40	" "
			10						
do. × 100 =	123	4	7	×3=	369	13	9	=300	" "
					425	2	$9\frac{3}{4}$	=345	" "

Here we multiply *twice* by 10, there being three figures in the multiplier.

We then multiply £1, 4*s.* $7\frac{3}{4}$*d.* by 5, which makes 5 times the multiplicand; the next line, £12, 6*s.* $5\frac{1}{2}$*d.*, which is equal to 10 times, is multiplied by 4, to produce 40 times the multiplicand; and the last line, £123, 4*s.* 7*d.*, which is equal to 100 times, is multiplied by 3, to produce 300 times the multiplicand. The products of 5 times, 40 times, and 300 times the multiplicand, = 345 times, added together, form the answer, £425, 2*s.* $9\frac{3}{4}$*d.*

The following is another Method:

1. Multiply the *highest* denomination of the multiplicand, as in Simple Multiplication.

2. Multiply separately the lower denominations in succession, taking such portions of them at a time as can be multiplied most easily. Convert the products as they arise, to their highest denomination, then add them all together for the answer.

Example.—Multiply £2, 12*s.* $4\frac{1}{2}$*d.* by 325.

				£	s.	d.
				2	12	$4\frac{1}{2}$
				325		
				650	0	0
10*s.* 0*d.*	× 325	= 3250*s.*	=	162	10	0
2*s.* 0*d.*	× "	= 650*s.*	=	32	10	0
4*d.*	× "	= 1300*d.*	=	5	8	4
$0\frac{1}{2}$*d.*	× "	= $162\frac{1}{2}$*d.*	=	0	13	$6\frac{1}{2}$
12*s.* $4\frac{1}{2}$*d.*			Answer,	£851	1	$10\frac{1}{2}$

Here we first multiply £2 by 325; then, for convenience, multiply successively 10*s.*, 2*s.*, 4*d.*, and $\frac{1}{2}$*d.* by 325.

Exercises.

19.	Multiply	£1	19	$5\frac{1}{4}$	by	379,	845.
20.	"	7	3	$10\frac{3}{4}$	"	932,	691.
21.	"	2	16	$9\frac{1}{2}$	"	702,	396.
22.	"	0	17	$9\frac{1}{4}$	"	1234,	5678.
23.	"	4	4	$6\frac{1}{2}$	"	3005,	7082.
24.	"	3	15	$11\frac{1}{4}$	"	9803,	5840.

V. When the Multiplier contains a Fraction—as $8\frac{3}{5}$.

Rule.—Multiply first by the fraction *; then by the integer or *whole* number of the multiplier, and add the products together for the answer.

* See Simple Multiplication, Rule IV., page 25.

Example.—Multiply £3, 15*s.* $4\frac{1}{2}$*d.* by $8\frac{3}{5}$.

£	s.	d.
3	15	$4\frac{1}{2}$
		$8\frac{3}{5}$
5)11	6	$1\frac{1}{2}$
2	5	$2\frac{1}{2}\frac{4}{5}$
30	3	0
£32	8	$2\frac{1}{2}\frac{4}{5}$ *Answer.*

Here we first multiply the given sum by $\frac{3}{5}$, then by 8, and add the two products for the answer. For the method of *dividing* £11, 6*s.* $1\frac{1}{2}$*d.* by 5, see page 65.

Exercises.

25.	Multiply	£2	16	6	by	$5\frac{1}{4}$,	. . . Ans.	£ 14	16	$7\frac{1}{2}$
26.	"	8	12	$4\frac{1}{2}$	"	$6\frac{1}{2}$,	. . . "	56	0	$5\frac{1}{4}$
27.	"	7	9	8	"	$7\frac{3}{4}$,	. . . "	57	19	11
28.	"	28	12	$6\frac{1}{4}$	"	$8\frac{1}{6}$,	. . . "	192	18	$11\frac{2}{4}\frac{1}{6}$
29.	"	36	14	$10\frac{1}{2}$	"	$9\frac{4}{5}$,	. . . "	360	1	$9\frac{1}{4}\frac{1}{5}$
30.	"	73	6	5	"	$7\frac{5}{8}$,	. . . "	574	6	$11\frac{2}{4}\frac{2}{3}$
31.	"	354	15	$7\frac{3}{4}$	"	$12\frac{5}{7}$,	. . . "	4510	16	$0\frac{3}{4}\frac{2}{7}$

32. " 16 cwt. 3 qr. 12 lb. by $27\frac{2}{5}$, Ans. 461 cwt. 3 qr. $15\frac{1}{5}$ lb.

Miscellaneous Exercises.

33. Multiply 45 tons 3 cwt. 1 qr. by 72, Ans. 3251 tons 14 cwt.
34. " 23 cwt. 2 qr. 27 lb. by 97, Ans. 2302 cwt. 3 qr. 15 lb.

Answers.

19.	20.	21.
£ 747 6 $9\frac{3}{4}$	£6705 10 11	£1993 7 9
1666 4 $8\frac{1}{4}$	4971 12 $0\frac{1}{4}$	1124 9 6

22.	23.	24.
£1096 9 $2\frac{1}{2}$	£12702 7 $8\frac{1}{2}$	£37220 15 $8\frac{3}{4}$
5045 2 $9\frac{1}{4}$	29936 4 1	22173 15 0

35. Multiply 7 qr. 3 bush. 2 pecks by 123,
Ans. 914 qr. 6 bush. 2 pecks.
36. " 73 ac. 1 ro. 29 po. by 35, . Ans. 2570 ac. 15 po.
37. " 2 lb. 3 oz. 15 dwt. by 16, . . Ans. 37 lb.
38. " 7 cwt. 3 qr. 12 lb. by 26, . Ans. 204 cwt. 1 qr. 4 lb.
39. What is the price of 27 lb. of tea at 5*s.* 6½*d.* per lb.?
Ans. £7, 9*s.* 7½*d.*
40. What is the price of 47 lb. of sugar at 8¼*d.* per lb.?
Ans. £1, 13*s.* 3¾*d.*
41. What is the price of 123 gallons of brandy at £1, 9*s.* 3¾*d.* per gall.? . . . . . . Ans. £180, 5*s.* 5¼*d.*
42. What is the price of 75 yards of broad cloth at £1, 3*s.* 5*d.* per yard? . . . . . . Ans. £87, 16*s.* 3*d.*
43. What is the price of 63 tons of coals at 13*s.* 6*d.* per ton?
Ans. £42, 10*s.* 6*d.*
44. What is the price of 13 rows of potatoes at 7*s.* 3¼*d.* per row?
Ans. £4, 14*s.* 6¼*d.*
45. What is the price of 96 yards of silk at 8*s.* 4¾*d.* per yard?
Ans. £40, 6*s.*
46. What is the price of 35 casks of rum at £2, 5*s.* 11½*d.* per cask? . . . . Ans. £80, 8*s.* 6½*d.*
47. How much will 15 hats cost at 15*s.* 4½*d.* each?
Ans. £11, 10*s.* 7½*d.*
48. How much will 103 stones of wool cost at 14*s.* 7¼*d.* per stone?
Ans. £75, 6*s.* 4¼*d.*
49. What is the cost of 106 cwt. of tallow at £1, 17*s.* 9½*d.* per cwt.?
Ans. £200, 5*s.* 11*d.*
50. What is the cost of 39 tons of guano at £5, 16*s.* 10¼*d.* per ton?
Ans. £227, 18*s.* 1¾*d.*
51. What is the cost of 99 barrels of figs at £1, 18*s.* 9¼*d.* per barrel? . . . . Ans. £191, 18*s.* 3¾*d.*
52. What is the cost of 32 pipes of wine at £12, 13*s.* 2½*d.* per pipe? . . . . Ans. £405, 2*s.* 8*d.*
53. What is the cost of 936 qr. of wheat at £2, 9*s.* 6½*d.* per qr.?
Ans. £2318, 11*s.*
54. What is the cost of 218 sacks of oatmeal at £1, 19*s.* 11¼*d.* per sack? . . . . . . Ans. £425, 6*s.* 8½*d.*
55. What is the cost of 549 bags of cotton at £12, 1*s.* 2¾*d.* per bag?
Ans. £6621, 14*s.* 9¾*d.*
56. What is the cost of 71 cwt. of butter at £4, 13*s.* 4¼*d.* per cwt.?
Ans. £331, 8*s.* 1¾*d.*
57. What is the cost of 89 qr. of barley at £1, 13*s.* 7*d.* per qr.?
Ans. £149, 8*s.* 11*d.*
58. What is the cost of 69 oz. of gold at £3, 17*s.* 11¼*d.* per oz.?
Ans. £268, 19*s.* 1¼*d.*
59. What is the weight of 47 pieces of lead, each 25 lb. 6 oz. 12 dr.? . . . . . Ans. 1194 lb. 13 oz. 4 dr.
60. What is the weight of 93 ingots of silver, each 3 lb. 11 oz. 13 dwt.? . . . . . . Ans. 369 lb. 3 oz. 9 dwt.

61. A farm consists of 9 fields, each 12 ac. 1 ro. 32 po.; what is the extent of the farm? . . Ans. 112 ac. 8 po.
62. The rent of my house for a week is £2, 5*s.* 7¾*d.*; what is the rent of it for a year at the same rate? Ans. £118, 13*s.* 7*d.*
63. There is a hospital which contains 180 boys, and the yearly maintenance, clothing, and education of each comes to £18, 15*s.* 6¾*d.*; what sum of money is required to defray the expenses of the hospital for a year?
Ans. £3380, 1*s.* 3*d.*
64. A farmer has a field containing 15 acres, which is sown with wheat; how much money will he receive for the crop, if each acre produces 9 qr., and the wheat is sold at £2, 16*s.* 5¼*d.* per qr.? . . . . . Ans. £380, 19*s.* 0¾*d.*
65. A farmer went to market and sold 28 qr. of wheat at £2, 19*s.* 4½*d.* per qr., 19 qr. of barley at £2, 4*s.* 6¼*d.*, and 30 qr. of oats at £1, 9*s.* 10¾*d.*; how much money did he receive for his grain? . . . Ans. £170, 4*s.* 8¼*d.*
66. What sum will a manufacturer pay in wages annually, who employs 637 men, each of whom has a weekly wage of £1, 3*s.* 6*d.*, ... Ans. £38,920, 14*s.*
67. A charitable institution gives relief to 170 individuals, to the amount of 3*s.* 3*d.* per week each. What is its yearly expenditure? . . . . . . Ans. £1436, 10*s.*
68. A baker uses 4 qr. 5 bush. 3 pecks of wheat in a week. How much does he require for a year? . Ans. 245 qr. 3 bush.
69. How far will a steamer sail in 312 hours, at the rate of 9 miles 6 furlongs 20 perches an hour, . Ans. 3061½ miles.
70. A cubic yard is 27 cubic feet, a cubic foot of gold weighs 10 cwt. 2 qr. 27 lb. 11 oz.; find the weight of a cubic yard of gold in tons, . . Ans. 14 tons 10 cwt. 19 lb. 9 oz.
71. The mean length of a lunar month is 29 days 12 hours 44 minutes 2⅞ seconds; find the length of 235 lunar months,
Ans. 6939 days 16 h. 30 m. 58 sec.
72. The length of a Julian year is 365 days and 6 hours; find the length of 19 Julian years, and find how much it differs from 235 lunar months as given in the last question,
Ans. 6939 days 18 hours. Diff. 1 hour 29 m. 2 sec.
73. In 1857 the number of acres used in the growing of hops in England was 50975, and the average duty paid per acre was £8, 3*s.* 9¾*d.*; what was the revenue derived that year from English hops? . . . Ans. £417,517, 2*s.* 2¼*d.*
74. In 1857 the pure silk manufactures exported from the United Kingdom, consisting of stuffs, handkerchiefs, and ribbons, was 624753 lb., and the value per pound was £1, 5*s.* 8½*d.*; find the total value, . . Ans. £803,067, 18*s.* 4½*d.*
75. The average annual expense of keeping a prisoner in Britain is £23, 10*s.* 3*d.*, in 1857 the daily average number of prisoners throughout the year was 19,686; what was the expense that year for confining criminals? . . Ans. £462,867, 1*s.* 6*d.*

COMPOUND DIVISION.

RULES.

I. When the Divisor does not exceed 12.

Rule.—1. Write the divisor on the left of the dividend, and then divide the highest denomination of the dividend, as in Simple Division, Rule I.

2. If there is no remainder, proceed to divide the next lower denomination. But if there is a remainder, reduce it to the next lower denomination, and add to it any of that denomination in the given quantity; then divide this sum as before.

3. Proceed with the next denomination in the same way, and so on till all the denominations have been divided. The various quotients will be of the same denomination as the dividends from which they arise, and form the answer required.

The Method of Proof is the same as in Simple Division.

Example.—Divide £23, 15*s.* 8¾*d.* by 9.

	£.	s.	d.
9)	23	15	8¾
	£2	12	10¼ $\frac{2}{9}$

Pupil. Nines in 23? 2 times and 5. I put down £2, and change £5 to shillings; £5 are 100*s.*, and 15*s.* *taken in*, make 115*s.* Nines in 115? 12 times and 7. I put down 12*s.*, and change the 7*s.* that remain to pence; 7*s.* are 84*d.*, and 8*d.* make 92*d.* Nines in 92? 10 times and 2. I put down 10*d.*, and change the 2*d.* that remain to farthings; 2*d.* are 8*f.*, and 3*f.* make 11*f.* Nines in 11? 1 time and 2. I put down ¼*d.* Therefore, the whole quotient is £2, 12*s.* 10¼*d.* $\frac{2}{9}$.

Exercises.

1.	Divide	£34	13	4½	by	2,	Ans.	£17	6	8¼
2.	"	17	12	8¼	"	8,	"	5	17	6¾
3.	"	23	15	8¾	"	9,	"	2	12	10¼ $\frac{2}{9}$
4.	"	83	9	5	"	5,	"	16	13	10½ $\frac{2}{5}$
5.	"	57	17	3¾	"	8,	"	7	4	7¾ $\frac{7}{8}$
6.	"	93	19	7½	"	9,	"	10	8	10$\frac{0}{4}$ $\frac{2}{3}$
7.	"	124	13	8½	"	5,	"	24	18	8¾ $\frac{3}{5}$
8.	"	137	16	4	"	10,	"	13	15	7½ $\frac{2}{5}$
9.	"	345	8	6¼	"	11,	"	31	8	0½ $\frac{3}{11}$
10.	"	417	13	4¾	"	12,	"	34	16	1¼ $\frac{7}{12}$

11.	"	85 lb. 8 oz. 10 dr.	by 8,	Ans.	10 lb. 11 oz. 1¼ dr.
12.	"	136 tons 14 cwt. 2 qr.	" 9,	"	15 tons 3 cwt. 8⅓ qr.
13.	"	158 miles 7 fur. 26 per.	" 7,	"	22 miles 5 fur. 26$\frac{4}{7}$ per.
14.	"	76 acres 3 ro. 30 per.	" 12,	"	6 acres 1 ro. 25⅚ per.
15.	"	216 yd. 2 qr. 3 nails	" 8,	"	27 yd. 0 qr. 1⅜ nail.
16.	"	167 days 14 h. 34 min.	" 7,	"	23 days 22 h. 39$\frac{1}{7}$ m.

II. When the Divisor exceeds 12.

Rule.—Divide as in Rule I., only employ *long* instead of short division.

Example.—Divide 324 cwt. 2 qr. 3 lb. by 93.

```
           4   28
   cwt.   qr.  lb.  cwt.  qr.  lb.
93)324     2   3 ( 3     1    26 77/93
   279
    45
     4
   182(1
    93
    89
    28
   715
  1780
  2495(26
  186
   635
   558
    77
    93
```

In this example the numbers are written as in *long* division of simple numbers. *Pupil.* I first divide 324 cwt. by 93; the quotient is 3 cwt., and the remainder is 45 cwt. This remainder I convert to quarters by multiplying by 4, and taking in 2 qr.; the sum, 182 qr., I divide by 93; the quotient is 1 qr., and the remainder 89 qr. This remainder I change to pounds by multiplying by 28, and taking in 3 lb., the sum is 2495 lb.: this sum I divide by 93; the quotient is 26 lb., and the remainder 77, which, as usual, is written with the divisor below it. Therefore, the whole quotient is 3 cwt. 1 qr. 26 $\frac{77}{93}$ lb.

Exercises.

17.	Divide £	837	1	10½	by	17,	51,	126.
18.	"	1234	5	6¼	"	19,	59,	325.
19.	"	325	7	10½	"	23,	87,	712.
20.	"	697	8	11	"	37,	93,	491.
21.	"	7329	16	5¼	"	842,	8317,	912.
22.	"	8456	12	3¼	"	7856,	392,	573.

Answers.

17.			18.			19.		
£49	4	9¾ $\frac{3}{17}$	£64	19	2¾ $\frac{8}{19}$	£14	2	11¼ $\frac{15}{23}$
16	8	8¼ $\frac{3}{51}$	20	18	4¾ $\frac{8}{59}$	3	14	9½ $\frac{48}{87}$
6	12	10¼ $\frac{108}{126}$	3	15	11¼ $\frac{280}{325}$	0	9	1½ $\frac{522}{712}$

20.			21.			22.		
£18	16	10¼ $\frac{19}{37}$	£8	14	1¼ $\frac{36}{842}$	£ 1	1	6¼ $\frac{3101}{7856}$
7	9	11$\frac{0}{4}$ $\frac{80}{93}$	0	17	7½ $\frac{448}{8317}$	21	11	5½ $\frac{39}{392}$
1	8	4¾ $\frac{75}{491}$	8	0	8¾ $\frac{560}{912}$	14	15	2$\frac{0}{4}$ $\frac{85}{573}$

23. Divide £5678 19 11½ by 9357, 125, 691.
24. 〃 9876 14 7¾ 〃 731, 10048.

25. Divide 312 cwt. 1 qr. 21 lb. by 63,
Ans. 4 cwt. 3 qr. 23$\frac{4}{9}$ lb.
26. 〃 67 tons 13 cwt. 2 qr. by 97,
Ans. 13 cwt. 3 qr. 22$\frac{78}{97}$ lb.
27. 〃 456 qr. 7 bush. 2 pecks by 117,
Ans. 3 qr. 7 bush. 0$\frac{114}{117}$ pecks.
28. 〃 123 cwt. 3 qr. 14 lb. by 456, . Ans. 1 qr. 2¼$\frac{80}{456}$ lb.
29. 〃 365 days 5 hours 48 min. 51 sec. by 12,
Ans. 30 days 10 hr. 29 min. 4$\frac{3}{12}$ sec.

WHEN THE DIVISOR IS THE PRODUCT OF TWO NUMBERS, NEITHER OF WHICH EXCEEDS 12, *short* Division may be employed.

RULE.—Resolve the divisor into its factors; divide the dividend by one of them, and then the resulting quotient by the other. The true remainder is found as in Simple Division.

Example.—Divide £124, 13*s.* 8½*d.* by 25.

£	s.	d.	
5)124	13	8½	
5)24	18	8¾	$\frac{3}{5}$
£4	19	8¾	$\frac{23}{25}$

Pupil.—In this example, 25 = 5 × 5: therefore, I divide by 5; this gives £24, 18*s.* 8¾*d.* I again divide by 5; this gives £4, 19*s.* 8¾*d.* $\frac{23}{25}$ for the quotient required.

Exercises.

30. Divide £329 17 6½ by 14, 24, 33.
31. 〃 735 13 7¾ 〃 15, 25, 40.
32. 〃 964 2 1¼ 〃 16, 27, 42.
33. 〃 333 3 3¾ 〃 18, 28, 44.

Answers.

23.

£ 0 12 1½ $\frac{6064}{9357}$
45 8 7½ $\frac{88}{125}$
8 4 4¼ $\frac{539}{691}$

24.

£13 10 2½ $\frac{593}{731}$
0 19 7¾ $\frac{6399}{10048}$

30.

£23 11 3$\frac{0}{4}$ $\frac{2}{14}$
13 14 10¾ $\frac{2}{24}$
9 19 11$\frac{0}{4}$ $\frac{14}{33}$

31.

£49 0 10¾ $\frac{10}{15}$
29 8 6½ $\frac{5}{25}$
18 7 10$\frac{0}{4}$ $\frac{15}{40}$

32.

£60 5 1½ $\frac{5}{16}$
35 14 1¾ $\frac{8}{27}$
22 19 1$\frac{0}{4}$ $\frac{29}{42}$

33.

£18 10 2$\frac{0}{4}$ $\frac{15}{18}$
11 17 11½ $\frac{23}{28}$
7 11 5¼ $\frac{3}{44}$

34.	Divide	£129	15	11	by	20, 32,	48.
35.	"	32	7	8¼	"	21, 30,	49.
36.	"	6	5	3¼	"	22, 36,	50.
37.	"	853	12	8½	"	54, 70,	96.
38.	"	757	17	9¼	"	56, 72,	99.
39.	"	26	13	4½	"	60, 77,	100.
40.	"	645	9	10¾	"	63, 80,	108.
41.	"	327	19	11¾	"	64, 90,	121.
42.	"	68	17	2¼	"	55, 81,	120.
43.	"	249	14	11½	"	66, 84,	144.
44.	"	79	15	3½	"	56, 84,	25.
45.	"	465	16	8½	"	49, 64,	81.

III. When the Divisor is 10, 100, 1000, or 1 with any other number of nothings annexed.

Rule.—1. Point off as many figures from the *right* of the highest denomination of the dividend, as there are nothings in the divisor; the remaining figures are the quotient of the denomination divided.

2. Reduce the figures pointed off to the next lower denomination, and add any of the same denomination in the given sum; then point off as before for a further quotient, and reduce the figures pointed off to the next lower denomination; and so on.

The figures that remain at each stage, after the pointing off, form the answer required.

Answers.

34.			35.			36.			37.		
£6	9	$9\frac{1}{2}\,\frac{4}{20}$	£1	10	$10\frac{0}{4}\,\frac{10}{21}$	£0	5	$8\frac{1}{4}\,\frac{7}{22}$	£15	16	$1\frac{3}{4}\,\frac{40}{54}$
4	1	$1\frac{1}{4}\,\frac{28}{32}$	1	1	$7\frac{0}{4}\,\frac{10}{30}$	0	3	$5\frac{3}{4}\,\frac{1}{36}$	12	3	$10\frac{3}{4}$
2	14	$0\frac{3}{4}\,\frac{44}{48}$	0	13	$2\frac{1}{2}\,\frac{24}{49}$	0	2	$6\frac{0}{4}\,\frac{13}{50}$	8	17	$10\frac{0}{4}\,\frac{34}{96}$

38.			39.			40.			41.		
£13	10	$8\frac{0}{4}\,\frac{21}{56}$	£0	8	$10\frac{1}{2}\,\frac{42}{60}$	£10	4	$11\frac{0}{4}\,\frac{7}{63}$	£5	2	$5\frac{3}{4}\,\frac{63}{64}$
10	10	$6\frac{1}{4}\,\frac{12}{72}$	0	6	$11\frac{0}{4}\,\frac{38}{77}$	8	1	$4\frac{1}{4}\,\frac{78}{80}$	8	12	$10\frac{1}{2}\,\frac{58}{90}$
7	13	$1\frac{1}{4}\,\frac{22}{99}$	0	5	$4\frac{0}{4}\,\frac{2}{100}$	5	19	$6\frac{1}{4}\,\frac{72}{108}$	2	14	$2\frac{1}{2}\,\frac{37}{121}$

42.			43.			44.			45.		
£1	3	$2\frac{1}{2}\,\frac{35}{55}$	£3	15	$8\frac{0}{4}\,\frac{46}{66}$	£1	8	$5\frac{3}{4}\,\frac{22}{56}$	£9	10	$1\frac{1}{2}\,\frac{28}{49}$
0	15	$9\frac{0}{4}\,\frac{60}{81}$	2	19	$5\frac{1}{2}\,\frac{22}{84}$	0	18	$11\frac{3}{4}\,\frac{50}{84}$	7	5	$6\frac{3}{4}\,\frac{34}{64}$
0	10	$7\frac{1}{2}\,\frac{105}{120}$	1	14	$8\frac{0}{4}\,\frac{142}{144}$	3	3	$9\frac{1}{2}\,\frac{24}{25}$	5	15	$0\frac{1}{4}\,\frac{1}{81}$

Example.—Divide £3642, 15*s*. 6*d*. by 100.

£	*s*.	*d*.
36,42	15	6
20		
8,55		
12		
6,66		
4		
2,64 *Ans.* £36	8	$6\frac{2}{4}\ \frac{64}{100}$

Here there being two nothings in the divisor, 2 figures are pointed off, from the *right* of the dividend, at each stage of the process. The figures that remain after the pointings off are—£36, then 8*s*., then 6*d*., and 2 farthings, with a remainder of 64, and form the answer.

Exercises.

46. Divide £	496	17	$8\frac{1}{2}$	by	10,	Ans. £	49	13	$9\frac{1}{4}$	
47. "	879	18	$11\frac{3}{4}$	"	10,	"	87	19	$4\frac{2}{4}$	$\frac{1}{10}$
48. "	2984	12	$7\frac{3}{4}$	"	100,	"	29	16	$11\frac{2}{4}$	$\frac{47}{100}$
49. "	46895	9	$8\frac{1}{4}$	"	100,	"	468	19	$1\frac{2}{4}$	$\frac{85}{100}$
50. "	98400	19	$7\frac{1}{2}$	"	100,	"	984	0	$2\frac{1}{4}$	$\frac{42}{100}$
51. "	378421	18	11	"	1000,	"	378	8	$5\frac{1}{4}$	$\frac{88}{1000}$

IV. When the Divisor contains a Fraction—as $3\frac{2}{5}$.

Rule.—Multiply the integer or *whole* number of the divisor, by the under figure of the fraction, and add the upper figure to the product; multiply the dividend also by the under figure of the fraction: then divide the one product by the other.

Example.—Divide £82, 15*s*. 8*d*. by $5\frac{3}{4}d$.

	£	*s*.	*d*.			
$5\frac{3}{4}$)	82	15	8			
4			4	£	*s*.	*d*.
23)	331	2	8(	14	7	$11\frac{1}{4}\ \frac{1}{23}$ *Ans.*

Here the integer of the divisor is multiplied by 4, the under figure of the fraction, and 3, the upper figure, is added to the product, making 23 for the divisor; the multiplicand is also multiplied by 4, making £331, 2*s*. 8*d*., the division is then proceeded with as in Rule II.

Exercises.

52. Divide £	36	13	$5\frac{1}{4}$	by	$8\frac{1}{2}$,	Ans. £	10	9	$6\frac{1}{2}$	$\frac{4}{7}$
53. "	58	6	$7\frac{3}{4}$	"	$4\frac{1}{4}$,	"	13	14	$6\frac{2}{4}$	$\frac{4}{17}$
54. "	87	12	$10\frac{1}{2}$	"	$6\frac{3}{4}$,	"	12	19	$8\frac{2}{4}$	$\frac{8}{9}$
55. "	71	4	8	"	$7\frac{3}{8}$,	"	9	5	$9\frac{3}{4}$	$\frac{14}{59}$
56. "	'138	11	$8\frac{3}{4}$	"	$12\frac{3}{7}$,	"	11	2	$11\frac{3}{4}$	
57. "	754	10	6	"	$26\frac{4}{5}$,	"	28	8	$0\frac{1}{4}$	44

V. When the Divisor is a Compound Number.

Rule.—Reduce both divisor and dividend to the lowest denomination that either of them contains; thus, if the lowest denomination in either be pence, reduce both to pence. Having now two simple numbers, proceed by long or short division, as the case may require.

Example.—How many times does £268, 2*s.* 6*d.* contain £3, 14*s.* 5¾*d.*?

```
 £   s.  d.    £   s. d.
 3   14  5¾ ) 268  2  6
 20            20
 --           ----
 74           5362
 12             12
 --          -----
 893         64350
   4             4
 ----       ------
 3575       )257400(72
             25025
             -----
              7150
              7150
```

Pupil. In this example the lowest denomination is farthings; therefore I change both my divisor and dividend to farthings, and then divide the one simple number by the other, and the quotient is 72.

Exercises.

58.	Divide	£ 85	8	4	by	£ 2	1	8
59.	"	40	4	9	"	1	7	9
60.	"	453	12	0	"	4	14	6
61.	"	43	16	10½	"	0	12	8½
62.	"	401	4	8	"	2	11	5¼
63.	"	47	15	6	"	0	18	4½
64.	"	277	3	5¼	"	0	7	7¼
65.	"	20015	15	1¼	"	57	18	7¾

66. How many times does 82 cwt. 3 qr. 7 lb. contain 3 qr. 21 lb.? . . . . Ans. 35.

67. How many times does 361 acres 3 roods 13 poles contain 1 acre 2 roods 37 poles? . . . . Ans. 209.

68. How many times is £2, 12*s.* 7¾*d.* contained in £960, 15*s.* 8¾*d.*? . . . . Ans. 365.

Miscellaneous Exercises.

69. If 36 cwt. of cheese cost £80, 2*s.*, how much will 1 cwt. cost? . . . . Ans. £2, 4*s.* 6*d.*

70. If 47 qr. of wheat cost £123, 11*s.* 9*d.*, what is the price per quarter? . . . . Ans. £2, 12*s.* 7$\frac{0}{4}$*d.* $\frac{16}{47}$.

71. A gentleman's income is £548 a year, how much is his income for a month, a week, and a day? . Ans. £45, 13*s.* 4*d.* per month; £10, 10*s.* 9$\frac{0}{4}$*d.* $\frac{48}{52}$ per week; £1, 10*s.* 0¼*d.* $\frac{115}{365}$ per day.

Answers.

58. 41	60. 96	62. 156	64. 729
59. 29	61. 69	63. 52	65. 347

72. Divide £723, 14*s.* 7½*d.* equally among 76 men,
Ans. £9, 10*s.* 5¼*d.* $\frac{66}{76}$.

73. I bought 69 sheep for £119, 14*s.*, how much was that a head?
Ans. £1, 14*s.* 8¼*d.* $\frac{27}{69}$.

74. What is the price of a gallon of brandy, when 96 gallons cost £145, 17*s.* 4*d.*? . . . . . Ans. £1, 10*s.* 4½*d.* $\frac{64}{96}$.

75. The rent of a farm containing 738 acres is £232, 18*s.* 7½*d.*, what is the rent of an acre? . . . . . . Ans. 6*s.* 3¾*d.*

76. If 435 lb. of tea cost £106, 17*s.* 8*d.*, how much is the price of a pound? . . . . Ans. 4*s.* 10¾*d.* $\frac{383}{435}$.

77. If 112 ingots of gold are worth £77878, 5*s.* 4*d.*, what is the value of one? . . . . Ans. £695, 6*s.* 10*d.*

78. The clothing of 90 charity-boys came to £146, 11*s.* 5¾*d.*, what was the expense of clothing one boy? Ans. £1, 12*s.* 6¾*d.* $\frac{41}{90}$.

79. How many tons of coal can be purchased for £315, 8*s.* 10¼*d.*, at 17*s.* 8¼*d.* per ton? . . . . Ans. 365.

80. Prize-money to the amount of £683, 3*s.* 10½*d.* is to be equally divided among 83 seamen, how much will each receive?
Ans. £8, 4*s.* 7½*d.*

81. How many stones of flour, at 3*s.* 4½*d.* per stone, may I buy with £18, 18*s.*? . . . . Ans. 112.

82. A gentleman distributed £31, 2*s.* 10½*d.* among a number of poor people, and gave each 6*s.* 3½*d.*; how many poor people were there? . . . . Ans. 99.

83. If the produce of 2 acres 1 rood 15 poles maintain a horse, how many horses may be maintained on the produce of 150 acres?
Ans. 64.

84. A quantity of tea, consisting of 19 parcels, contains 332 lb. 8 oz., what is the weight of a single parcel? . Ans. 17 lb. 8 oz.

85. The total cost of the London police in 1855 was £405566, 12*s.* 3*d.*, and the number of persons employed was 5783. What was the average cost per man for one year?
Ans. £70, 2*s.* 7¼*d.* $\frac{3473}{5783}$.

86. The quantity of tea consumed in the United Kingdom in 1855 was 63430693 lb., and its value was £3980463. What was the average value of tea per pound? . Ans. 1*s.* 3*d.* $\frac{3850725}{63430693}$.

87. The gross amount of custom-duty derived from tea in the year 1857 was £5060048, and the population of the United Kingdom was estimated on the 1st January of the same year, 28416058. What was the average duty paid per head for tea during that year? . . . . . . Ans. 3*s.* 6¾*d.* nearly.

88. The total quantity of tea entered for home consumption in 1857 was 69159848 lb. Find from this and the facts given in last question, the average duty per pound, and the average quantity consumed per head of the population,
Ans. Duty, 1*s.* 5½*d.*; Quantity, 2 lb. 6 oz. 15 dr. nearly.

89. In 1857 the total permanent debt of Great Britain was £736009272, and the annual interest was £22075136. What was the average rate of interest paid per pound sterling?

Ans. $7d.\ \frac{145967736}{736009272}$ or $7\frac{1}{4}d.$ nearly.

90. In 1857 the total permanent debt of the United Kingdom of Great Britain and Ireland was £779701417, and the interest paid on it was £23601929. What was the average interest paid per pound sterling? . . . . . . Ans. $7\frac{1}{4}d.\ \frac{46510747}{779701417}$.

91. The quantity of unrefined sugar imported into the United Kingdom from British possessions in 1856 was 276142 tons, and the duty paid on it £3752746. What was the duty paid per ton?

Ans. £13, 11*s*. $9\frac{1}{4}d.\ \frac{87628}{276142}$.

Exercises on Compound Addition, Subtraction, Multiplication, and Division.

1. A purchases from B a hogshead of sugar, of which the value was £73, 10*s*.; a box of tea, £54, 16*s*. 8*d*.; and a pipe of wine, £93, 5*s*. In return, B receives from A £50 in money, and 3 pieces of cloth, valued at £67, 15*s*. How much does A still owe B? . . . . Ans. £103, 16*s*. 8*d*.

A purchases from B—A hhd. of sugar, ...	£73 10 0		
A box of tea, ...	54 16 8		
A pipe of wine, ...	93 5 0		
Value purchased, ...	. =		
B receives from A—In money, ...	£50 0 0		
3 pieces of cloth, ...	67 15 0		
Value received, ...	. =		
A still owes B, ...			£103 16 8

2. A lady went to market with £5, 3*s*. 11*d*., and laid out on groceries, 18*s*. $4\frac{1}{2}d.$; on bread, 12*s*. $5\frac{1}{4}d.$; on beef, £1, 6*s*. 3*d*.; and on various other articles, 4*s*. 11*d*. How much money should she have remaining? . . . . Ans. £2, 1*s*. $11\frac{1}{4}d.$

3. A tradesman being insolvent, called all his creditors together, and found he owed to A, £53, 7*s*. 6*d*.; to B, £105, 10*s*.; to C, £34, 5*s*. 2*d*.; to D, £28, 16*s*. 5*d*.; to E, £14, 15*s*. 8*d*.; to F, £112, 9*s*.; to G, £143, 12*s*. 9*d*. The value of his stock was £212, 6*s*.; the debts due to him amounted to £112, 8*s*. 3*d*., besides £21, 10*s*. 5*d*. money in hand. How much would his creditors lose by taking the whole of his effects? . Ans. £146, 11*s*. 10*d*.

4. A gentleman's income is £1200 a year, and he spends on an average £1, 17*s*. $5\frac{3}{4}d.$ every day. How much does he save in a year?—a week?—a day? . . Ans. Yearly, £516, 0*s*. $1\frac{1}{4}d.$; weekly, £9, 18*s*. $5\frac{1}{2}d.\ \frac{13}{52}$; daily, £1, 8*s*. $3\frac{1}{4}d.\ \frac{60}{365}$.

5. A gentleman's income is £460 a year; he wishes to save £53, 19*s*. 6*d*. annually. How much may he spend a week?—per day? . Ans. Weekly, £7, 16*s*. $1\frac{3}{4}d.\ \frac{44}{52}$; daily, £1, 2*s*. $2\frac{3}{4}d.\ \frac{228}{365}$.

6. A farmer sold 39 quarters of wheat at £2, 16*s*. 11*d*. per quarter, and 28 quarters at £2, 4*s*. $7\frac{3}{4}d.$ How much money did he receive?—and what was the average price per quarter?

Ans. £162, 6*s*. $7\frac{1}{4}d.$; average price, £2, 12*s*. $4\frac{1}{4}d.\ \frac{31}{67}$.

7. A person bought 45 shares in the Edinburgh and Glasgow Railway, at £73, 10*s*. 6*d*. per share, and was afterwards obliged to sell out when the price per share was £65, 17*s*. 9*d*. How much did he lose? . . . . . . Ans. £343, 13*s*. 9*d*.

8. A vessel came into Leith harbour with 67 tons of guano; 10 tons were disposed of at £7, 6*s*. 3*d*. per ton, 8 tons at £7, 1*s*. 9½*d*. per ton, 17 tons at £6, 15*s*. per ton, 23 tons at £6, 11*s*. 4½*d*. per ton, and the remainder at £5, 18*s*. 7*d*. per ton. Required the value of the cargo, and the average price per ton, Ans. Value of cargo, £449, 0*s*. 8½*d*.; average price, £6, 14*s*. 0¼*d*. $\frac{64}{67}$.

9. A manufacturer sold 789 yards of calico at 5¾*d*. per yard, 69 yards of tartan at 4*s*. 7½*d*. per yard, 78 yards of carpeting at 3*s*. 9*d*. per yard. How much money did he draw?

Ans. £47, 6*s*. 11¾*d*.

10. A gentleman's income is £160 a year; his house-rent amounts to £19, 10*s*.; his taxes are £5, 3*s*. 7½*d*.; his grocer's account, £15, 11*s*. 3*d*.; his baker's account, £7, 17*s*. 5*d*.; his butcher's, £14, 1*s*. 3¾*d*.; his shoemaker's, £6, 5*s*. 9*d*.; his tailor's, £8, 1*s*. 5½*d*.; his draper's, £5, 13*s*. 11¾*d*.; his income-tax, £4, 13*s*. 4*d*.; and his miscellaneous expenses are £46, 14*s*. 10½*d*. How much money may he save annually? . . . . Ans. £26, 7*s*.

11. What is the value of 39 boxes of oranges at £5, 3*s*. 11*d*. per box? . . . . Ans. £202, 12*s*. 9*d*.

12. A man's daily wages are 3*s*. 10½*d*.; he requires to save £23 for house-rent and clothes. How much may he spend per day?

Ans. 2*s*. 7¼*d*. $\frac{185}{365}$.

13. A merchant's clerk being sent to collect payment of accounts, took £5, 3*s*. 6*d*. in his pocket; at York he received £49, 13*s*. 9*d*., his expenses there were 15*s*. 3¼*d*.; at Sunderland he received £51, 11*s*. 5½*d*., his expenses there were £2, 5*s*. 9*d*.; at Durham he received £179, 13*s*. 10¾*d*., his expenses there were £2, 10*s*. 6*d*.; at Newcastle he received £91, 7*s*. 9¼*d*., his expenses there were £1, 3*s*. 9*d*.; and at Carlisle he received £275, 10*s*. 11¼*d*., and his expenses there were £3, 6*s*. 4*d*. How much money does he have at Carlisle? . . . . . Ans. £642, 19*s*. 8¾*d*.

14. A grocer gave his boy £50 to pay a debt of £34, 11*s*. 5*d*. How much money must he get back? . . Ans. £15, 8*s*. 7*d*.

15. A purse contains £95, 14 crowns, 23 half-crowns, 19 shillings, 14 sixpences, and 12 fourpenny-pieces, and the money is equally divided among 12 individuals. How much does each receive?

Ans. £8, 11*s*. 5½*d*.

16. If a workman gain 23*s*. 7*d*. per week, and spend 18*s*. 3½*d*. per week, how much does he save in 52 weeks? Ans. £13, 15*s*. 2*d*.

17. A gentleman's annual income is £1000, and his daily expenses are £1, 17*s*. 3½*d*. How much does he save in 9 years?

Ans. £2874, 16*s*. 10½*d*.

18. If 1869 sovereigns are coined from forty troy pounds of gold, what is the weight of a sovereign? Ans. 5 dwt. 3 $\frac{513}{1869}$ gr.

INVOICES.

An Invoice* is a list or account of the particulars and prices of goods, &c., that have been sold on a certain day by one person to another. The following is an example:

Manchester, *January* 1, 1854.

Mr John Adams,
Edinburgh. *Bought of* Edward Johnston & Co.

	£	s.	d.
26 Pieces Printed Cotton, 825 yards,....*7d.*	24	1	3
2 " " " 120 " ....*8d.*	4	0	0
12 " Twilled " 504 " ....*7d.*	14	14	0
Wrapper,....................	0	2	6
per Rail.	42	17	9

Invoices to be written out and priced.

1. Manchester, *June* 4, 1846.

Mr David Wilson,

Bought of Melrose & Co.

						£	s.	d.
24 lb.	of fine green tea,	at	12*s.*	11½*d.*	per lb.			
17 "	of best hyson,	"	7	4	"			
8 "	of coffee,	"	2	1¼	"			
29 "	of refined sugar,	"	0	10	"			
35 "	of raw sugar,	"	0	6½	"			
13 "	of raisins,	"	1	2¾	"			
						25	11	7¼

2. Bristol, *May* 15, 1846.

Mr James Fairbairn,

Bought of P. Forbes & Co.

						£	s.	d.
18 dozen	of sherry wine,	at	39*s.*	11*d.*	per doz.			
23 "	of port wine,	"	41	5½	"			
5 "	of claret,	"	89	10¼	"			
19 gallons	of brandy,	"	23	4½	per gal.			
25 "	of rum,	"	18	10	"			
10 "	of whisky,	"	12	2¾	"			
						157	18	6¾

* Sometimes called a Bill of Parcels.

3. GLASGOW, *August* 14, 1846.

MR WILLIAM PANTON,

Bought of ADAM ROBINSON.

						£	s.	d.
36	yards of superfine cloth,	at	23s.	4½d.	per yd.			
100	〃 of cassimere,	〃	8	7¼	〃			
49	pair of Eng. blankets,	〃	15	10¾	per pr.			
65	〃 of Scotch 〃	〃	8	3⅛	〃			
235	yards of carpeting,	〃	8	5	per yd.			
79	〃 of flannel,	〃	1	8	〃			
						197	14	4¼

Exercises.

4. *Edinburgh, June* 3, 1846.—Adam Black, Esq. bought of Cowan & Co. 24 ream of demy at 52*s*. 6*d*. per ream, 75 ream of wove post at 40*s*., 27 ream of crown at 33*s*., 13 ream of hot-pressed at 43*s*., 52 ream of foolscap at 25*s*. 6*d*., 70 ream of thin post at 28*s*. 9*d*. Write out, and find the amount of this bill, Ans. £452, 8*s*. 6*d*.

5. *London, February* 4, 1847.—Mrs George Scott bought of John Duncan 73 yards of silk at 14*s*. 3*d*. per yard, 25 yards of lawn at 2*s*. 6*d*., 31 yards of cambric at 5*s*. 3½*d*., 86 yards of velvet at £1, 4*s*. 9¾*d*., 75 yards of lace at 12*s*. 3½*d*., and 19 yards of brocade at 16*s*. 9*d*. Write out, and find the amount of this bill, Ans. £232, 0*s*. 9½*d*.

6. *Liverpool, March* 21, 1847.—Mr William Moffat bought of Peter Johnston 57 feet of wainscot sashes at 10½*d*. per foot, 560 feet of ash at 3½*d*., 79 cubic feet of oak at 4*s*. 2*d*., 136 feet of framed deal at 3*s*. 9*d*., 95 feet of mahogany at 5*s*. 6*d*., and 9 men's labour for 17 days at 4*s*. 9*d*. each per day. What is the amount of this bill? . . . . Ans. £115, 1*s*. 7½*d*.

7. *Dublin, February* 28, 1847.—Mr John Veitch bought of Walter Watson 36 yards of Italian crape at 1*s*. 10½*d*. per yard, 17 yards of silk velvet at 12*s*. 4¾*d*., 19 yards of Irish tabinet at 3*s*. 3½*d*., 29 yards scarlet cloth at 19*s*. 4*d*., 31 yards French cambric at 18*s*. 3½*d*., and 18 Indian shawls at £8, 10*s*. 6*d*. each. Write out the bill, and calculate its amount, . Ans. £226, 17*s*. 5¾*d*.

8. *Sheffield, March* 13, 1847.—Mr David Reid bought of Alexander Roger 7 dozen razors at 1*s*. 4½*d*. each, 5 dozen ditto at 1*s*. 1*d*., 2 dozen ditto at 8¾*d*., 6 dozen knives and forks at £1, 4*s*. 10*d*. per dozen, 3 dozen ditto at 18*s*. 6*d*., 8 dozen scissors at 7*s*. 2*d*. per dozen, and 12 cases of lancets at 17*s*. 3¾*d*. per case. Required a copy of the bill and its amount, . Ans. £33, 7s. 7d.

THE GREATEST COMMON MEASURE.

A MEASURE or FACTOR of any number is a number that *divides* it exactly without a remainder: thus, 3 is a measure of 24.

A COMMON MEASURE of two or more numbers is any number that will divide each of them without a remainder: thus, 4 is a common measure of 16 and 24.

THE GREATEST COMMON MEASURE of two or more numbers is the *greatest* number that measures each of them: thus, 5 is the greatest common measure of 15, 20, and 25.

A *Prime* number is one that can be divided without a remainder only by itself or by 1: thus, 1, 2, 3, 5, 7, 29, are prime numbers. They are sometimes called *Primes*.

A *Composite* number is one which is the product of two or more factors: thus, 32, 40, 64, are composite numbers.

A Number that measures two other numbers, measures also their sum and difference.

Thus, since 4 measures 36 and 28, it also will measure $36 + 28$, or 64, and $36 - 28$, or 8.

For $36 + 28 = 4 \times 9 + 4 \times 7 = 4 \times 16 = 64$; that is, 4 taken 9 times, and then 4 taken 7 times in addition, is equal to 4 taken 16 times at once.

And $36 - 28 = 4 \times 9 - 4 \times 7 = 4 \times 2 = 8$; that is, 4 taken 9 times, diminished by 4 taken 7 times, is equal to 4 taken 2 times at once.

A Number that measures any other number, measures also any multiple of that other number.

Thus, since 3 measures 6, it will also measure 12, 18, 24, or any other multiple of 6.

For $12 = 6 \times 2 = 3 \times 2 \times 2 = 3 \times 4$; that is, 12, which is a multiple of 6, is also a multiple of 3, and is therefore measured by 3.

To find the Greatest Common Measure of two Numbers.

Rule.—Divide the greater number by the less; then the divisor by the remainder, if any; and so on, continually dividing the preceding divisor by the last remainder, till nothing remains. The *last divisor* will be the greatest common measure.

Example.—Find the greatest common measure of 696 and 1305.

```
696)1305(1
    696
    ---
    609)696(1
        609
        ---
         87)609(7
            609
```

Pupil. I first divide 1305 by 696; the quotient is 1, and the remainder 609. I next divide 696, the last divisor, by 609; the quotient is 1, and the remainder 87. I next divide 609, the last divisor, by 87; the quotient is 7, and the remainder nothing; therefore, 87 is the greatest common measure.

Reason of the Rule.—Referring to the foregoing example, it is obvious that the greatest common measure cannot exceed 696; we therefore try if 696 is the measure sought. 1305 divided by 696 leaves a remainder of 609; therefore, 696 is not the greatest common measure.

Now, the greatest common measure of 696 and 1305 must measure the remainder 609, therefore, the greatest common measure cannot exceed 609; we therefore try if 609 is the measure sought. 696 divided by 609 leaves a remainder of 87; therefore, 609 is not the greatest common measure.

Now, since the greatest common measure of 696 and 1305 is also the greatest common measure of 609 and 696, therefore, the measure sought cannot exceed their difference, 87; we therefore try if 87 measures 609. 609 divided by 87 leaves no remainder; therefore, 87 is the greatest common measure required.

Exercises.

1. Find the greatest common measure of 252 and 348, Ans. 12.
2. " " " 493 and 899, Ans. 29.
3. " " " 620 and 2108, Ans. 124.
4. " " " 6023 and 15466, Ans. 19.
5. " " " 5865 and 69180, Ans. 15.
6. " " " 4081 and 5141, Ans. 53.

VULGAR FRACTIONS.

A FRACTION means a part of a whole: the term is derived from a Latin word signifying *broken*.

Whole or unbroken numbers, as 1, 2, 3, &c., are termed *integers;* broken numbers, as, $\frac{1}{2}$, a half; $\frac{1}{3}$, a third, &c., are termed *fractions.*

Fractions are of two kinds: *vulgar* fractions, from a Latin word signifying common; and *decimals*, from a word signifying ten.

VULGAR FRACTIONS are the common fractions of halves, thirds, fourths, and so on; the term is applied to all fractions when expressed by figures in this form—$\frac{2}{3}$, two-thirds; $\frac{5}{6}$, five-sixths, &c. They are called *vulgar* fractions, in distinction from *decimal* fractions (see page 135).

If we suppose a loaf to be divided into two equal parts, each of the parts is a half, and forms a fraction of the whole; in figures, it is written as a Vulgar Fraction, thus—$\frac{1}{2}$.

Again, if the loaf is divided into four equal parts; each of these is called a *fourth* or a *quarter*, and is written thus—$\frac{1}{4}$; two of them may be expressed as $\frac{2}{4}$, but as two-fourths are the same as one-half, they are written, $\frac{1}{2}$; three of them are written, $\frac{3}{4}$, expressing three-fourths. If the whole be divided into three equal parts, each part is called a *third;* if into five, each is called a *fifth;* if into six, a *sixth;* and so on, according to the number of parts into which the whole is divided; thus—$\frac{2}{3}$ means two-thirds of a whole; $\frac{3}{5}$, three-fifths; $\frac{5}{6}$, five-sixths.

To represent a Vulgar Fraction, therefore, two numbers are required, which are written the one above the other, with a short line between. The number *under* the line shews into how many parts the *whole* of the article, whatever it may be, is divided; and the number *above* the line shews how many of these parts we mean to express.

The upper number is called the *numerator*; because it shews the *number* of the parts—as *three*-fourths, *six*-sevenths; the lower number is called the *denominator*, because it denominates the nature of the fraction—such as *thirds*, *eighths*, &

All the parts are together equal to the *whole*. Thus—two-halves, or three-thirds, or four-quarters, make each a *whole.*

The *numerator* and *denominator* are called the *terms* of the fraction.

A *fraction* may therefore be considered as the result of two indicated operations; namely, *division* and *multiplication.* For

example, $\frac{2}{3}$ indicates that the *principal* unit is first *divided* into three equal parts; and, secondly, that one of these parts is taken two times, or *multiplied* by 2, and in this view is considered as *two-thirds* of 1. Now, when multiplication and division are to be performed in succession, it is a matter of no consequence whether we first *multiply* and then *divide*, or first *divide* and then *multiply*. Hence, taking the number 1, and multiplying it by 2, we have 2; and *indicating* the division of 2 by 3, we have $\frac{2}{3}$, which, in this view, is considered as the *one-third* of 2; hence the $\frac{2}{3}$ of 1 and the $\frac{1}{3}$ of 2 are equal.

Hence, a fraction may be viewed as *indicating* the division of the numerator by the denominator.

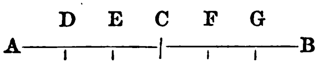
From the nature of the notation of a fraction, a fraction is *multiplied* by any number, by *multiplying* the *numerator* of the fraction by that number; and a fraction is *divided* by any number, by multiplying the denominator by that number.

The value of a fraction is not altered by *multiplying* or *dividing* both of its terms by the same number. Thus, $\frac{3}{4}$ is equal in value to $\frac{6}{8}$; for $\frac{3}{4}$ multiplied by 2 gives $\frac{3 \times 2}{4}$, and this product divided by 2 gives $\frac{3 \times 2}{4 \times 2}$; now, when a number is first multiplied by 2, and then the product divided by 2, the value of the number is not altered, $\therefore \frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$. If a class of 4 boys have 3 oranges divided among them, and a second class of 8 boys have 6 oranges divided among them, it is obvious that one of the first class and one of the second would receive the same part of an orange.

As the principles now stated are of the utmost importance, we shall establish their truth in another manner.

Twice the third part of 1 is the same in value as the third part of 2, or $\frac{2}{3}$ of $1 = \frac{1}{3}$ of 2.*

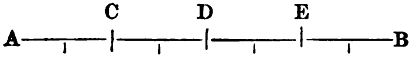
Let AB be taken equal to 2 feet, for example, and divide each of the feet AC and CB into 3 equal parts. It is obvious that AE is equal to twice AD; but AD is equal to the third part of AC; therefore, AE is equal to twice the third part of 1 foot. Again, AE, EF, and FB are all equal to one another; therefore, AE is equal to the third part of 2 feet. Hence, to obtain the length $\frac{2}{3}$, it makes no difference whether we first divide 1 foot into 3 equal parts, and then take 2 of them, or divide 2 feet into 3 equal parts, and take 1 of them.



* De Morgan.

The value of a fraction is not altered by multiplying or dividing both its terms by the same number: thus, $\frac{3}{4} = \frac{6}{8}$.

Let AB represent a foot; divide it into 4 equal parts, AC, CD, DE, EB, and divide each of these into 2 equal parts. Then AE is $\frac{3}{4}$; but the second division divides AB into 8 equal parts, of which AE contains 6; it is therefore $\frac{6}{8}$. Hence $\frac{3}{4} = \frac{6}{8}$. That $\frac{6}{8} = \frac{3}{4}$ is obvious.



When the numerator of a fraction is *equal to* its denominator, the fraction is *equal to* 1. For example, the fractions $\frac{5}{5}$, $\frac{8}{8}$, $\frac{11}{11}$, are each equal to 1: this is very obvious.

When the numerator of a fraction is *less* than its denominator, the fraction is *less* than 1. For example, $\frac{7}{9}$ is less than 1, for $\frac{7}{9}$ is less than $\frac{9}{9}$; but $\frac{9}{9}$ is equal to 1, therefore, $\frac{7}{9}$ is less than 1.

When the numerator is *greater* than the denominator, the fraction is *greater* than 1. For example, $\frac{11}{3}$ is greater than 1, for $\frac{11}{3}$ is greater than $\frac{3}{3}$; but $\frac{3}{3}$ is equal to 1, therefore, $\frac{11}{3}$ is greater than 1.

A PROPER FRACTION is one whose numerator is less than its denominator; as $\frac{3}{7}$.

An IMPROPER FRACTION is one whose numcrator is *not* less than its denominator; as $\frac{6}{5}$, $\frac{9}{4}$.

NUMBERS that are not fractional are called INTEGERS, or WHOLE NUMBERS, to distinguish them from fractions.

A MIXED NUMBER consists of a *whole number* and a *fraction*, and is expressed by writing the whole number before the fraction; as $5\frac{3}{10}$.

Since a fraction may be divided into a number of equal parts, and any number of these parts be taken, we obtain an idea of the fraction of a fraction. *Two-thirds* of *five-eighths*, written $\frac{2}{3}$ of $\frac{5}{8}$, means that the fraction $\frac{5}{8}$ is to be divided into *three* equal parts, and that *two* of them are taken. Such a fraction is termed a *compound fraction.*

A COMPOUND FRACTION, then, is the fraction of a fraction, or any number of fractions connected by the word *of;* as $\frac{3}{7}$ of $\frac{5}{11}$ of $\frac{6}{13}$.

A COMPLEX FRACTION is one which has a fraction or mixed number for its numerator, or denominator, or both; as $\frac{3\frac{2}{5}}{7}$, $\frac{6}{4\frac{1}{2}}$, $\frac{5\frac{4}{7}}{9\frac{3}{4}}$.

Note. An integer may be reduced to a fractional form by putting 1 for its denominator; thus, 3, when changed to a fractional form, becomes $\frac{3}{1}$.

One whole number may be considered as the fraction of another; thus, we say 3 is the *fourth* of 12, that 5 is the *five-sevenths* of 7, &c.

REDUCTION OF VULGAR FRACTIONS.

I. To reduce a Fraction (as $\frac{36}{84}$) to its lowest terms.

Rule.—1. Divide the numerator and denominator by any number that will divide both without a remainder, thus making an equivalent fraction in lower terms.

2. Divide this new fraction in a similar way; and continue the process till the fraction cannot be reduced any lower: this last fraction is the answer.

Or, when a divisor cannot be readily got, find the *greatest common measure* (see page 77) of the numerator and denominator; then divide both by it, and the result will be the fraction in its lowest terms.

Example.—Reduce $\frac{36}{84}$ to its lowest terms.

$$\overset{(4)}{\frac{36}{84}} = \overset{(3)}{\frac{9}{21}} = \frac{3}{7}$$

Pupil. 36 and 84 are both divisible by 4; I divide therefore by 4, and obtain the fraction $\frac{9}{21}$ in lower terms. Again, 9 and 21 are both divisible by 3; I divide by 3, and obtain $\frac{3}{7}$, which is equal to $\frac{36}{84}$ in its lowest terms.

$$12 \left| \frac{36}{84} \right| \frac{3}{7}$$

Or, since 12 is the greatest common measure of 36 and 84, I divide both the terms by 12, and the resulting fraction, $\frac{3}{7}$, is in its lowest terms.

The rules are founded on the principle formerly stated, that when both the terms of a fraction are divided by the same number, the value of the fraction is not altered.

The division of the terms of a fraction by any **measure** is called *cancelling*.

In applying the Rule, the following remarks will be found useful: 1st, If the terms of the given fractions are *even* numbers, *cancel* by 2. 2d, If the number expressed by the last *two* figures of the terms of the fraction is divisible by 4, *cancel* by 4; if the number expressed by the last *three* figures of the terms is measured by 8, *cancel* by 8. 3d, If both the terms end in 5, or one in 5, and the other in 0, *cancel* by 5. 4th, If both the terms end in ciphers, cut an equal number from both. 5th, If the sum of the figures in each be measured by 3, *cancel* by 3; if by 9, *cancel* by 9.

These properties of numbers will be found demonstrated in **Algebra**.

When the terms of a fraction are resolved into factors, the *cancelling* is performed by dividing a factor in the numerator and a factor in the denominator by the same number; a dash is drawn through the cancelled factors, and the quotients written above or below them, thus:

$$\frac{6 \times \overset{\overset{2}{\cancel{4}}}{\cancel{16}} \times \overset{4}{\cancel{20}}}{\cancel{4} \times \cancel{5} \times \cancel{2} \times 7} = \frac{48}{7}$$

The factor 16 in the numerator is divisible by 4, a factor in the denominator; draw lines through 16 and 4, and write the quotient 4 above 16. Do the same thing with the 5 and 20; and with 2 and 4. Multiply together all those factors in the numerator that are not cancelled, and write the product

48 for the numerator; and all those in the denominator, and write the product 7 in the denominator.

Exercises.

Reduce the following fractions to their simplest terms :

1. $\frac{64}{128}$,	Ans. $\frac{1}{2}$.	6. $\frac{171}{189}$,	Ans. $\frac{19}{21}$.
2. $\frac{120}{360}$,	" $\frac{1}{3}$.	7. $\frac{142857}{999999}$,	" $\frac{1}{7}$.
3. $\frac{785}{3250}$,	" $\frac{157}{650}$.	8. $\frac{964194}{1139502}$,	" $\frac{11}{13}$.
4. $\frac{875}{1000}$,	" $\frac{7}{8}$.	9. $\frac{855}{1881}$,	" $\frac{5}{11}$.
5. $\frac{3200}{5640}$,	" $\frac{80}{141}$.	10. $\frac{1702}{1886}$,	" $\frac{37}{41}$.

II. To reduce a mixed number (as $2\frac{3}{5}$) to a fractional Form.

RULE.—Multiply the integer by the *denominator* of the fraction, and to the product add the numerator; then write below this sum the denominator of the fraction.

Example.—Reduce $4\frac{3}{7}$ to its fractional form.

$$\begin{array}{l} 4\frac{3}{7} \\ 7 \\ \hline 31 \\ \hline 7 \end{array}$$

Pupil. Since 7 sevenths make one unit, therefore, to change any number of units to *sevenths*, I must multiply by 7; therefore, 4 units multiplied by 7 gives 28 *sevenths*, and taking in the 3 *sevenths*, the sum is 31 *sevenths*, which is written $\frac{31}{7}$.

Exercises.

11. Reduce $9\frac{3}{5}$, $16\frac{4}{9}$, $10\frac{5}{11}$, to their fractional forms,
Ans. $\frac{48}{5}$, $\frac{148}{9}$, $\frac{115}{11}$.

12. " $14\frac{5}{99}$, $12\frac{11}{17}$, $115\frac{3}{125}$, to their fractional forms,
Ans. $\frac{1391}{99}$, $\frac{215}{17}$, $\frac{14378}{125}$.

13. " $9\frac{3}{8}$, $15\frac{12}{31}$, $365\frac{1}{4}$ to their fractional forms,
Ans. $\frac{75}{8}$, $\frac{477}{31}$, $\frac{1461}{4}$.

III. To reduce an improper Fraction (as $\frac{17}{6}$) to a whole or a mixed number.

RULE.—Divide the numerator by the denominator, and the quotient is either a whole or a mixed number, according as the numerator is divisible by the denominator exactly, or not.

Example.—Reduce $\frac{17}{6}$ to a mixed number.

$$\begin{array}{r} 6)17 \\ \hline 2\frac{5}{6} \end{array} \quad \textit{Answer.}$$

Exercises.—Reduce the following to whole or mixed numbers:

14. $\frac{12}{6}$,	Ans. 2	16. $\frac{651}{7}$,	Ans. 93	18. $\frac{213}{95}$,	Ans. $2\frac{23}{95}$
15. $\frac{319}{5}$,	" $68\frac{4}{5}$	17. $\frac{347}{12}$,	" $28\frac{11}{12}$	19. $\frac{476}{17}$,	" 28

IV. To reduce a Compound to a Simple Fraction.

Rule.—Multiply all the numerators together for the numerator, and all the denominators together for the denominator of the simple fraction. Reduce, when necessary, the resulting fraction to its lowest terms.

When possible, *cancel* before multiplying.

If part of the compound fraction be whole or mixed numbers, they must be changed to their fractional forms before the rule can be applied.

Example 1.—Reduce $\frac{5}{7}$ of $\frac{3}{11}$ to a simple fraction.

$$\frac{5 \times 3}{7 \times 11} = \frac{15}{77}$$

In this example the numerators are 5 and 3, which, multiplied together, give 15 for the numerator of the simple fraction; and the denominators 7 and 11, multiplied together, give 77 for its denominator.

Example 2.—Reduce $\frac{4}{7}$ of $\frac{14}{12}$ of $\frac{6}{5}$ of $\frac{10}{13}$ to a simple fraction.

$$\frac{\cancel{4}}{\cancel{7}} \times \frac{\overset{2}{\cancel{14}}}{\underset{\cancel{3}}{\cancel{12}}} \times \frac{\overset{2}{\cancel{6}}}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{13} = \frac{8}{13}$$

First cancel by 4, then by 7, then by 3, and then by 5.

Exercises.

20. Reduce $\frac{3}{7}$ of $\frac{5}{9}$, $\frac{1}{2}$ of $\frac{6}{9}$, $\frac{2}{3}$ of $\frac{3}{5}$, to simple fractions, Ans. $\frac{5}{21}$, $\frac{1}{3}$, $\frac{2}{5}$.
21. " $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{7}{11}$ of $\frac{11}{5}$ of $\frac{5}{7}$ to a simple fraction, Ans. $\frac{3}{7}$.
22. " $\frac{2}{3}$ of $5\frac{1}{2}$ of 6, $\frac{3}{7}$ of $11\frac{3}{4}$ of $10\frac{1}{2}$, to simple fractions, Ans. $\frac{22}{1}$, $\frac{423}{8}$.
23. " $\frac{3}{5}$ of $4\frac{1}{6}$ of $\frac{4}{9}$ of $14\frac{4}{7}$ to a simple fraction, Ans. $\frac{340}{21}$.

It may be proper to remind the pupil that the answers are always in the lowest terms.

V. To reduce a Complex to a Simple Fraction.

Rule.—Reduce both the numerator and denominator of the complex fraction to their fractional forms, then multiply the numerator of the *upper* fraction by the denominator of the *lower*, for the *numerator* of the simple fraction, and the numerator of the *lower* by the denominator of the *upper*, for the *denominator* of the simple fraction.

Or, multiply both numerator and denominator by the least common multiple of the denominators of the given fractions. Change the simple fraction, when necessary, to its lowest terms.

Example.—Reduce the complex fractions $\frac{5\frac{1}{2}}{4}$, and $\frac{6\frac{1}{3}}{3\frac{1}{7}}$, to simple fractions.

$$\text{Here } \frac{5\frac{1}{2}}{4} = \frac{\frac{11}{2}}{\frac{4}{1}} = \frac{11}{8}, \text{ and } \frac{6\frac{1}{3}}{3\frac{1}{7}} = \frac{\frac{19}{3}}{\frac{22}{7}} = \frac{133}{66}.$$

$$\text{Or, } \frac{5\frac{1}{2} \times 2}{4 \times 2} = \frac{11}{8}, \text{ and } \frac{6\frac{1}{3} \times 21}{3\frac{1}{7} \times 21} = \frac{133}{66}.$$

The Reason of the first method is the same as that for the division of fractions, and the reason of the second is obvious.

Exercises.—Reduce the following complex to simple fractions:

24. $\frac{3\frac{1}{3}}{10}$, $\frac{4\frac{3}{7}}{31}$, $\frac{6}{25\frac{1}{2}}$, ... Ans. $\frac{1}{3}$, $\frac{1}{7}$, $\frac{4}{17}$.

25. $\frac{7\frac{1}{9}}{8\frac{8}{11}}$, $\frac{17\frac{1}{2}}{25\frac{3}{7}}$, $\frac{8\frac{1}{4}}{12\frac{2}{3}}$, ... Ans. $\frac{22}{27}$, $\frac{245}{356}$, $\frac{99}{152}$.

VI. To convert Fractions having *different* Denominators, to equivalent Fractions having a *common* Denominator.

Rule.—1. Find the least common multiple (p. 78) of all the denominators of the different fractions.

2. Divide this common multiple by the *denominator* of the first fraction, and then multiply *both* terms of the fraction by the quotient. Do the same thing with all the other fractions in succession.

When the process is completed, equivalent fractions have been obtained, having a common denominator.

Mixed numbers must be changed to their fractional form, compound and complex fractions to simple ones, before the rule can be applied.

Example.—Convert $\frac{5}{9}$, $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{11}{12}$ to fractions having a common denominator.

$$\frac{5}{9} \times \frac{8}{8} = \frac{40}{72}$$

$$\frac{2}{3} \times \frac{24}{24} = \frac{48}{72}$$

$$\frac{7}{8} \times \frac{9}{9} = \frac{63}{72}$$

$$\frac{11}{12} \times \frac{6}{6} = \frac{66}{72}$$

Here the least common multiple of the denominators 9, 3, 8, 12, is found to be 72.*

We then divide 72 by 9, the denominator of $\frac{5}{9}$, and multiply the terms of the fraction by 8, the quotient, which gives $\frac{40}{72}$; 72 is next divided by 3, the denominator of the second fraction, and the terms of the fraction are multiplied by 24, the quotient, which gives $\frac{48}{72}$; in the same manner, 72 is divided successively by 8 and 12, and the terms of the third and fourth fractions are multiplied by the quotients 9 and 6, which gives the equivalent fractions $\frac{63}{72}$ and $\frac{66}{72}$.

* *Note.*—It is to be observed that this least common multiple is also the *common* denominator of the fractions.

Reason of the Rule.—In the example, it is obvious that the equivalent fractions have a common denominator; and also that the original fractions have not been altered in value by the process, because the terms of each fraction have been multiplied by the same number (p. 81); hence the Rule.

Exercises.

Convert to fractions having a common denominator:

26. $\frac{2}{3}$, $\frac{2}{7}$, and $\frac{1}{2}$, . . . Ans. $\frac{28}{42}$, $\frac{12}{42}$, $\frac{21}{42}$.
27. $\frac{7}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$, . . " $\frac{21}{24}$, $\frac{16}{24}$, $\frac{18}{24}$, $\frac{20}{24}$.
28. $\frac{1}{3}$, $\frac{7}{12}$, $\frac{2}{9}$, and $\frac{11}{18}$, . . " $\frac{12}{36}$, $\frac{21}{36}$, $\frac{8}{36}$, $\frac{22}{36}$.
29. $\frac{5}{6}$, $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{15}{16}$, . " $\frac{40}{48}$, $\frac{18}{48}$, $\frac{12}{48}$, $\frac{45}{48}$.
30. $\frac{5}{12}$, $\frac{2}{9}$, $\frac{17}{48}$, and $\frac{37}{72}$, . " $\frac{60}{144}$, $\frac{32}{144}$, $\frac{51}{144}$, $\frac{74}{144}$.
31. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{1}{12}$, . " $\frac{6}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, $\frac{1}{12}$.
32. $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{9}$, and $\frac{5}{11}$, . " $\frac{1155}{3465}$, $\frac{1386}{3465}$, $\frac{1485}{3465}$, $\frac{1540}{3465}$, $\frac{1575}{3465}$.
33. $\frac{7}{8}$, $\frac{1}{12}$, $\frac{23}{24}$, and 4, . " $\frac{21}{24}$, $\frac{2}{24}$, $\frac{23}{24}$, $\frac{96}{24}$.
34. $\frac{1}{2}$ of $\frac{3}{7}$, $\frac{6}{11}$ of $\frac{10}{15}$ of $7\frac{1}{4}$, . " $\frac{33}{154}$, $\frac{406}{154}$.
35. $\frac{7}{11}$, $4\frac{1}{2}$, $\frac{2}{33}$, and $7\frac{3}{8}$, . " $\frac{168}{264}$, $\frac{1188}{264}$, $\frac{16}{264}$, $\frac{1947}{264}$.

VII. To convert a Fraction of one Denomination into an equivalent Fraction of another Denomination.

Rule.—Ascertain how many of the smaller denomination make *one* of the greater: if the conversion is from a higher to a lower denomination, multiply the *numerator* of the fraction by that number; if from a lower to a higher, multiply the *denominator*.

Convert the resulting fraction, when necessary, to its lowest terms.

Example 1.—Convert $\frac{3}{7}$ of a pound to the fraction of a penny.

$$\frac{3 \times 240}{7} = \frac{720}{7} \text{ of } 1d. \textit{ Ans.}$$

Here, as the change is from a higher to a lower, we multiply the *numerator* by 240, the number of pence in a pound.

Example 2.—Convert $\frac{5}{9}$ of a farthing to the fraction of a guinea.

$$\frac{5}{9 \times 1008} = \frac{5}{9072} \text{ guin.}$$

Here, as the change is from a lower to a higher, we multiply the *denominator* by 1008, the number of farthings in a guinea.

Exercises.

36. Convert $\frac{3}{5}$ of a pound to the fraction of a farthing, Ans. $\frac{576}{1}$ f.
37. " $\frac{2}{9}$ of a shilling to the fraction of a farthing, Ans. $\frac{32}{3}$ f.
38. " $\frac{3}{7}$ of a guinea to the fraction of a penny, . Ans. $\frac{108}{1}$ *d.*
39. " $\frac{5}{6}$ of a penny to the fraction of a pound, . Ans. $\frac{1}{288}$ £.
40. " $\frac{4}{11}$ of a guinea to the fraction of a pound, Ans. $\frac{21}{55}$ £.
41. " $\frac{7}{12}$ of a farthing to the fraction of a pound, Ans. $\frac{7}{11520}$ £.
42. " $\frac{3}{5}$ of a pole to the fraction of an acre, . Ans. $\frac{3}{800}$ ac.
43. " $\frac{5}{21}$ of a grain to the fraction of a lb., . Ans. $\frac{1}{24192}$ lb.
44. " $\frac{3}{11}$ of a cwt. to the fraction of a lb., . Ans. $\frac{336}{11}$ lb.
45. " $\frac{5}{13}$ of a lb. to the fraction of a ton, . Ans. $\frac{1}{5824}$ tons.

VIII. To convert a Compound Number (as 14*s.* 6*d.*) to the Fraction of any Denomination of the same kind.

Rule.—Convert the given sum into its lowest denomination, and place it as the numerator; then convert *one* of the proposed denomination into the same denomination as the other, and place it as the denominator: the resulting fraction may then be reduced to its lowest terms.

Example.—Convert 5*s.* 3¼*d.* to the fraction of a pound.

$$\left.\begin{array}{ll} 5s.\ 3\tfrac{1}{4}d.\text{, reduced to farthings, its lowest denomination} & = 253 \\ \pounds 1,\quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} & = 960 \end{array}\right\} \frac{253}{960} \text{ of } \pounds 1.$$

This rule is the same in principle as the last: for 5*s.* 3¼*d.* = 253 farthings = $\frac{253}{1}$ farthings; and to change this fraction to the fraction of a pound, I divide by 4, 12, and 20;

therefore, $5s.\ 3\tfrac{1}{4}d. = \dfrac{253}{1 \times 4 \times 12 \times 20} = \dfrac{253}{960}$ £.

Exercises.

46. Convert 14*s.* 7½*d.* to the fraction of a pound, . Ans. $\frac{117}{160}$ £.
47. " 16*s.* 8*d.* to the fraction of a pound, . Ans. $\frac{5}{6}$ £.
48. " 17*s.* 3¼*d.* to the fraction of a guinea, Ans. $\frac{829}{1008}$.
49. " 2 oz. 3 dwt. 12½ gr. to the fraction of a lb., Ans. $\frac{2089}{11520}$.
50. " 23 lb. 13⅕ oz. to the fraction of a cwt., Ans. $\frac{953}{4480}$.
51. " 5 hr. 48 min. 48 sec. to the fraction of a day, Ans. $\frac{109}{450}$.
52. " 3 ro. 14 po. to the fraction of an acre, . Ans. $\frac{67}{80}$.
53. " 4*s.* 4¾*d.* to the fraction of a shilling, . Ans. $\frac{211}{48}$.
54. " 1 bush. 3 pecks to the fraction of a qr., Ans. $\frac{7}{32}$.

IX. To find the Value of a Fraction of a given Denomination.

Rule.—Reckon the numerator of the fraction as so many of the given denomination, and then divide it by the denominator, as in Compound Division.

Note.—The pupil ought to remember that a fraction only *indicates* the division of the numerator by the denominator (p. 81); when this indicated division is *actually performed*, the fraction is said to be *valued*.

Example.—What is the value of $\frac{2}{7}$ of a pound?

£2	
20	Here the 2 is reckoned as £2, and divided
7)40	by 7, as in Compound Division.
5s. $8\frac{1}{2}$*d.* $\frac{2}{7}$. *Ans.*	

Exercises.

55.	Find the value of $\frac{2}{3}$	of a	shilling,	Ans.	8*d.*
56.	〃 〃 $\frac{5}{8}$	〃	pound,	〃	12*s.* 6*d.*
57.	〃 〃 $\frac{5}{24}$	〃	〃	〃	4*s.* 2*d.*
58.	〃 〃 $\frac{3}{5}$	〃	shilling,	〃	$7\frac{0}{4}$*d.* $\frac{4}{5}$.
59.	〃 〃 $\frac{3}{11}$	〃	pound,	〃	5*s.* $5\frac{1}{4}$*d.* $\frac{9}{11}$.
60.	〃 〃 $\frac{4}{7}$	〃	shilling,	〃	$6\frac{3}{4}$*d.* $\frac{3}{7}$.
61.	〃 〃 $\frac{3}{11}$	〃	pound,	〃	5*s.* $5\frac{1}{4}$*d.* $\frac{9}{11}$.
62.	〃 〃 $\frac{2}{9}$	〃	guinea,	〃	4*s.* 8*d.*
63.	〃 〃 $\frac{7}{9}$	〃	ton,	〃	15 cwt. 2 qr. $6\frac{2}{9}$ lb.
64.	〃 〃 $\frac{113}{51}$	〃	pound,	〃	£2, 4*s.* $3\frac{3}{4}$*d.* $\frac{1}{17}$.
65.	〃 〃 $\frac{3}{16}$	〃	acre,	〃	30 poles.
66.	〃 〃 $\frac{11}{18}$	〃	day,	〃	14 hr. 40 min.
67.	〃 〃 $\frac{5}{9}$	〃	quarter,	〃	4 bush. $1\frac{7}{9}$ pk.
68.	〃 〃 $\frac{20}{13}$		mile,	Ans.	1 m. 4 fur. 67 yd. $2\frac{1}{13}$ ft.

69. What is the $\frac{5}{19}$ of £26, 17*s.* 5*d.*, . Ans. £7, 1*s.* $5\frac{0}{4}$*d.* $\frac{8}{19}$.

70. The height of Arthur's Seat is 821 feet: if a person has ascended five-sevenths of this height, through what space must he still ascend before he arrive at the top? . Ans. $234\frac{4}{7}$ feet.

71. A person's debts amounted to £654, 13*s.* 4*d.*; the elder of two sons agrees to pay $\frac{5}{8}$ of the amount, the second son the other $\frac{3}{8}$. How much did each son pay?

Ans. The elder pays £409, 3*s.* 4*d.*; the younger, £245, 10*s.*

ADDITION OF VULGAR FRACTIONS.

RULES.—I. When the fractions have a common denominator; add together the numerators, and under the sum write the common denominator. If this sum is an improper fraction, reduce it to a whole or a mixed number.

II. When the fractions have not a common denominator; convert them to equivalent fractions having a common denominator, then add the numerators as before.

III. When mixed numbers are to be added; first add the fractions, then the integers.

IV. When the fractions are not of the same denomination; find their values (by Rule IX., p. 89), and add as in Compound Addition.

Or, convert the fractions to equivalent ones of the same denomination; then convert the resulting fractions to a common denominator, and proceed as before.

THE REASON of the rule is obvious from the following examples:

Example 1.—Add together $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$. In this example the fractions have a common denominator, the numerators therefore express units of the same kind, and consequently can be added together. Hence—$\frac{3}{8}+\frac{5}{8}+\frac{7}{8}=\frac{15}{8}=1\frac{7}{8}$.

Example 2.—Find the sum of the fractions $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{9}{16}$.

$$\begin{aligned} \tfrac{3}{4} \times \tfrac{12}{12} &= \tfrac{36}{48} \\ \tfrac{5}{6} \times \tfrac{8}{8} &= \tfrac{40}{48} \\ \tfrac{9}{16} \times \tfrac{3}{3} &= \tfrac{27}{48} \\ \hline \tfrac{103}{48} &= 2\tfrac{7}{48} \end{aligned}$$

Pupil. In this example the fractions have not a common denominator, and consequently must be changed to fractions having a common denominator, that they may be numbers of the same name. Converting them, therefore, to a common denominator, and adding the numerators together, I find the sum is equal to $\frac{103}{48}$; the answer being an improper fraction, is converted into the mixed number, $2\frac{7}{48}$.

Example 3.—Add together $15\frac{3}{8}$, $2\frac{11}{12}$, and $27\frac{1}{4}$.

$$\begin{aligned} 15\tfrac{3}{8} &= \tfrac{9}{24} \\ 2\tfrac{11}{12} &= \tfrac{22}{24} \\ 27\tfrac{1}{4} &= \tfrac{6}{24} \\ \hline 45\tfrac{13}{24} \quad & \tfrac{37}{24} = 1\tfrac{13}{24} \end{aligned}$$

Pupil. Here I first add the fractions, and find their sum to be equal to $1\frac{13}{24}$; I therefore put down $\frac{13}{24}$ under the fractions, and carrying 1 to the whole numbers, I find the sum required to be $45\frac{13}{24}$.

Example 4.—Add together £$\frac{3}{7}$, $\frac{5}{9}$*s.* and $\frac{2}{3}$ guin.

	£	*s.*	*d.*	
£$\frac{3}{7}$ =	0	8	$6\frac{3}{4}\;\frac{3}{7}$	$= \frac{9}{21}$
$\frac{5}{9}$*s.* =	0	0	$6\frac{1}{2}\;\frac{2}{3}$	$= \frac{14}{21}$
$\frac{2}{3}$ guin. =	0	14	0	
	1	3	$1\frac{1}{2}\;\frac{2}{21}$	$\frac{23}{21} = 1\frac{2}{21}$

Pupil. Here I find the value of £$\frac{3}{7}$ = 8*s.* $6\frac{3}{4}$*d.* $\frac{3}{7}$—of $\frac{5}{9}$*s.* = $6\frac{1}{2}$*d.* $\frac{2}{3}$—of $\frac{2}{3}$ guin. = 14*s.* I now add the fractions, and the rest as in addition of compound numbers.

	£				
$£\frac{3}{7} =$	$\frac{540}{1260}$				
$\frac{5}{9}s. = £\frac{1}{36} =$	$\frac{35}{1260}$				
$\frac{2}{3}g. = £\frac{7}{10} =$	$\frac{882}{1260}$	£	s.	d.	
	$\frac{1457}{1260}$)1457(	1	3	$1\frac{1}{2}$	$\frac{2}{21}$

Or thus: I convert $\frac{5}{9}s.$ to the fraction of a pound, and find it to be equal to $£\frac{1}{36}$; then $\frac{2}{3}$g. to the fraction of a pound, and find it equal to $£\frac{7}{10}$. I now change the fractions $\frac{3}{7}$, $\frac{1}{36}$, and $\frac{7}{10}$, to a common denominator, then add them together, and find the sum to be $£\frac{1457}{1260}$, which, valued, gives £1, 3s. $1\frac{1}{2}d.\ \frac{2}{21}$, the same as before.

Exercises.

1.	$\frac{3}{7}+\frac{3}{7}+\frac{5}{7}+\frac{6}{7}$, . . .	Ans. $2\frac{3}{7}$.
2.	$\frac{1}{3}+\frac{2}{9}+\frac{3}{8}+\frac{5}{16}$, . . .	" $1\frac{35}{144}$.
3.	$\frac{9}{11}+\frac{7}{14}+\frac{3}{28}+\frac{5}{22}$, . .	" $1\frac{201}{308}$.
4.	$\frac{1}{4}+\frac{3}{8}+\frac{5}{6}+\frac{23}{24}$, . . .	" $2\frac{5}{12}$.
5.	$\frac{2}{3}$ of $\frac{1}{7}+\frac{3}{7}$ of $\frac{5}{12}$ of $4\frac{1}{2}$, . .	" $\frac{151}{168}$.
6.	$12\frac{1}{2}+24\frac{3}{4}+17\frac{5}{8}$, . . .	" $54\frac{7}{8}$.
7.	$19\frac{5}{6}+203\frac{7}{9}+151\frac{23}{36}$, . .	" $375\frac{1}{4}$.
8.	$23\frac{5}{8}+9\frac{3}{10}+17\frac{1}{20}+\frac{39}{40}$, . .	" $50\frac{19}{20}$.
9.	$£\frac{5}{12}+\frac{3}{7}s.+\frac{5}{6}$ guin. . .	" £1, 6s. $3\frac{0}{4}d.\ \frac{4}{7}$.
10.	$\frac{5}{8}$ lb. $+\frac{6}{7}$ oz. $+\frac{3}{5}$ dwt. $+\frac{1}{7}$ gr. .	" 8 oz. 7 dwt. $17\frac{34}{35}$ gr.
11.	$12\frac{3}{4}$ ac. $+1\frac{7}{8}$ ro. $+19\frac{5}{18}$ po. .	" 13 ac. 1 ro. $14\frac{5}{18}$ po.
12.	$6\frac{2}{3}$ bush. $+2\frac{1}{6}$ qr. $+3\frac{1}{12}$ pk. .	" 3 qr. 0 bush. $1\frac{3}{4}$ pk.

SUBTRACTION OF VULGAR FRACTIONS.

RULES.—I. When the fractions have a common denominator; subtract the less numerator from the greater, and under the difference write the common denominator. The resulting fraction may then be reduced to its lowest terms.

II. When the fractions have not a common denominator; convert them to equivalent fractions having a common denominator, then subtract as before.

III. When mixed numbers are given; convert the fractions, when necessary, to equivalent fractions having a common denominator, subtract, if possible, the lower numerator from the upper, and under the difference write the common denominator. Find the difference of the integers, as in Simple Subtraction.

But if the lower exceeds the upper numerator, add the *upper* numerator and denominator together;* from the sum subtract

* This is merely an easy way of *borrowing* 1 integer and converting it to the denominator of the given fraction, on the same principle as in Compound Subtraction. The 1 is *carried* to the integer in the lower line for the same reason.

the *lower* numerator, and write the common denominator below the remainder. Then *carry* 1 to the integer in the lower line, and subtract the sum from the integer in the upper line.

IV. When the fractions are of different denominations, find their values, then proceed as in Compound Subtraction.

THE REASON of the rule is evident from the illustrations given under Simple Subtraction, and Addition of Fractions.

Examples.

1.	2.	3.	4.	5.
From $\frac{5}{9}$	$\frac{2}{3} = \frac{8}{12}$	$84\frac{11}{12} = \frac{22}{24}$	$21\frac{1}{7} = \frac{4}{28}$	£$\frac{3}{5}$ = 12*s*. 0*d*.
Take $\frac{2}{9}$	$\frac{5}{12} = \frac{5}{12}$	$26\frac{3}{8} = \frac{9}{24}$	$16\frac{3}{4} = \frac{21}{28}$	$\frac{5}{9}$*s*. = 0 $6\frac{1}{2}\,\frac{2}{3}$
Rem. $\frac{3}{9} = \frac{1}{3}$	$\frac{3}{12} = \frac{1}{4}$	$58\frac{13}{24}$ $\frac{13}{24}$	$4\frac{11}{28}$ $\frac{11}{28}$	11 $5\frac{1}{4}\,\frac{1}{3}$

Exercises.

1.	From	$\frac{5}{12}$ take $\frac{4}{15}$, . . .	Ans.	$\frac{3}{20}$.
2.	"	$\frac{2}{3}$ take $\frac{3}{5}$, . .	"	$\frac{1}{15}$.
3.	"	$12\frac{1}{2}$ take $9\frac{3}{4}$, . .	"	$2\frac{3}{4}$.
4.	"	$29\frac{5}{8}$ take $16\frac{5}{9}$, .	"	$13\frac{5}{72}$.
5.	"	$\frac{2}{3}$ of $\frac{9}{10}$ take $\frac{1}{2}$ of $\frac{3}{7}$, .	"	$\frac{27}{70}$.
6.	"	49 take $23\frac{5}{8}$, .	"	$25\frac{3}{8}$.
7.	"	$50\frac{1}{7}$ take $18\frac{3}{5}$, . .	"	$31\frac{19}{35}$.
8.	"	30 take $22\frac{5}{11}$, .	"	$7\frac{6}{11}$.
9.	"	£$\frac{3}{7}$ take $\frac{2}{9}$ guin. .	"	3*s*. $10\frac{3}{4}$*d*. $\frac{3}{7}$.
10.	"	£$13\frac{4}{11}$ take £$8\frac{7}{12}$,	"	£4, 15*s*. $7\frac{1}{4}$*d*. $\frac{1}{11}$.
11.	"	$8\frac{3}{8}$ ton take $19\frac{5}{13}$ cwt.	"	7 ton 8 cwt. 0 qr. $12\frac{12}{13}$ lb.
12.	"	$18\frac{3}{5}$ ac. take $2\frac{3}{17}$ ro.	"	18 ac. 0 ro. $8\frac{16}{17}$ po.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.—Multiply all the numerators of the given fractions together, for the numerator, and all the denominators together for the denominator of the product; when necessary, reduce the resulting fraction to its lowest terms.

Cancel when possible. Compound and complex fractions must be changed to simple ones, and mixed numbers to their fractional form, before the rule can be applied.

If a fraction be multiplied by its denominator, the product is the numerator; thus, $\frac{7}{13}$ multiplied by 13 becomes $\frac{7 \times 13}{13}$; and cancelling by 13, the product is 7, the numerator of the fraction.

A fraction is multiplied by an integer, by multiplying its numerator only: thus $\frac{3}{12} \times 6 = \frac{18}{12} = 1\frac{1}{2}$: also, by dividing its denominator by the integer, thus $\frac{3}{12} \div 6 = \frac{3}{2}$, or $1\frac{1}{2}$.

The result of multiplying by a proper fraction is to *lessen* the multiplicand. This will be obvious if we consider that to multiply by 1 does not increase a number, but leaves it the same as before, and consequently to multiply by *less* than 1—that is, by a fraction—must diminish it.

Example 1.—Multiply $\frac{4}{7}$ by $\frac{5}{9}$.

$$\frac{4 \times 5}{7 \times 9} = \frac{20}{63}$$

Pupil. Here I am required to multiply $\frac{4}{7}$ by $\frac{5}{9}$. I first multiply $\frac{4}{7}$ by 5: that is, I put 5 as a factor above the line; but since the multiplier is not 5, but the $\frac{1}{9}$ of 5, this product must be 9 times too *great*; hence, to obtain the true product, I must divide by 9; that is, I put 9 as a factor below the line. Hence the Rule, Multiply, &c.

Example 2.—Multiply $17\frac{3}{4}$ by $14\frac{2}{5}$.

$$\frac{71}{\cancel{4}} \times \frac{\overset{18}{\cancel{72}}}{5} = \frac{1278}{5} \qquad 5\,)\,1278 \;\; \underline{\quad} \;\; 255\frac{3}{5}$$

Pupil. These mixed numbers, when changed to their fractional form, become $\frac{71}{4}$ and $\frac{72}{5}$; hence, indicating their multiplication, and cancelling by 4, then multiplying out, I find the product to be $\frac{1278}{5}$; and valuing this fraction, the product becomes $255\frac{3}{5}$.

Exercises.

1. Multiply 728354 by $16\frac{2}{3}$, $123\frac{5}{9}$, $75\frac{4}{11}$,
   Ans. $12189288\frac{1}{3}$, $89992183\frac{1}{9}$, 54891406.
2. 〃 964758 by $27\frac{11}{12}$, $76\frac{6}{19}$, $43\frac{11}{23}$,
   Ans. $26932827\frac{1}{2}$, $73626268\frac{8}{19}$, 41946000.
3. 〃 $\frac{5}{9}$ by $\frac{6}{7}$, ... Ans. $\frac{10}{21}$.
4. 〃 $\frac{3}{4}$ of $\frac{3}{5}$ by $12\frac{1}{2}$, ... 〃 $5\frac{5}{8}$.
5. 〃 $\frac{7}{11}$ of $\frac{22}{23}$ by $\frac{1}{3}$ of $9\frac{3}{5}$, ... 〃 $1\frac{109}{115}$.
6. 〃 $123\frac{5}{7}$ by $26\frac{7}{8}$, ... 〃 $3324\frac{23}{28}$.
7. 〃 $79\frac{3}{16}$ by $9\frac{1}{3}$, ... 〃 $739\frac{1}{12}$.
8. 〃 $23\frac{3}{7}$ by 46, ... 〃 $1077\frac{5}{7}$.
9. 〃 $12\frac{1}{5}$ by $23\frac{2}{9}$, ... 〃 $283\frac{14}{45}$.
10. 〃 £6, 15*s*. $4\frac{1}{2}$*d*. by $19\frac{3}{4}$, . . Ans. £133, 13*s*. $7\frac{3}{4}$*d*. $\frac{1}{2}$.
11. 〃 £12, 17*s*. $5\frac{3}{4}$*d*. by $25\frac{1}{2}$, 〃 328, 5*s*. $8\frac{1}{2}$*d*. $\frac{1}{2}$.
12. 〃 £24, 19*s*. 11*d*. by $35\frac{3}{5}$, . 〃 889, 17*s*. $0\frac{1}{4}$*d*. $\frac{3}{5}$.
13. What is the value of $27\frac{3}{4}$ yd. of broad cloth at £1, 3*s*. $9\frac{1}{2}$*d*. per yd.? . . . . Ans. £33, 0*s*. $2\frac{1}{2}$*d*. $\frac{1}{8}$.
14. What is the price of $26\frac{3}{4}$ qr. of wheat at £2, 10*s*. $3\frac{3}{4}$*d*. per qr.?
    Ans. £67, 5*s*. $10\frac{1}{4}$*d*. $\frac{1}{4}$.
15. What cost 4 cwt. 2 qr. 7 lb. of sugar at 67*s*. $3\frac{1}{4}$*d*. per cwt.?
    Ans. £15, 6*s*. $11\frac{0}{4}$*d*. $\frac{5}{16}$.

16. How many yards are there in $12\frac{3}{4}$ pieces of cloth, each containing $26\frac{1}{2}$ yd.? . . . . . . Ans. $337\frac{7}{8}$ yd.

17. What is the weight of $37\frac{1}{2}$ bags of guano, each bag weighing $2\frac{1}{8}$ cwt.? . . . . . Ans. 79 cwt. 2 qr. 21 lb.

DIVISION OF VULGAR FRACTIONS.

Rule.—Invert the given divisor—thus, if it is $\frac{3}{7}$, write it as $\frac{7}{3}$; then multiply the two fractions together, and the resulting fraction is the quotient: it may then be reduced to its lowest terms.

Before proceeding, convert any mixed numbers into their fractional forms, and compound or complex into simple fractions.

A fraction is divided by an integer, by dividing its numerator only; thus $\frac{8}{9} \div 4 = \frac{2}{9}$: also, by multiplying its denominator by the integer, thus $\frac{8}{9 \times 4} = \frac{8}{36}$, or $\frac{2}{9}$.

The result of dividing by a fraction is to *increase* a number, on the same principle that multiplying lessens it. See Multiplication, page 93.

If the fractions are changed to a common denominator, then the *numerator* of the dividend divided by the *numerator* of the divisor will give the quotient required.

When the product of any two numbers is 1, the one number is called the *reciprocal* of the other; thus $7 \times \frac{1}{7} = 1$, therefore $\frac{1}{7}$ is the reciprocal of 7: $\frac{2}{3} \times \frac{3}{2} = 1$, therefore $\frac{2}{3}$ is the *reciprocal* of $\frac{3}{2}$.

Hence *division* by any number gives the same result as *multiplication* by its reciprocal; thus the division of 7 by 3 is indicated by $\frac{7}{3}$, and the multiplication of 7 by $\frac{1}{3}$, the reciprocal of 3, gives $\frac{7}{3}$, the same as before.

Example.—Divide $\frac{4}{7}$ by $\frac{5}{9}$.

$$\frac{4 \times 9}{7 \times 5} = \frac{36}{35}$$

$$35)36(1\tfrac{1}{35}$$
$$\underline{35}$$
$$\tfrac{1}{35}$$

Pupil. Here I am required to divide $\frac{4}{7}$ by $\frac{5}{9}$. I first divide $\frac{4}{7}$ by 5; that is, I put 5 as a factor below the line; but since my divisor is not 5, but the $\frac{1}{9}$ of 5, this quotient must be 9 times too *little*; hence, to obtain the true quotient, I must multiply by 9; that is, I write 9 as a factor above the line. Hence the Rule, Invert the divisor, &c.

Exercises.

1. Divide 7839562 by $9\frac{1}{2}$, $27\frac{1}{3}$, $81\frac{3}{4}$,
Ans. $825217\frac{1}{19}$, $286813\frac{10}{41}$, $95896\frac{256}{327}$.
2. " 1234567 by $29\frac{5}{21}$, $37\frac{1}{9}$, $26\frac{2}{15}$,
Ans. $42224\frac{371}{614}$, $33266\frac{259}{334}$, $47241\frac{33}{392}$.
3. " $\frac{8}{9}$ by $\frac{2}{3}$, ... Ans. $1\frac{1}{3}$.
4. " $\frac{11}{12}$ by $\frac{22}{23}$, ... " $\frac{23}{24}$.
5. " $\frac{7}{9}$ by $\frac{21}{23}$, ... " $\frac{23}{27}$.
6. " $\frac{7}{9}$ of $4\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{5}{6}$, ... " $6\frac{3}{10}$.
7. " $46\frac{1}{2}$ by $12\frac{3}{4}$, ... " $3\frac{11}{17}$.
8. " $\frac{2}{5}$ of $17\frac{3}{4}$ by $\frac{1}{7}$ of 4, ... " $12\frac{17}{40}$.
9. " £23, 11*s.* $4\frac{1}{2}$*d.* by $4\frac{3}{4}$, . . . " £4, 19*s.* $2\frac{3}{4}$*d.* $\frac{7}{19}$.
10. " £17, 2*s.* 9*d.* by $11\frac{5}{12}$, . . . " £1, 10*s.* $0\frac{1}{4}$*d.* $\frac{7}{137}$.
11. " £5, 2*s.* $3\frac{1}{4}$*d.* by $23\frac{5}{6}$, . . . " £0, 4*s.* $3\frac{1}{4}$*d.* $\frac{139}{143}$.
12. " £29, 15*s.* $11\frac{3}{4}$*d.* by $116\frac{2}{7}$, . . " £0, 5*s.* $1\frac{1}{2}$*d.* $\frac{5}{814}$.
13. If $13\frac{3}{4}$ lb. of tea cost £$6\frac{7}{9}$, what is the value of 1 lb.?
Ans. 9*s.* $10\frac{1}{4}$*d.* $\frac{7}{33}$.
14. What is the value of a gallon of brandy, if $27\frac{3}{4}$ gallons cost £30, 16*s.* 11*d.*? . . . . . Ans. £1, 2*s.* $2\frac{3}{4}$*d.* $\frac{11}{111}$.
15. Divide £$27\frac{11}{19}$ among 12 men and 13 boys, allowing each boy $\frac{1}{3}$ of a man's share,
Ans. A man's share, £1, 13*s.* $9\frac{0}{4}$*d.* $\frac{900}{931}$; a boy's, 11*s.* $3\frac{0}{4}$*d.* $\frac{300}{931}$.
16. If 5 pair of gloves cost $8\frac{12}{13}$*s.*, what is the price per pair?
Ans. 1*s.* $9\frac{1}{4}$*d.* $\frac{43}{65}$.

Miscellaneous Exercises on Fractions.

1. What is the sum of $\frac{5}{12}$, $\frac{4}{15}$, and $\frac{3}{20}$? . . . . . Ans. $\frac{5}{6}$.
2. How many sevenths in nine and three-sevenths?
Ans. 66 sevenths, or $\frac{66}{7}$.
3. What part of £3, 11*s.* 3*d.* is £1, 19*s.* $6\frac{1}{4}$*d.*? . Ans. $\frac{1897}{3420}$.
4. A possesses the $\frac{1}{2}$ of a ship, B the $\frac{1}{3}$, C the $\frac{1}{7}$, and D the rest. What part of the ship belongs to D? . . Ans. $\frac{1}{42}$.
5. What is the price of a yard of cloth, when $15\frac{2}{5}$ yd. cost £$10\frac{3}{7}$?
Ans. 13*s.* $6\frac{1}{2}$*d.* $\frac{50}{539}$.
6. From a cask which contained 20 gallons, there was drawn the $\frac{1}{2}$ of $\frac{2}{11}$ of $34\frac{3}{7}$ gallons. How many gallons remained?
Ans. $16\frac{67}{77}$ gallons.
7. What is the value of $\frac{3}{5}$ lb. at $\frac{11}{15}$*s.* per lb.? . Ans. $5\frac{1}{4}$*d.* $\frac{3}{25}$.
8. If the whole value of a ship is £48000, what sum of money must I pay for $\frac{3}{16}$ of her? . . . . . Ans. £9000.
9. What part of 9 is 2? . . . . Ans. $\frac{2}{9}$.
10. What number multiplied by $\frac{3}{5}$ gives $\frac{4}{9}$? . . Ans. $\frac{20}{27}$.
11. What is the difference between sixteen and a fourth, and sixteen-fourths? . . . . Ans. $12\frac{1}{4}$.
12. A boy gave $\frac{1}{2}$ of an orange to one companion, and $\frac{1}{3}$ of what remained to another. How much did he keep to himself?
Ans. $\frac{1}{3}$.

13. The $\frac{2}{5}$ of a sum of money is £160. What is the sum of money? . . . . Ans. £400.
14. A man can complete a piece of work in 4 days, a boy the same piece of work in 9 days. What part of the work will they accomplish in 2 days, when working together? Ans. $\frac{13}{18}$.
15. What is the value of $\frac{5}{8}$ yd. of silk at £$\frac{17}{18}$ per yard?
Ans. 11*s.* 9½*d.* $\frac{2}{3}$.
16. Divide £276$\frac{5}{8}$ into 12$\frac{5}{12}$ shares, . Ans. £22, 5*s.* 5½*d.* $\frac{2}{149}$.
17. A gentleman who possessed the $\frac{2}{3}$ of a copper-mine, sells $\frac{6}{7}$ of his share for £453. What was the whole mine worth?
Ans. £792, 15*s.*
18. A shipowner sold $\frac{2}{3}$ of $\frac{1}{4}$ of a vessel to one person, and $\frac{6}{7}$ of $\frac{11}{12}$ to another. What part had he remaining? . Ans. $\frac{1}{21}$.
19. If 17¾ lb. of coffee cost £2$\frac{53}{60}$, what is the value per lb.?
Ans. 3*s.* 2¾*d.* $\frac{67}{71}$.
20. What fraction of a cwt. is $\frac{3}{11}$ lb.? . . Ans. $\frac{3}{1232}$ cwt.
21. If a gallon of Cogniac brandy cost 22*s.* 11½*d.*, how much will 27$\frac{3}{5}$ gallons cost at the same rate? Ans. £31, 13*s.* 7¾*d.* $\frac{1}{5}$.
22. A person has read $\frac{2}{9}$ of a book which contains 549 pages. How many pages has he still to read? . . Ans. 427.
23. What is the sum of £2$\frac{3}{11}$, 5$\frac{3}{7}$ guineas, 4$\frac{5}{7}$*s.*, and 11$\frac{3}{8}$*d.*?
Ans. £8, 5*s.* 1¼*d.* $\frac{93}{154}$.
24. A gentleman bequeathed the $\frac{3}{5}$ of his estate, which was worth £7200, to his son, $\frac{2}{7}$ of what remained to his daughter, and the rest to his widow. What sum of money did each receive?
Ans. Son's share, £4320; daughter's, £822, 17*s.* 1½*d.* $\frac{6}{7}$; widow's, £2057, 2*s.* 10¼*d.* $\frac{1}{7}$.
25. Add together the sum, difference, and product of $\frac{3}{17}$ and $\frac{12}{19}$,
Ans. 1$\frac{121}{323}$.
26. Add together 12½, 29$\frac{5}{8}$, 17$\frac{11}{15}$, 36$\frac{29}{30}$, and 302$\frac{4}{5}$, . Ans. 399$\frac{5}{8}$.
27. A minister's glebe consists of 4 fields; the first contains 2$\frac{5}{8}$ ac.; the second, 3½ ac.; the third, 1$\frac{7}{12}$ ac.; the fourth, 5$\frac{3}{32}$ ac. How much does the glebe measure?
Ans. 12 ac. 3 ro. 8⅓ po.
28. What is the value of 1 lb. of silver, when a piece weighing 15$\frac{5}{8}$ lb. is sold for £60, 11*s.* 4*d.*? . Ans. £3, 17*s.* 6¼*d.* $\frac{27}{125}$.
29. *Bradford, March* 11, 1847.—Mr John Anderson bought of Samuel Horsburgh 17$\frac{5}{8}$ yd. of Coburgs at 1*s.* 3½*d.* per yd., 12$\frac{3}{7}$ yd. oriental check at 2*s.* 1¾*d.*, 26$\frac{5}{9}$ yd. Orleans at 1*s.* 11½*d.*, 16½ yd. paramata at 2*s.* 9*d.*, 31¾ yd. French merino at 4*s.* 5½*d.*, 49$\frac{3}{17}$ yd. gros de Naples at 4*s.* 11½*d.*, 35$\frac{7}{19}$ yd. lustring at 7*s.* 5½*d.*, 10$\frac{8}{25}$ yd. watered silk at 10*s.* 3*d.*, 17½ yd. tartan at 11¾*d.* Required the amount of the bill, ... Ans. £45, 19*s.* 3¾*d.* $\frac{15539}{119700}$.
30. If the circumference of the driving-wheel of a locomotive be 16½ feet, how many revolutions will it make between Bristol and Exeter, the distance being 75½ miles?
Ans. 24160.

PRACTICE.

Practice consists of various expeditious methods of calculating the value of goods, &c., by means of fractional or aliquot parts.

It is so named because the Rules are generally employed in actual *practice* for calculating the prices of goods, &c., instead of the rules of Compound Multiplication, as the working is generally easier and shorter.

One number is said to be an *aliquot part* of another, when it is contained in it an exact number of times: thus 4 is an aliquot part of 16—namely, $\frac{1}{4}$—as it is contained in it exactly 4 times.

The calculations in Practice are performed by means of such tables of aliquot parts as the following, which should be carefully committed to memory.

TABLES OF ALIQUOT PARTS.

MONEY.

10/	= $\frac{1}{2}$ of £1	3/4	= $\frac{1}{3}$ of 10/	9*d.*	= $\frac{1}{10}$ of 7/6
6/8	= $\frac{1}{3}$ ″	2/6	= $\frac{1}{4}$ ″	8*d.*	= $\frac{1}{6}$ ″ 4/
5/	= $\frac{1}{4}$ ″	″	= $\frac{1}{2}$ of 5/	″	= $\frac{1}{5}$ ″ 3/4
4/	= $\frac{1}{5}$ ″	2/	= $\frac{1}{5}$ ″ 10/	″	= $\frac{1}{3}$ ″ 2/
3/4	= $\frac{1}{6}$ ″	″	= $\frac{1}{2}$ ″ 4/	″	= $\frac{1}{2}$ ″ 1/4
2/6	= $\frac{1}{8}$ ″	1/8	= $\frac{1}{6}$ ″ 10/	7$\frac{1}{2}$*d.*	= $\frac{1}{8}$ ″ 5/
2/	= $\frac{1}{10}$ ″	″	= $\frac{1}{4}$ ″ 6/8	5*d.*	= $\frac{1}{12}$ ″ 5/
1/8	= $\frac{1}{12}$ ″	″	= $\frac{1}{3}$ ″ 5/	″	= $\frac{1}{8}$ ″ 3/4
1/4	= $\frac{1}{15}$ ″	″	= $\frac{1}{2}$ ″ 3/4	″	= $\frac{1}{6}$ ″ 2/6
1/3	= $\frac{1}{16}$ ″	1/4	= $\frac{1}{5}$ ″ 6/8	″	= $\frac{1}{4}$ ″ 1/8
1/	= $\frac{1}{20}$ ″	″	= $\frac{1}{3}$ ″ 4/	4*d.*	= $\frac{1}{12}$ ″ 4/
6*d.*	= $\frac{1}{40}$ ″	1/3	= $\frac{1}{8}$ ″ 10/	″	= $\frac{1}{10}$ ″ 3/4
6*d.*	= $\frac{1}{2}$ of 1/	″	= $\frac{1}{4}$ ″ 5/	″	= $\frac{1}{6}$ ″ 2/
4*d.*	= $\frac{1}{3}$ ″	″	= $\frac{1}{2}$ ″ 2/6	3*d.*	= $\frac{1}{10}$ ″ 2/6
3*d.*	= $\frac{1}{4}$ ″	10*d.*	= $\frac{1}{12}$ ″ 10/	″	= $\frac{1}{8}$ ″ 2/
2*d.*	= $\frac{1}{6}$ ″	″	= $\frac{1}{8}$ ″ 6/8	″	= $\frac{1}{5}$ ″ 1/3
1$\frac{1}{2}$*d.*	= $\frac{1}{8}$ ″	″	= $\frac{1}{6}$ ″ 5/	1$\frac{1}{2}$*d.*	= $\frac{1}{4}$ ″ 6*d.*
1*d.*	= $\frac{1}{12}$ ″	″	= $\frac{1}{4}$ ″ 3/4	0$\frac{3}{4}$*d.*	= $\frac{1}{8}$ ″ 6*d.*
0$\frac{1}{2}$*d.*	= $\frac{1}{2}$ of 1*d.*	″	= $\frac{1}{3}$ ″ 2/6	″	= $\frac{1}{4}$ ″ 3*d.*
0$\frac{1}{4}$*d.*	= $\frac{1}{4}$ ″	″	= $\frac{1}{2}$ ″ 1/8	″	= $\frac{1}{2}$ ″ 1$\frac{1}{2}$*d.*

The pupil should construct this table for himself, and then commit it to memory.

WEIGHT.

2 qr.	$=\frac{1}{2}$	of 1 cwt.	16 lb.	$=\frac{1}{7}$	of 1 cwt.	7 lb.	$=\frac{1}{16}$	of 1 cwt.
1 qr.	$=\frac{1}{4}$	"	14 lb.	$=\frac{1}{8}$	"	"	$=\frac{1}{8}$	" 2 qr.
14 lb.	$=\frac{1}{2}$	of 1 qr.	"	$=\frac{1}{4}$	of 2 qr.	"	$=\frac{1}{2}$	" 14 lb.
7 "	$=\frac{1}{4}$	"	8 lb.	$=\frac{1}{14}$	" 1 cwt.	4 lb.	$=\frac{1}{28}$	" 1 cwt.
4 "	$=\frac{1}{7}$	"	"	$=\frac{1}{7}$	" 2 qr.	"	$=\frac{1}{14}$	" 2 qr.
$3\frac{1}{2}$ "	$=\frac{1}{8}$	"	"	$=\frac{1}{2}$	" 16 lb.	"	$=\frac{1}{4}$	" 16 lb.

SURFACE.

2 ro.	$=\frac{1}{2}$	of 1 ac.	8 po.	$=\frac{1}{5}$	of 1 ro.	20 po.	$=\frac{1}{4}$	of 2 ro.
1 "	$=\frac{1}{4}$	"	5 "	$=\frac{1}{8}$	"	16 "	$=\frac{1}{10}$	of 1 ac.
20 po.	$=\frac{1}{2}$	of 1 ro.	4 "	$=\frac{1}{10}$	"		$\frac{1}{5}$	" 2 ro.
10 "	$=\frac{1}{4}$	"	2 "	$=\frac{1}{20}$	"	10 "	$=\frac{1}{16}$	" 1 ac.
			20 "	$=\frac{1}{8}$	of 1 ac.		$\frac{1}{8}$	" 2 ro.

The same principle of aliquot parts can be applied to any other weights or measures.

I. When the given quantity is a simple number, and the price is an aliquot part of £1, or 1*s*., or 1*d*.

Rule—1. Consider the quantity as so many pounds, if the price be an aliquot part of a pound; as so many shillings, if part of a shilling; or as so many pence, if part of a penny.

2. Divide the sum as in Compound Division, by 2, if the aliquot part is $\frac{1}{2}$; by 3, if it is $\frac{1}{3}$; and so on; converting the quotient, when necessary, into its highest denomination.

Example 1.—What is the price of 718 yards of silk, at 3*s*. 4*d*. a yard?

```
 s       £
3/4 = 1/6 )718
        £118 16 8 Ans.
```

Here the price of 713 yd. at £1 is obviously £713. Now, since 3*s*. 4*d*. is equal to $\frac{1}{6}$ of a pound, the value of 713 yd. at 3*s*. 4*d*. will be equal to the $\frac{1}{6}$ of this amount; that is, £118, 16*s*. 8*d*.

Example 2.—What is the value of 1256 yards of cotton cloth at 4d. per yard?

```
       s.   s.
4d. = 1/3 )1256
      2,0 )418  8
        £20  18  8 Ans.
```

Here the value of 1256 yd. at 1*s*. per yd. is 1256*s*. Now, since 4*d*. = $\frac{1}{3}$ of 1*s*., it is plain that the value of 1256 yd. at 4*d*. will be equal to the $\frac{1}{3}$ of this sum; therefore, dividing by 3, the answer will be 418*s*. 8*d*., which, by Reduction, becomes £20, 18*s*. 8*d*.

Exercises.

1.	Find the value of 3475 yd. at 10*s*.	Ans.	£1737	10	0
2.	" " 1693 lb. at 4*d*.	"	28	4	4
3.	" " 7234 yd. at 6*s*. 8*d*.	"	2411	6	8
4.	" " 1395 " 3*s*. 4*d*.	"	232	10	0
5.	" " 2162 " 6*d*.	"	54	1	0
6.	" " 917 " 1*s*. 8*d*.	"	76	8	4
7.	" " 273 " $1\frac{1}{2}d$.	"	1	14	$1\frac{1}{2}$
8.	" " 1845 lb. at 2*s*. 6*d*.	"	230	12	6
9.	" " 347 cwt. at 5*s*.	"	86	15	0
10.	" " 2968 lb. at 2*d*.	"	24	14	8
11.	" " 1724 yd. at 1*s*.	"	86	4	0
12.	" " 939 yd. at 1*d*.	"	3	18	3

II. When the price is any number of times an aliquot part of £1, or 1*s*.

Rule.—Find the price at the aliquot part, and multiply the result by the number of times that the part is contained in the given price.

Example.—Find the price of 758 lb. at 7*s*. 6*d*.

	s	£	
2/6 = $\frac{1}{8}$)		758	
		94 15	= price at 2/6
Multiply by		3	
		£284 5 *Ans.* =	" 7/6

Here we first find the price of the number of lb. at 2*s*. 6*d*., which is the third of 7*s*. 6*d*., and also $\frac{1}{8}$ of a pound; we then multiply by 3, because the price at 7*s*. 6*d*. will be three times the price at 2*s*. 6*d*.

Exercises.

Find the value of the following:

13.	587 yds. at	$10\frac{1}{2}d$. per yd.	Ans.	£25	13	$7\frac{1}{2}$
14.	489 "	9*d*. "	"	18	6	9
15.	643 "	11*s*. 8*d*. "	"	375	1	8
16.	712 "	12*s*. 6*d*. "	"	445	0	0
17.	648 lbs. at	8*s*. 4*d*. per lb.	"	270	0	0
18.	825 "	13*s*. 4*d*. "	"	550	0	0
19.	385 yds. at	16*s*. 8*d*. per yd.	"	320	16	8
20.	918 "	22*s*. 6*d*. "	"	1032	15	0
21.	811 "	23*s*. 4*d*. "	"	946	3	4
22.	517 "	26*s*. 8*d*. "	"	689	6	8
23.	718 cwts.	27*s*. 6*d*. per cwt.	"	987	5	0
24.	872 "	25*s*. "	"	1090	0	0
25.	587 "	21*s*. 8*d*. "	"	635	18	4
26.	784 "	33*s*. 4*d*. "	"	1306	13	4
27.	418 yds. at	18*s*. 4*d*. per yd.	"	383	3	4
28.	576 "	28*s*. 4*d*. "	"	816	0	0

III. When the quantity is a simple number, but the price is *not* an aliquot part of £1, or 1*s*., or 1*d*.

Rule—1. Find the nearest or most convenient aliquot part of the price, and divide the quantity as in Rule I.

2. Take such other parts as will make up the rest of the price, and divide in each case, that sum of which the aliquot part is taken; then add all the quotients together for the answer.

When there are pounds in the price, first multiply the quantity by the number of pounds, and then add to the product the value of the aliquot parts.

Note.—Before beginning to divide, the sums represented by the different aliquot parts to be used, should be written opposite the parts, and then added together, that it may be seen if the whole of the price has been taken into account.

The reason of the Rule will appear from the following examples:

Example 1.—What is the price of 360 yards at 15*s.*?

			£360
360 at	10/	= ½ of £1,	180
"	5/	= ½ " 10/	90
	15/		£270 *Ans.*

Here we reckon the quantity as £360, and divide it by 2, as 10*s*. is the ½ of £1; and then the quotient by 2, as 5*s*. is the ½ of 10*s*. The answer is £270.

Example 2.—What is the price of 549 yards at 18*s*. 9½*d*. a yard?

			£549		
549 at	10/	= ½ of £1,	274	10	0
"	5/	= ½ " 10/	137	5	0
"	3/4	= ⅓ " 10/	91	10	0
"	5*d*.	= ⅛ " 3/4	11	8	9
"	0½*d*.	= $\frac{1}{10}$ " 0/5	1	2	10½
	18/9½		£515	16	7½ *Ans.*

Here we take the various sums that make up 18*s*. 9½*d*. (adding them together to see that the sum is correct), and write opposite them the respective aliquot parts that they represent. We then divide the given quantity, which is considered as £549, by 2 for the first aliquot part; the quotient by 2 for the next; and so on; always dividing that sum of which the aliquot part is taken. In the present example, it will be noticed that 3*s*. 4*d*., which is ⅓ of 10*s*., divides the quotient of 10*s*., and not that which immediately precedes it, because it is ⅓ of 10*s*. that is taken. When all the aliquot parts have been used for division, the quotients are added together for the answer.

Example 3.—What is the price of 549 qr. of wheat at £2, 7*s*. 6*d*. per qr.?

		£		
		549		
		2		
549 qr. at £2 0/0 =	. .	1098	*s.*	*d.*
" 0 5/0 =	¼ of £1,	137	5	0
" 0 2/6 =	½ " 5/	68	12	6
£2 7/6		£1303	17	6

In this example, the price of 549 qr. at £1 per qr. is equal to £549. Now, the price at £2 will obviously be equal to 2 times this sum, therefore, £549 is multiplied by 2: 5*s*. is = ¼ of £1, therefore, the price of 549 qr. at 5*s*. must be equal the ¼ of £549, therefore, £549 is divided by 4; the result, £137, 5*s*., is the price of 549 qr. at 5*s*.: 2*s*. 6*d*. = ½ of 5*s*., therefore, the price of 549 qr. at 2*s*. 6*d*. must be equal to the ½ of their price at 5*s*., hence £137, 5*s*. is divided by 2; the result, £68, 12*s*. 6*d*., is the price of 549 qr. at 2*s*. 6*d*. The sum of these, £1303, 17*s*. 6*d*., is their value at £2, 7*s*. 6*d*.

After the pupil is familiar with the method of working the exercises, he may be required to go over any example in the following manner:

Pupil. Since £2, 7*s*. 6*d*. is the price per qr., I shall take the aliquot parts of £1, therefore, I write down £549, being the price of 549 qr. at £1 per qr. Now, the price at £2 per qr. must obviously be 2 times as great, therefore, I multiply by 2; this gives £1098 = the price of 549 qr. at £2 per qr. Again, since 5*s*. = ¼ of £1, the value of 549 qr. at 5*s*. will be equal to ¼ of their value at £1; this gives £137, 5*s*. = the value of 549 qr. at 5*s*. Lastly, since 2*s*. 6*d*. = ½ of 5*s*., the value of 549 qr. at 2*s*. 6*d*. will be equal to the ½ of their value at 5*s*.; this gives £68, 12*s*. 6*d*. Now, it is plain that if I add the price of 549 qr. at £2, their value at 5*s*., and their value at 2*s*. 6*d*. per qr., the sum will be the value of 549 qr. at £2, 7*s*. 6*d*. per qr., therefore, £1303, 17*s*. 6*d*. is the answer required.

Example 4.—What is the price of 725 yards of silk at 3*s*. 9*d*. per yard?

		s.	
		725	
		3	
725 yd. at 3/0 =	. .	2175	
" 9 =	¼ of 3/	543	9
3 9	2,0)	271,8	9
		£135	18 9

It is obvious that the price of 725 yards at 1*s*. per yard is 725 shillings, and this price, multiplied by 3, gives the price at 3*s*. per yd.; and since 9*d*. = ¼ of 3*s*., the price at 9*d*. will be the ¼ of 2175*s*.; that is, 543*s*. 9*d*. The sum of these prices, 2718*s*. 9*d*., is the price at 3*s*. 9*d*., which, by reduction, becomes £135, 18*s*. 9*d*.

By taking the aliquot parts of £1, the operation will stand thus—

			£		
725 yd. at £1	= . . .		725		
"	3/4 = ⅙ of £1,		120	16	8
"	5 = ⅛ " 3/4		15	2	1
	3/9		£135	18	9

This example may also be wrought as below: the value at 4*s.* is first calculated, then the value at 3*d.*, the difference between 4*s.* and 3*s.* 9*d.* is subtracted; the remainder is the answer.

			s.	
	725 yd. at 1/	= . . .	725	
			4	
	" 4/	= . .	2900	
Subtract	" 3*d.*	= ¼ of 1/	181	3
			2,0)271,8	9
			£135 18	9

Example 5.—What is the price of 937 barrels of ale at £6, 19*s.* 7¾*d.* per barrel?

			£937		
			6		
937 barrels at £6 0/0	= . . .		5622		
" 0 10/0	= ½ of £1		468	10	0
" 0 5/0	= ½ " 10/		234	5	0
" 0 4/0	= ⅕ " £1		187	8	0
" 0 0/6	= ⅛ " 4/		23	8	6
" 0 0/1½	= ¼ " 0/6		5	17	1½
" 0 0/0¼	= ⅙ " 0/1½		0	19	6¼
£6 19/7¾			£6542	8	1¾

Or thus—£6, 19*s.* 7¾*d.* = 139*s.* 7¾*d.*

		s.	
		937	
		139	
		8433	
		28110	
		93700	
937 barrels at 139/ = £6 19/0	= . . .	130243	
" at 0 0/6	= ½ of 1/0	468	6
" " 0 0/1½	= ¼ " /6	117	1½
" " 0 0/0¼	= ⅙ " 0/1½	19	6¼
£6 19/7¾		2,0)13084,8	1¾
		£6542 8	1¾

Note.—Any question in Practice may be proved by working it out a different way. The last example is an illustration of this remark.

All the exercises under this rule may be very readily calculated as follows:

RULE.—Multiply the number of articles by the number in the lowest denomination in the price, and divide the product by the number of that denomination which makes one of the next higher; then multiply the number of articles by the number of the next higher denomination of the price, and add this product to the former quotient; convert this sum to the next higher denomination; and so on to the highest.

The last example wrought by this rule would stand as under—namely, the price of 937 articles at £6, 19*s*. 7¾*d*.

Multiply	937						
By	3 farthings.						
	4)2811						
	702¾	=	price of	937	at	¾*d*.	in pence.
Add 937 × 7*d*.	6559	=	"	937	"	7*d*.	"
	12)7261¾						
	605 1¾	=	"	937	"	7¾*d*.	in shillings.
Add 937 × 9/	8433	=	"	937	"	19*s*.	"
× 10/	9370						
	2,0)1840,8 1¾						
	920 8 1¾	=	"	937	"	19*s*. 7¾*d*.	in pounds.
Add 937 × £6	5622		"	937	"	£6	"
	£6542 8 1¾	=	"	937	"	£6, 19s. 7¾*d*.	

This rule avoids altogether the labour of breaking up the price into aliquot parts, and is very easily learned.

Exercises.

Find the value of the following:

			£	s.	d.
29.	964 yd. at 2*s*. 9*d*.	Ans.	£132	11	0
30.	567 lb. at £1, 7*s*. 6*d*.	"	779	12	6
31.	4351 cwt. at 5*s*. 6*d*.	"	1196	10	6
32.	9327 qr. at 2*s*. 4*d*.	"	1088	3	0
33.	764 yd. at 3½*d*.	"	11	2	10
34.	3997 yd. at 7*s*. 9*d*.	"	1548	16	9
35.	482 cwt. at £7, 2*s*. 3*d*.	"	3428	4	6
36.	9647 oz. at 15*s*. 6*d*.	"	7476	8	6
37.	4825 gal. at £1, 16*s*. 8*d*.	"	7929	3	4
38.	189 yd. at £3, 4*s*. 7*d*.	"	610	6	3
39.	3716 lb. at 7½*d*.	"	116	2	6
40.	2943 oz. at 9*d*.	"	110	7	3
41.	1121 cwt. at 2*s*. 3½*d*.	"	128	8	11½
42.	3916 qr. at 3*s*. 9*d*.	"	734	5	0

43.	475 gal. at £5, 6*s.* 8*d.*	Ans. £2533	6	8
44.	968 cwt. at £3, 2*s.* 4*d.* . . .	" 3016	18	8
45.	1234 cwt. at 6¾*d.*	" 34	14	1½
46.	5678 tons at 1*s.* 9½*d.*	" 508	13	1
47.	9012 tons at £7, 15*s.* 10*d.* . . .	" 70218	10	0
48.	751 lb. at £1, 12*s.* 6*d.* . . .	" 1220	7	6
49.	3964 ac. at 8*s.* 8*d.*	" 1717	14	8
50.	1009 ac. at £1, 5*s.* 10*d.* . . .	" 1303	5	10
51.	374 yd. at 5½*d.*	" 8	11	5
52.	2166 tons at 6*s.* 9¾*d.*	" 737	15	10½
53.	2772 cwt. at £2, 7*s.* 5¼*d.* . . .	" 6574	16	9
54.	4165 qr. at £3, 1*s.* 11½*d.* . . .	" 12902	16	5½
55.	1893 lb. at 2*s.* 10¾*d.*	" 274	1	9¾
56.	2714 oz. at £1, 16*s.* 11*d.* . . .	" 5009	11	10
57.	1862 yd. at £2, 3*s.* 5¾*d.*	" 4047	18	2½
58.	3931 yd. at 2*s.* 11*d.*	" 573	5	5
59.	5624 yd. at £1, 15*s.* 9½*d.* . . .	" 10064	12	4
60.	793 lb. at £2, 6*s.* 10¾*d.* . . .	" 1859	8	4¾
61.	125 cwt. at 17*s.* 5½*d.*	" 109	2	3½
62.	937 tons at 19*s.* 6*d.*	" 913	11	6
63.	1824 qr. at 13*s.* 4¾*d.*	" 1221	14	0
64.	2793 oz. at £2, 1*s.* 3½*d.* . . .	" 5766	7	7½
65.	1395 ac. at £1, 13*s.* 11¼*d.* . . .	" 2367	2	9¾
66.	2068 yd. at 12*s.* 10½*d.*	" 1331	5	6
67.	1754 tons at £1, 15*s.* 11*d.* . . .	" 3149	17	10
68.	8967 cwt. at 6*s.* 9¾*d.*	" 3054	7	8¼
69.	391 yd. at £5, 8*s.* 6½*d.*	" 2121	19	9½
70.	7184 bar. at 5*s.* 10½*d.*	" 2110	6	0
71.	2743 cwt. at £1, 12*s.* 7½*d.* . . .	" 4474	10	4½
72.	9165 yd. at 15*s.* 4½*d.*	" 7045	11	10½
73.	753 oz. at 3*s.* 10¾*d.*	" 146	13	6¾
74.	1792 ac. at £1, 17*s.* 11¾*d.* . . .	" 3402	18	8
75.	1846 tons at £9, 18*s.*	" 18275	8	0
76.	3915 lb. at £5, 18*s.* 6¼*d.* . . .	" 23200	9	0¾
77.	713 yd. at 19*s.* 4¼*d.*	" 689	19	6¼
78.	6415 yd. at £3, 7*s.* 6*d.* . . .	" 21650	12	6
79.	9374 tons at 11*s.* 9¾*d.*	" 5536	10	4½
80.	6132 qr. at 13*s.* 7½*d.*	" 4177	8	6
81.	567 yd. at 16*s.* 11*d.*	" 479	11	9
82.	3952 ac. at £3, 11*s.* 5¼*d.* . . .	" 14116	1	0
83.	2174 cwt. at £2, 16*s.* 8*d.*	" 6159	13	4
84.	683 lb. at 1*s.* 4½*d.*	" 46	19	1½
85.	845 yd. at £7, 11*s.* 3¾*d.*	" 6392	19	0¾
86.	1796 yd. at 1*s.* 6½*d.*	" 138	8	10
87.	8354 lb. at 16*s.* 11½*d.*	" 7083	9	11
88.	7912 lb. at 14*s.* 10¼*d.*	" 5876	6	2

IV. When the Price is greater or less than £1 or 1s., and the *difference* is the Aliquot Part of £1 or 1s.

Rule.—Consider the number of articles as so many pounds, or shillings, then find the price for the aliquot part by Rule I., and add or subtract, according as the price is greater or less than a pound or a shilling.

Example 1.—Find the price of 736 yards at 1*s*. 3*d*. each.

The price at 1/	=	736*s*.
Add, price at 3*d*. = ¼*s*.		184
	Ans.	920 = £46.

Example 2.—Find the price of 1056 yards at 18*s*. 4*d*. each.

1056 at £1,	=	£1056
Subtract price at 1/8 = $\frac{1}{12}$ of £1,	=	88
Price at 18/4,	=	£968

Here, if the price were £1 per yard, the answer, it is evident, would be just as many pounds as yards; but as the price is 1*s*. 8*d*. less than a pound—that is, $\frac{1}{12}$th part less—1-12th of the number, considered as pounds, is subtracted from it, and the remainder is the price of so many yards at 18*s*. 4*d*.

Exercises.

89.	Find the value of	397	articles at	1*s*. 4*d*.	Ans.	£26	9	4
90.	"	876	"	1*s*. 1½*d*.	"	49	5	6
91.	"	581	"	10½*d*.	"	25	8	4½
92.	"	798	"	£1, 2*s*. 6*d*.	"	897	15	0
93.	"	476	"	17*s*. 6*d*.	"	416	10	0
94.	"	384	"	22*s*.	"	422	8	0
95.	"	781	"	23*s*. 4*d*.	"	911	3	4
96.	"	647	"	16*s*. 8*d*.	"	539	3	4
97.	"	875	"	26*s*. 8*d*.	"	1166	13	4
98.	"	946	"	15*s*.	"	709	10	0

V. When the Price is an even Number of Shillings.

Rule.—Multiply the number of articles by *half* the shillings in the price, *double* the unit's figure of the product for shillings, the other figures are pounds.

Example.—Find the value of 723 articles at 14*s*. each.

	723	
½ of 14/ =	7	
	5061	= £506 2 0

Here we multiply by 7, which is half the number of shillings, or the number of times 2*s*. in the price; the product 5061, being the number of times 2*s*. in the price, becomes pounds on dividing by 10, the quotient being all the figures of the product except the last; this last figure, which is the remainder, being doubled, becomes shillings.

From the above illustration, the reason of the rule is obvious.

ANOTHER METHOD.—Find the price of the whole at one pound, or one shilling, multiply this price by the pounds or shillings in the given price, and take *parts* for the lower denominations in the price; the sum of the product and the parts will be the price sought.

	ac.	ro.	po.			£	s.	d.
	35	0	0	at 20/ per acre	=	£35	0	0
	0	3	0	〃 〃	=	0	15	0
	0	0	20	〃 〃	=	0	2	6
Price of 35 ac. 3 ro. 20 po. at 20/ per ac.					=	£35	17	6
								3

					£	s.	d.
Price at	£3 0/0	per acre	=		107	12	6
〃	0 10/0	〃	=	$\frac{1}{2}$ of £1	17	18	9
〃	0 4/0	〃	=	$\frac{1}{5}$ 〃 £1	7	3	6
〃	0 0/3	〃	=	$\frac{1}{16}$ 〃 4/	0	8	$11\frac{1}{2}$ $\frac{1}{2}$
〃	£3 14/3	〃	=		£133	3	$8\frac{1}{2}$ $\frac{1}{2}$

Pupil. Since 35 acres at £1 is £35, and 3 roods at £1 per acre is 15s., and 20 poles, or $\frac{1}{8}$ of an acre, at £1 per acre, is 2s. 6d.; 35 ac. 3 ro. 20 po. at £1 per acre is £35, 17s. 6d.; this sum multiplied by 3 will give £107, 12s. 6d., the price at £3 per acre. Again, the half of said sum, or £17, 18s. 9d., is the price of the whole at 10s. per acre; in the same way we find £7, 3s. 6d. to be the price of the whole at 4s. per acre, and 8s. $11\frac{1}{2}$d. $\frac{1}{2}$ the price of the whole at 3d. per acre. These being added, give the price of 35 ac. 3 ro. 20 po. at £3, 14s. 3d. = £133, 3s. $8\frac{1}{2}$d. $\frac{1}{2}$.

Exercises.

Find the value of the following:

			£	s.	d.
122.	27 cwt. 1 qr. 14 lb. at £2, 3s. $7\frac{1}{2}$d. .	Ans.	59	13	$7\frac{3}{4}$ $\frac{7}{8}$
123.	39 cwt. 2 qr. 8 lb. at £5, 6s. 1d. .	〃	209	17	$10\frac{1}{4}$ $\frac{6}{7}$
124.	135 cwt. 3 qr. 4 lb. at 17s. $3\frac{3}{4}$d. .	〃	117	10	$9\frac{1}{4}$ $\frac{12}{14}$
125.	79 cwt. 3 qr. 21 lb. at £7, 15s. $10\frac{1}{2}$d.	〃	623	0	$3\frac{0}{4}$ $\frac{3}{8}$
126.	85 cwt. 0 qr. 24 lb. at £1, 13s. 4d. .	〃	142	0	$5\frac{1}{2}$ $\frac{6}{7}$
127.	23 cwt. 2 qr. 7 lb. at 12s. $11\frac{1}{4}$d. .	〃	15	4	$10\frac{0}{4}$ $\frac{5}{16}$
128.	742 cwt. 1 qr. 23 lb. at £3, 19s. 4d.	〃	2945	1	$5\frac{1}{2}$
129.	51 ac. 2 ro. 30 po. at £26, 17s. $1\frac{1}{4}$d.	〃	1388	2	$7\frac{3}{4}$ $\frac{1}{8}$
130.	12 ac. 3 ro. 25 po. at £108, 13s. 11d.	〃	1338	6	$5\frac{3}{4}$ $\frac{3}{8}$
131.	96 ac. 1 ro. 10 po. at £17, 2s. $10\frac{1}{4}$d.	〃	1651	1	$1\frac{1}{2}$ $\frac{13}{16}$
132.	82 ac. 0 ro. 15 po. at £12, 14s. 6d. .	〃	1044	12	$10\frac{1}{4}$ $\frac{1}{4}$
133.	525 ft. 7 in. at 7s. 6d. . . .	〃	197	1	$10\frac{1}{2}$
134.	432 ft. 11 in. at 2s. $3\frac{1}{2}$d. . . .	〃	49	12	$1\frac{0}{4}$ $\frac{4}{8}$
135.	745 ft. $10\frac{1}{2}$ in. at 4s. $9\frac{1}{2}$d. . . .	〃	178	13	$11\frac{3}{4}$ $\frac{1}{4}$
136.	65 ft. $3\frac{1}{4}$ in. at 3s. $7\frac{3}{4}$d.	〃	11	17	$11\frac{1}{2}$ $\frac{10}{48}$

For further applications of Practice, see 'Mental Arithmetic.'

MENTAL ARITHMETIC.

Mental Arithmetic differs from slate arithmetic only in being performed in the mind, and is often found very useful, as individuals are not always in a situation where the mechanical means of pen and pencil can be obtained.

As it is difficult, however, to remember long processes, the chief object of a treatise on Mental Arithmetic is to investigate short methods, by which correct results can be obtained.

These are generally found by tracing certain relations which subsist between the divisions of weights and measures, and certain specific numbers, and the divisions of money, as they at present exist, and how the one can conveniently be turned into the other.

Mental Arithmetic is not to be taught at any particular part of the course, as if it were a separate subject. *Adding mentally* should be taught in Addition, subtracting in Subtraction, &c.

As a preparatory exercise, the teacher should cause the pupil to answer a great many questions, such as the following:

What is the sum of 3, 5, and 7? What is the difference of 30 and 23? What must be added to 18 that the sum may be 25? What is the product of 16 by 9?—of 18 by 6?—of 43 by 7?—of 47 by 6?—of 112 by 11? What are the number of pence which make 2*s.* 3*d.*, 3*s.* 9*d.*, 4*s.* 2*d.*, 4*s.* 11*d.*, 5*s.* 5*d.*, 5*s.* 9*d.*, 6*s.* 8*d.*, 7*s.* 4*d.*, 8*s.* 4*d.*, 9*s.* 6*d.*, 10*s.* 1*d.*? How often does 24 contain 2? How often does 30 contain 2, 3, 4, 5, 6, 7, 8, 9?—and so on with other numbers, till he can divide them readily. How many shillings are in 27*d.*, 32*d.*, 37*d.*, 42*d.*, 50*d.*, 56*d.*, 64*d.*, 70*d.*, 84*d.*, 90*d.*, 104*d.*, 120*d.*, 132*d.*, 156*d.*? How many pounds are in 24*s.*, 36*s.*, 40*s.*, 48*s.*, 52*s.*, 60*s.*, 73*s.*, 84*s.*, 96*s.*, 120*s.*, 170*s.*, 200*s.*? How many shillings are in £2, 4*s.*, £3, 11*s.*, £4, 6*s.*, £4, 17*s.*, £5, £6, 13*s.*, £7, 11*s.*, £8, 15*s.*? What is the sum of 4*d.*, 7*d.*, and 9*d.*?—of 8*d.*, 7*d.*, and 9*d.*?—of 10*d.*, 7*d.*, and 11*d.*? If 8*d.* be paid from 1*s.* 3*d.*, how much will be left? If 15*s.* be paid from £1, 4*s.*, what will be left? What is the price of 4 articles at 9*d.* each? What is the price of 4 yards at 10*d.* per yard? How many articles can be bought for 6*s.* at 1*s.* each, at 9*d.* each, at 8*d.* each, at 6*d.* each?

These, and an unlimited number of similar questions, the teacher can easily ask without the assistance of any book, and the pupil as easily answer without any rule. There are, however, a great many short and simple methods of calculating the value of particular numbers at any price, and of any number at particular prices, which the pupil cannot be expected to discover for himself, and these we shall now proceed to explain.

I. To find the price of 12 Articles.

Rule.—Reckon every penny in the given price as 1*s.*, and every farthing as 3*d.*

The price of 3, or 6, may be found by taking the ¼ or ½ of the price of 12.

Since there are 12 pence in a shilling, it is evident that 12 articles at 1*d.* each will come to 1 shilling, and at 2*d.* to 2 shillings, and so on, while 12 at 1 farthing each will come to 3 pence.

Exercises.—What is the price of

12	at	2½*d.*	Ans. £0	2	6	12	at	0*s.*	11¼*d.*	Ans. £0	11	3
12	"	3¼*d.*	" 0	3	3	12	"	1*s.*	1¼*d.*	" 0	13	3
12	"	3¾*d.*	" 0	3	9	12	"	1*s.*	5¾*d.*	" 0	17	9
12	"	4½*d.*	" 0	4	6	12	"	1*s.*	10½*d.*	" 1	2	6
12	"	5¼*d.*	" 0	5	3	12	"	2*s.*	3*d.*	" 1	7	0
12	"	7¾*d.*	" 0	7	9	12	"	3*s.*	4¼*d.*	" 2	0	3
12	"	9¼*d.*	" 0	9	3	12	"	4*s.*	1½*d.*	" 2	9	6
12	"	10¾*d.*	" 0	10	9	12	"	5*s.*	9¾*d.*	" 3	9	9

II. To find the price of 24 Articles.

Rule.—Reckon every penny in the given price as 2*s.*, and every farthing as 6*d.*

Since 24 pence make 2 shillings, and 24 farthings are 6*d.*, there will evidently be twice as many shillings in the price of 24 articles, as there are pence in the price of one, and every farthing will make 6*d.*

Exercises.—What is the price of

24	at	2¼*d.*	Ans. £0	4	6	24	at	1*s.* 1¾*d.*	Ans. £1	7	6
24	"	3¾*d.*	" 0	7	6	24	"	1*s.* 4½*d.*	" 1	13	0
24	"	4½*d.*	" 0	9	0	24	"	1*s.* 7¾*d.*	" 1	19	6
24	"	5¼*d.*	" 0	10	6	24	"	1*s.* 9¼*d.*	" 2	2	6
24	"	6½*d.*	" 0	13	0	24	"	2*s.* 1½*d.*	" 2	11	0
24	"	8¼*d.*	" 0	16	6	24	"	2*s.* 7¼*d.*	" 3	2	6
24	"	9½*d.*	" 0	19	0	24	"	3*s.* 5¾*d.*	" 4	3	6
24	"	11¾*d.*	" 1	3	6	24	"	4*s.* 2¾*d*	" 5	1	6

III. To find the price of 48 Articles.

Rule.—Convert the price of one to farthings, and call the result shillings, and it will be the price of 48.

Since 48 farthings are equivalent to one shilling, there will evidently be as many shillings in the price of 48 articles as there are farthings in the price of one.

Exercises.—What is the price of

48 at	1¾d.	Ans. £0	7	0	48 at	0s.	10¼d.	Ans. £2	1	0
48 „	2½d.	„ 0	10	0	48 „	0s.	11½d.	„ 2	6	0
48 „	3¾d.	„ 0	15	0	48 „	1s.	1½d.	„ 2	14	0
48 „	4¼d.	„ 0	17	0	48 „	1s.	4¼d.	„ 3	5	0
48 „	5½d.	„ 1	2	0	48 „	1s.	7¾d.	„ 3	19	0
48 „	6¾d.	„ 1	7	0	48 „	1s.	11¼d.	„ 4	13	0
48 „	8¼d.	„ 1	13	0	48 „	2s.	1¼d.	„ 5	1	0
48 „	9½d.	„ 1	18	0	48 „	3s.	4¾d.	„ 8	3	0

IV. To find the price of 96 Articles.

Rule.—Convert the price of one to farthings, double the right-hand figure for shillings, and call the remaining figures pounds, and the result will be the price of 96.

Since 96 farthings are 2s., there will be as many 2s. in the price of 96 articles as there are farthings in the price of one; but ten 2s. make a pound, and any number is divided by 10 by taking away the right-hand figure, and writing it as tenths; and since in this case the figure taken away is 2s., if we double it, the result will be shillings.

Exercises.—What is the price of

96 at	1¾d.	Ans. £0	14	0	96 at	0s.	9¼d.	Ans. £3	14	0
96 „	2¾d.	„ 1	2	0	96 „	0s.	10¾d.	„ 4	6	0
96 „	3½d.	„ 1	8	0	96 „	0s.	11½d.	„ 4	12	0
96 „	4¼d.	„ 1	14	0	96 „	1s.	4¼d.	„ 6	10	0
96 „	5¾d.	„ 2	6	0	96 „	1s.	7¾d.	„ 7	18	0
96 „	6½d.	„ 2	12	0	96 „	2s.	1¾d.	„ 10	6	0
96 „	7¾d.	„ 3	2	0	96 „	3s.	7¼d.	„ 17	6	0

V. To find the value of 240 Articles.

Rule.—Convert the price of one to pence, and call the pence pounds; and for one farthing take 5s., for ½d. take 10s., and for ¾d. take 15s.

Since there are 240 pence in a pound, it is evident that 240 articles at one penny each will be £1, and 240 at a farthing each will be 5s.

Exercises.—What is the price of

240 at	2¼d.	Ans. £2	5	0	240 at	0s.	11½d.	Ans. £11	10	0
240 „	3¾d.	„ 3	15	0	240 „	1s.	1d.	„ 13	0	0
240 „	4½d.	„ 4	10	0	240 „	1s.	4¼d.	„ 16	5	0
240 „	5¾d.	„ 5	15	0	240 „	2s.	3¾d.	„ 27	15	0
240 „	7¼d.	„ 7	5	0	240 „	3s.	7½d.	„ 43	10	0
240 „	8½d.	„ 8	10	0	240 „	5s.	9¼d.	„ 69	5	0
240 „	9¾d.	„ 9	15	0	240 „	7s.	5¾d.	„ 89	15	0
240 „	10¼d.	„ 10	5	0	240 „	11s.	4d.	„ 136	0	0

VI. To calculate the value of any number of Dozens.

Rule.—Find the price of one dozen, by Rule I., and multiply this price by the number of dozens in the given quantity.

The price of any number of dozens will evidently be as many times the price of a dozen as there are dozens in the quantity.

Exercises.—What is the value of

36	at	$1\frac{1}{2}d.$	Ans.	£0	4	6		84	at	0s.	$5\frac{1}{4}d.$	Ans.	£1	16	9
36	"	$5\frac{1}{4}d.$	"	0	15	9		96	"	0s.	$4\frac{3}{4}d.$	"	1	18	0
48	"	$7\frac{3}{4}d.$	"	1	11	0		96	"	1s.	$2\frac{1}{4}d.$	"	5	14	0
24	"	$9\frac{3}{4}d.$	"	0	19	6		108	"	0s.	$3\frac{3}{4}d.$	"	1	13	9
60	"	$10\frac{1}{4}d.$	"	2	11	3		108	"	0s.	$8\frac{1}{4}d.$	"	3	14	3
60	"	$11\frac{3}{4}d.$	"	2	18	9		120	"	0s.	$7\frac{3}{4}d.$	"	3	17	6
72	"	$4\frac{3}{4}d.$	"	1	8	6		132	"	0s.	$10\frac{1}{4}d.$	"	5	12	9
84	"	$7\frac{1}{2}d.$	"	2	12	6		144	"	0s.	$11\frac{1}{2}d.$	"	6	18	0
84	"	$8\frac{1}{4}d.$	"	2	17	9		156	"	0s.	$9\frac{1}{2}d.$	"	6	3	6

VII. To find the value of 16 Articles.

Rule.—Convert the price of one to farthings, and divide by 3; call the quotient shillings, and each unit in the remainder 4*d.*, and the result will be the value of 16.

Since there are 16 ounces in a pound, and 16 nails in a yard, it will be convenient to have a short method of obtaining the price of 16 articles. This may easily be obtained by observing that 16 is the third part of 48, and therefore its price will be the third part of the price of 48; or by observing that 16 at 1 farthing comes to 4 pence, which is the third part of a shilling; hence 16 at three farthings each comes to 1*s.*, and 16 at 1 farthing comes to 4*d.*

Exercises.—What is the price of

16	at	$1\frac{1}{4}d.$	Ans.	£0	1	8		16	at	0s.	$8\frac{3}{4}d.$	Ans.	£0	11	8
16	"	$2\frac{1}{2}d.$	"	0	3	4		16	"	0s.	$9\frac{1}{2}d.$	"	0	12	8
16	"	$3\frac{1}{4}d.$	"	0	4	4		16	"	0s.	$10\frac{3}{4}d.$	"	0	14	4
16	"	$4\frac{1}{2}d.$	"	0	6	0		16	"	0s.	$11\frac{1}{4}d.$	"	0	15	0
16	"	$5\frac{3}{4}d.$	"	0	7	8		16	"	1s.	$1d.$	"	0	17	4
16	"	$6\frac{1}{4}d.$	"	0	8	4		16	"	1s.	$2\frac{1}{2}d.$	"	0	19	4
16	"	$7\frac{3}{4}d.$	"	0	10	4		16	"	2s.	$5\frac{1}{2}d.$	"	1	19	4
16	"	$8\frac{1}{4}d.$	"	0	11	0		16	"	3s.	$5\frac{3}{4}d.$	"	2	15	8

VIII. To find the value of 28 Articles.

Rule.—Convert the price of one to halfpence, to which add their sixth part, and call the result shillings; it will be the price of 28.

If, in dividing by 6, there be a remainder, double it, and it will be pence.

The price of 28 may also be easily obtained by calculating separately the price of 12 and the price of 16, and adding the results.

Since 28 articles are equal to 24 articles and one-sixth part of 24, the price of 28 articles may easily be obtained from the price of 24.

Exercises.—What is the price of

28 at	3¼*d*.	Ans. £0	7	7	28 at	0*s*.	9½*d*.	Ans. £1	2	2
28 "	2*d*.	" 0	4	8	28 "	0*s*.	10½*d*.	" 1	4	6
28 "	3¾*d*.	" 0	8	9	28 "	0*s*.	11¾*d*.	" 1	7	5
28 "	4½*d*.	" 0	10	6	28 "	1*s*.	1½*d*.	" 1	11	6
28 "	5¼*d*.	" 0	12	3	28 "	1*s*.	3*d*.	" 1	15	0
28 "	6¾*d*.	" 0	15	9	28 "	1*s*.	4½*d*.	" 1	18	6
28 "	7½*d*.	" 0	17	6	28 "	1*s*.	6½*d*.	" 2	3	2
28 "	8¼*d*.	" 0	19	3	28 "	2*s*.	3*d*.	" 3	3	0

IX. To find the value of 112 Articles.

Rule.—Convert the price of one to farthings, to which add a sixth part of itself; then double the right-hand figure of the sum for shillings, and the remaining figures will be pounds.

Note.—Some may perhaps find it simpler to calculate in the following manner: Since 112 at one penny is 9*s*. 4*d*., and 112 at a farthing is 2*s*. 4*d*.; to find the value of 112, multiply 9*s*. 4*d*. by the number of pence in the price of one, to which add 2*s*. 4*d*. for 1 farthing, 4*s*. 8*d*. for a halfpenny, and 7*s*. for 3 farthings, in the price of one.

Another method of calculating the price of 112, the number of lbs. in a cwt., is find the price of 96 by Rule IV., and the price of 16 by Rule VII., and add them together, since 112 = 96 + 16.

Exercises.—What is the price of

112 at	1¾*d*.	Ans. £0	16	4	112 at	0*s*.	9¾*d*.	Ans. £4	11	0
112 "	2¼*d*.	" 1	1	0	112 "	0*s*.	10½*d*.	" 4	18	0
112 "	3*d*.	" 1	8	0	112 "	0*s*.	11¼*d*.	" 5	5	0
112 "	3¾*d*.	" 1	15	0	112 "	1*s*.	1½*d*.	" 6	6	0
112 "	5¼*d*.	" 2	9	0	112 "	1*s*.	4½*d*.	" 7	14	0
112 "	6½*d*.	" 3	0	8	112 "	1*s*.	6*d*.	" 8	8	0
112 "	7½*d*.	" 3	10	0	112 "	2*s*.	9*d*.	" 15	8	0
112 "	8¼*d*.	" 3	17	0	112 "	3*s*.	3*d*.	" 18	4	0

X. To find the value of 100 Articles.

Rule.—Convert the price of one to farthings; double the right-hand figure for shillings, and the other figures will be pounds, to which add as many pence as there are farthings in the price of one. If there be shillings in the price, take £5 for every shilling in the price of one.

Converting the price of one to farthings, and doubling the right-hand figure for shillings, &c., gives, by Rule IV. p. 111, the price of 96; and it is plain that calling the farthings pence gives the value of 4, but $96 + 4 = 100$; hence the Rule.

Exercises.—What is the price of

100	at	1¾d.	Ans. £0	14	7	100	at	0s.	8¾d.	Ans. £3	12	11
100	"	2½d.	"	1	0	10	100	"	0s.	9½d.	"	3	19	2
100	"	3¾d.	"	1	11	3	100	"	0s.	10¾d.	"	4	9	7
100	"	4½d.	"	1	17	6	100	"	1s.	1½d.	"	5	10	5
100	"	5¾d.	"	2	7	11	100	"	1s.	5¼d.	"	7	3	9
100	"	6¾d.	"	2	16	3	100	"	1s.	7½d.	"	8	2	6
100	"	7½d.	"	3	2	6	100	"	1s.	10d.	"	9	3	4

XI. To find the value of 365 Articles.

Rule.—Take as many times £18, 5s. as there are shillings in the price of one, as many times £1, 10s. 5d. as there are pence, and as many times 7s. 7¼d. as there are farthings.

Or, take as many pounds as there are pence in the price of one, and as many 5s. as there are farthings, to which add its half and the value of 5 articles; and the sum will be the price of 365.

Since $365 = 240 + 120 + 5$, and that 240 is the number of pence in a pound, and 120 half the number of pence in a pound, it is evident that 365 at 1d. each will come to £1, 10s. 5d., and that 365 at 1s. each comes to £18, 5s.

Exercises.—What is the price of

365	at	0s.	2½d.	Ans. £3	16	0½	365	at	1s.	4d.	Ans. £24	6	8
365	"	0s.	3¾d.	"	5	14	0¾	365	"	1s.	8d.	"	30	8	4
365	"	0s.	5d.	"	7	12	1	365	"	1s.	10½d.	"	34	4	4½
365	"	0s.	7½d.	"	11	8	1½	365	"	2s.	1d.	"	38	0	5
365	"	0s.	9¼d.	"	14	1	4¼	365	"	2s.	8d.	"	48	13	4
365	"	0s.	10½d.	"	15	19	4½	365	"	3s.	2d.	"	57	15	10
365	"	1s.	1d.	"	19	15	5	365	"	3s.	10d.	"	69	19	2

XII. To find the value of 313 Articles.

Rule.—Call the pence in the price of one, pounds, and the farthings, 5*s*., which increase by a fourth of itself, and the price of a dozen, and the price of one; and the result will be the value of 313.

Or, multiply £1, 6*s*. 1*d*. by the number of pence in the price of one, to which add as many times 6*s*. 6¼*d*. as there are farthings in the price of one.

The number of work-days in a year, 313, is = 240 + 60 + 12 + 1, and 60 is the fourth part of 240; hence the rule.

Exercises.—What is the price of

313	at	2*d*.	Ans.	£2	12	2	313	at	1*s*.	2*d*.	Ans.	£18	5	2
313	"	3½*d*.	"	4	11	3½	313	"	1*s*.	4*d*.	"	20	17	4
313	"	5*d*.	"	6	10	5	313	"	1*s*.	7*d*.	"	24	15	7
313	"	6¼*d*.	"	8	9	6¼	313	"	1*s*.	10*d*.	"	28	13	10
313	"	7¾*d*.	"	10	2	1¾	313	"	2*s*.	3*d*.	"	35	4	3
313	"	8¼*d*.	"	11	1	8¼	313	"	2*s*.	8*d*.	"	41	14	8
313	"	11*d*.	"	14	6	11	313	"	3*s*.	4*d*.	"	52	3	4
313	"	11¾*d*.	"	15	6	5¾	313	"	4*s*.	1*d*.	"	63	18	1

XIII. To find the price of any number of Articles at an even number of Shillings each.

Rule.—Multiply the number of articles by half the number of shillings in the price, then double the right-hand figure of the product for shillings, and the remaining figures will be pounds.

When the price of one exceeds a pound, it will generally be best to calculate by the rule for the excess of the price above one pound, and to the result add as many pounds as there are articles; and when the price exceeds any number of pounds, to calculate for the excess above the pounds by the rule, and for the pounds separately, and add the results.

When the price of one article is an even number of shillings, we may take half the even number of shillings, which will be a more easy multiplier than the whole price, but this product will evidently be the number of 2*s*. in the price of the whole, the tenth part of which will be the answer in pounds.

Exercises.—What is the price of

33	at	4*s*.	Ans.	£6	12	0	54	at	22*s*.	Ans.	£59	8	0
47	"	8*s*.	"	18	16	0	67	"	32*s*.	"	107	4	0
59	"	6*s*.	"	17	14	0	74	"	24*s*.	"	88	16	0
75	"	12*s*.	"	45	0	0	85	"	26*s*.	"	110	10	0
91	"	16*s*.	"	72	16	0	94	"	28*s*.	"	131	12	0
105	"	18*s*.	"	94	10	0	103	"	30*s*.	"	154	10	0
126	"	14*s*.	"	88	4	0	132	"	34*s*.	"	224	8	0
139	"	20*s*.	"	139	0	0	147	"	42*s*.	"	308	14	0

XIV. To find the price of any number of Articles when the price of one is an aliquot part of £1 or 1s.

Rule.—Take the same part of the number of articles that the price of one is of a pound or a shilling, and the quotient will be the answer in pounds or shillings.

Exercises.—What is the value of

54	at	3s. 4d.	Ans.	£9	0	0		53	at	10s. 0d.	Ans.	£26	10	0
36	〃	2s. 6d.	〃	4	10	0		64	〃	1s. 4d.	〃	4	5	4
68	〃	4s. 0d.	〃	13	12	0		76	〃	1s. 3d.	〃	4	15	0
106	〃	1s. 8d.	〃	8	16	8		89	〃	0s. 4d.	〃	1	9	8
120	〃	2s. 0d.	〃	12	0	0		126	〃	0s. 3d.	〃	1	11	6
140	〃	6s. 8d.	〃	46	13	4		255	〃	0s. 6d.	〃	6	7	6
135	〃	4s. 0d.	〃	27	0	0		343	〃	0s. 1½d.	〃	2	2	10½
148	〃	5s. 0d.	〃	37	0	0		485	〃	0s. 2d.	〃	4	0	10

XV. To calculate the Interest of any sum at 5 per cent. for a year.

Rule.—For every pound in the principal take one shilling; and for lower denominations in the principal, take the same part of a shilling that the lower denomination is of a pound.

Since 5 per cent. means £5 of interest for £100, that is, 100s. for £100, it is plain that the interest or commission on any sum at 5 per cent. is just as many shillings as there are pounds in the given sum; hence the Rule.

Exercises.—Find the interest for one year at 5 per cent. of

£ 49,	0s.	0d.	Ans. £	2	9	0		£ 53,	5s.	0d.	Ans. £	2	13	3
87,	0s.	0d.	〃	4	7	0		93,	10s.	0d.	〃	4	13	6
123,	10s.	0d.	〃	6	3	6		118,	15s.	0d.	〃	5	18	9
135,	15s.	0d.	〃	6	15	9		145,	7s.	6d.	〃	7	5	4½
156,	6s.	8d.	〃	7	16	4		164,	12s.	6d.	〃	8	4	7½
185,	11s.	8d.	〃	9	5	7		191,	10s.	0d.	〃	9	11	6
210,	17s.	6d.	〃	10	10	10½		224,	18s.	4d.	〃	11	4	11
234,	3s.	4d.	〃	11	14	2		486,	2s.	6d.	〃	24	6	1½

XVI. To find the Interest of any sum at 5 per cent. per annum for any number of months.

Rule.—Take one penny for every pound in the principal, and one farthing for every 5s., which multiply by the number of months for which the interest is required, and the result will be the answer.

When the rate of interest or commission is 2½ per cent., it can be calculated as above by first taking half the given sum, and then calculating by the rule; or the interest or commission may first be calculated at 5 per cent., and then its half taken for the answer at 2½ per cent. Interest at 5 per cent. is equal to 1*d*. per pound for a month.

Exercises.—What is the interest, at 5 per cent. per annum, of

	months.	Answer.		months.	Answer.
£ 54, 0*s*. for	3	£0 13 6	£ 87, 5*s*. for	7	£2 10 10¾
65, 0*s*. 〃	4	1 1 8	98, 10*s*. 〃	6	2 9 3
73, 0*s*. 〃	2	0 12 2	109, 15*s*. 〃	9	4 2 3¾
87, 10*s*. 〃	8	2 18 4	145, 10*s*. 〃	11	6 13 4½
91, 15*s*. 〃	10	3 16 5½	149, 10*s*. 〃	5	3 2 3½
105, 10*s*. 〃	5	2 3 11½	163, 5*s*. 〃	8	5 8 10
67, 10*s*. 〃	1	0 5 7½	150, 10*s*. 〃	3	1 17 7½
75, 15*s*. 〃	4	1 5 3	180, 0*s*. 〃	7	5 5 0

XVII. To find the Interest, Commission, Brokerage, Gain, &c., at any rate per cent.

Rule.—Convert the shillings and pence of the principal, if any, to the decimal of a pound by Rule III., p. 139; then multiply the result by the rate per cent., and remove the decimal point two places to the left, which will give the answer in pounds and decimals of a pound; the decimals in this result, valued by Rule IV., p. 141, will give the shillings, pence, and farthings in the answer.

The Reason of this rule is obvious.

Exercises.

Find the interest, commission, brokerage, gain, &c., on

	per cent.	Answer.		per cent.	Answer.
£ 50, 10*s*. at	3	£1 10 3½	£150, 0*s*. at	8	£12 0 0
64, 15*s*. 〃	4	2 11 9½	160, 0*s*. 〃	7½	12 0 0
74, 10*s*. 〃	6	4 9 4¾	174, 0*s*. 〃	6½	11 6 2½
83, 5*s*. 〃	2	1 13 3½	186, 0*s*. 〃	9	16 14 9½
92, 10*s*. 〃	3½	3 4 9	204, 0*s*. 〃	8½	17 6 9½
105, 15*s*. 〃	7	7 8 0½	225, 0*s*. 〃	1½	3 7 6
127, 10*s*. 〃	4½	5 14 9	250, 0*s*. 〃	10½	26 5 0
140, 10*s*. 〃	2½	3 10 3	340, 0*s*. 〃	12½	42 10 0

XVIII. Given the cost price and the rate of gain, to find the selling price.

Rule.—To the cost price add the same part of itself that the gain is of £100, or of £1, or of 1*s*., and the sum will be the selling price which will give that gain.

For the aliquot parts of 100, see table p. 161.

Exercises.—Find the selling price of the following:

Cost Price.	Rate of Gain.		Selling Price.
£0, 12*s*. 6*d*.	25 per cent.	Ans.	£0 15 $7\frac{1}{2}$
0, 10*s*. 10*d*.	20 "	"	0 13 0
0, 8*s*. 4*d*.	$12\frac{1}{2}$ "	"	0 9 $4\frac{1}{2}$
0, 15*s*. 6*d*.	25 "	"	0 19 $4\frac{1}{2}$
0, 15*s*. 9*d*.	$33\frac{1}{3}$ "	"	1 1 0
0, 14*s*. 6*d*.	2*d*. per shilling, . .	"	0 16 11
1, 5*s*. 0*d*.	2*s*. per pound, . . .	"	1 7 6
2, 8*s*. 0*d*.	$\frac{1}{3}$ of the buying price, .	"	3 4 0

Most of the previous rules may easily be inverted, and thereby the number of exercises doubled; as, for example, to find the price of one, when the price of a dozen is given, we will evidently have the following:

Rule.—Call the shillings in the price of one, *pence*, to which add as many farthings as there are threepences in the price of a dozen, and the sum will be the price of one.

Again, let there be given the price of a pound avoirdupois, to find the price of an ounce; or the price of a yard, to find the price of a nail: by inverting Rule VII. p. 112, we will evidently have the following for finding the price of an ounce or a nail:

Rule.—Multiply the price of 16 by 3, and call the shillings in the product, *farthings*, and this will give the price of one; which will be the price of an ounce if the price of a pound be given, or the price of a nail if the price of a yard be given.

In the same manner may many of the other rules be inverted, and the exercises given under the rule will easily be accommodated to the inverted rule.

Miscellaneous Exercises, to be solved Mentally.

1. Change to decimals* $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{8}$, . . Ans. ·5, ·25, ·75, ·625.
2. " " $\frac{1}{3}$, $\frac{2}{9}$, $\frac{5}{7}$, $\frac{4}{11}$, . Ans. $\cdot\dot{3}$, $\cdot\dot{2}$, $\cdot\dot{7}1428\dot{5}$, $\cdot\dot{3}\dot{6}$.
3. Find equivalent fractions to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{5}{6}$, having a common denominator 12, . . . Ans. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{3}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$.
4. If $\frac{1}{8}$ of a ship be worth £200, what is the whole ship worth? Ans. £1600.
5. Change the fractions $\frac{12}{36}$, $\frac{17}{68}$, $\frac{32}{48}$, and $\frac{75}{150}$, to their lowest terms, Ans. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, and $\frac{1}{2}$.
6. If the $\frac{2}{11}$ of a number be 34, what is the whole number? Ans. 187.
7. If the $\frac{5}{8}$ of a number be 25, what is its product by 3? Ans. 120.
8. How often does 30 contain the $\frac{3}{4}$ of 8? . . Ans. 5 times.
9. What is the $\frac{3}{4}$ of $\frac{5}{6}$ of 120? . . . . . . Ans. 75.
10. A man gave away 3*s*., which was $\frac{1}{5}$ of the money he had; how much had he? . . . . Ans. 15*s*.
11. If from one pound there be paid $\frac{1}{3}$ of $\frac{1}{2}$, and then $\frac{1}{5}$ of $\frac{3}{4}$; how much will remain? . . . . . . Ans. 13*s*. 8*d*.

* See Decimals, p. 135.

RATIO.

Ratio is a word used to express the comparative magnitude of two numbers or quantities of the same kind, or how many times the one number or quantity contains the other.

Thus, if one field contain 9 acres, and another 3 acres, we find that 9 acres contain 3 acres *three* times; the ratio of 9 acres to 3 acres is therefore *three*. Again, since 4 shillings is the one-half of 8 shillings, the ratio of 4 shillings to 8 shillings is *one-half.*

The two numbers are called the *terms* of the ratio: the first, the *antecedent;* the second, the *consequent.* Thus, in the ratio 9 to 3, the first term, 9, is the antecedent; the second, 3, is the consequent.

Ratio, being just another name for quotient, is expressed by writing the numbers in the form of a fraction; the antecedent for the numerator, and the consequent for the denominator: thus, the ratio of 9 to 3 is expressed by $\frac{9}{3}$, the ratio of 4 to 8 by $\frac{4}{8}$.

The ratio of two numbers is also sometimes expressed by placing a colon between them: thus, the ratio of 9 to 3 is expressed by 9 : 3.

A number that does not refer to any particular kind of unit is called an *abstract* number: thus, 9 is an abstract number.

A number that denotes so many of some particular kind of unit is called a *concrete* number: thus, 9 *acres* is a concrete number.

Now, though the terms of a ratio are concrete numbers of the same denomination, the ratio itself is an abstract number, whole or fractional.

When both the terms of a ratio are multiplied, or both divided, by the same number, the ratio is not altered.

This is very obvious, since it is only multiplying or dividing the terms of a fraction by the same number; which does not alter its value (p. 81). Thus, the ratio $\frac{9}{3} = \frac{3}{1}$, and the ratio $\frac{4}{8} = \frac{1}{2}$.

Exercises.

1. What is the ratio of 12 : 5? . . . . . Ans. $2\frac{2}{5}$.
2. " " 234 : 15? . . . . " $15\frac{3}{5}$.
3. " " 19 : 37? . . . . . . " $\frac{19}{37}$.
4. " " 625 : 23? . . . . " $27\frac{4}{23}$.
5. " " 1728 : 144? . . . . . " 12.
6. " " 18 : 24? . . . . " $\frac{3}{4}$.

PROPORTION.

Proportion, in Arithmetic, denotes the equality of two Ratios.

If two Ratios are equal; that is, if the two numbers forming the one Ratio are of the same comparative magnitude with regard to each other, as the two numbers forming the other Ratio, the four numbers which compose these Ratios, taken in order, make a *Proportion.*

The *first* and *fourth* terms of a proportion are called the *extremes;* the *second* and *third*, the *means.*

Thus, since the ratio of 12*s.* to 6*s.* is 2, and the ratio of 8 yards to 4 yards also 2, the ratios are equal, and the four numbers, 12, 6, 8, 4, make a proportion, which is written thus—$12:6::8:4$; or thus, $12:6=8:4$, and is read, 12 is to 6 as 8 is to 4. The numbers 12 and 4 are the extremes, 6 and 8 the means.

In any proportion, if the first term is greater than the second, equal to it, or less than it, the third term is also greater than the fourth, equal to it, or less than it.

For, the $\frac{\textit{first}}{\textit{second}} = \frac{\textit{third}}{\textit{fourth}}$; now, if the first be greater than the second, the first fraction is greater than unity, therefore, the second fraction is also greater than unity: hence its numerator, which is the third term, must be greater than the denominator, which is the fourth term; and in the same way when equal, equal; and when less, less.

In a Proportion, the first and second terms must express things of the same kind; thus, if the first is yards, the second must also be yards; the third and fourth must also be of the same kind. There can be no comparison between things that are *not* of the same kind, as, for instance, between yards and minutes.

When four numbers are in proportion, *the product of the extremes* is equal to *the product of the means.*

For, take any four numbers in proportion, as 6, 9, 8, 12; then

$$6 : 9 :: 8 : 12$$

$$\therefore \frac{6}{9} = \frac{8}{12}$$

$$\therefore 6 \times 12 = 8 \times 9$$

Pupil. Since 6 is to 9 as 8 is to 12, 6 divided by 9 must be equal to 8 divided by 12. I now multiply each of these equals by 9 and by 12, which gives 6 multiplied by 12 equal to 8 multiplied by 9; that is, the product of the extremes is equal to the product of the means.

From this it follows that the product of the second and third terms divided by the first, is equal to the fourth. For, since $9 \times 8 = 6 \times 12$, if each of these equals is divided by 6, we get $9 \times 8 \div 6 = 12$.

Hence, if the first, second, and third terms of a Proportion are given, the fourth can be found, by multiplying the second and third terms together, and dividing their product by the first.

It is on these principles that the Rule termed Simple Proportion is founded.

SIMPLE PROPORTION.

Simple Proportion is the Rule by which, when three numbers are given, we are enabled to find a fourth unknown number. Hence it is also called the Rule of Three.

The question, If 10 yards of cloth cost £5, what will 25 yards cost? is a case of Simple Proportion.

Two out of the three given numbers, which are called terms, are always of the same *kind*—as, for instance, 10 yards and 25 yards; and the third is the same in kind as the fourth number sought; thus, if the third is pounds, the fourth is also pounds.

RULES.

I. Rule for Stating.—1. Write down as the *third* of the three terms (which are all to be placed in one line) that number which is of the same kind as the answer sought.

2. Consider, from the nature of the question, whether the answer should be greater or less than the third term: if greater, place the *greater* of the other two numbers as the second term—if less, place the *less* as the second; then place the remaining term first.

Two dots, thus [:] are placed between the first and second term, four dots [::] between the second and the third; the terms when stated appear thus—3 : 6 :: 12.

II. Rule for Working.—1. Reduce the first and second terms, if compound, to the same simple denomination; also the third term, if compound, to its lowest denomination.

Note.—It is unnecessary to reduce the third term to a lower denomination, when the second term does not exceed [illegible] [illegible] and dividing, mentioned below, can be carried [illegible] [illegible] according to Compound Multiplication and [illegible]

[illegible] ly the [illegible] nd third terms tog[illegible] [illegible] ide [illegible] The quotient [illegible] nswer [illegible] ne kind as th[illegible]

[illegible] d under [illegible]

Example 1.—If 25 cwt. of sugar cost £75, how much will 16 cwt. cost at the same rate?

£	
25 : 16 :: 75	
16	
450	
750	
25 = 5)1200	
5)240	
£48 *Ans.*	

Pupil. In this question, it is obvious that the answer will be in money; therefore, I place £75 for the third term. Again, as 25 cwt. cost £75, 16 cwt. will cost less; I therefore place 16, the less of the two numbers, for the second term, and 25 for the first. I next multiply £75 by 16, and then divide the product, £1200, by 25, and the quotient, £48, is the answer required.

Note.—The reason of the operation will also be obvious from the consideration, that since £75 is the price of 25 cwt., the value of 1 cwt. will be equal to the 25th part of £75—namely, £3; and as 16 cwt. will cost 16 times as much as 1 cwt., the price of 1 cwt., multiplied by 16, will give the price of 16 cwt. = £48.

Example 2.—If a man can perform a certain piece of work in 28 days, working 9 hours each day, in how many days would he perform a similar piece of work, working 12 hours each day?

days.	
12 : 9 :: 28	
9	
12)252	
21 days.	

This is an example of what is called *inverse* proportion. By the application of the rule, the work will stand as in the margin. As in last example, the reason of the operation will be obvious from the consideration, that since the workman takes 28 days of 9 hours; that is, 28 × 9 hours, or 252 to complete the work, he must require 252 hours to finish a similar piece of work; and as he now works 12 hours a day, therefore, 252, divided by 12, will give the number of days; that is, 21 days.

Example 3.—If 4 cwt. 2 qr. 11 lb. of tobacco cost £25, 17*s.* 6¼*d.*, how much ought to be paid for 2 cwt. 1 qr.?

	4	28			4	28				
cwt.	qr.	lb.		cwt.	qr.	lb.		£	*s.*	*d.*
4	2	11	:	2	1	0	::	25	17	6¼
4				4				20		
18				9				517		
28				28				12		
145				252				6210		
370								4		
515								24841		

$$\therefore \frac{24841 \times 252}{515} = 12155\tfrac{107}{515} f, \quad = \quad £12,\ 13s.\ 2\tfrac{3}{4}d.\ \tfrac{107}{515}.$$

In this example, the question is stated as before. The third term, being a compound number, is converted to farthings, the lowest denomination it contains; the first and second terms, being also compound numbers, are converted to pounds, the lowest denomination that the

first contains. The second and third terms are then multiplied together, and their product divided by the first, and the quotient, 12155, is farthings, being that denomination to which the third term was converted. The 12155 farthings are then converted to pounds, and the answer required is £12, 13*s*. 2¾*d*. $\frac{107}{515}$.

CONTRACTIONS OF THIS RULE.—The working of the questions may often be much shortened as follows:

Divide the two first terms, or the first and last terms (but never the second and third) by any number that will divide both without a remainder; then proceed according to the rule, with the quotients, instead of the original terms, which are now said to be cancelled—a stroke being drawn across them to indicate this.

The cancelling does not alter the relative proportion between the two terms, because both have been divided by the same number, and the answer to the question will be the same as if the original terms had been employed; whilst, from the terms having been lessened, the calculations are more easily performed.

Example 1.—If 15 yd. cost £9, 12*s*., what will 55 yd. cost?

yd.		yd.		£	s.	d.	
~~15~~	:	~~55~~	::	9	12	0	
3		11				11	
				3)105	12	0	
				£35	4	0	*Ans.*

Here the first two terms are divided by 5, and the quotients employed instead of the original terms, which are cancelled. See Note as to the multiplication of the third term, p. 121.

Example 2.—If 5 lb. cost £1, 7*s*. 6*d*., what will 45 lb. cost?

lb.		lb.		£	s.	d.	
~~5~~	:	~~45~~	::	1	7	6	
		9				9	
				£12	7	6	*Ans.*

Here the first two terms are divided by 5; and the *first* being thus *entirely* cancelled, all that is necessary is to *multiply* the third term by 9, the cancel of the second.

Example 3.—If 34 yd. cost £4, 8*s*., how much will 11*s*. purchase?

£	s.		s.		yd.
4	8	:	~~11~~	::	34
20					
~~88~~					8)34
8					4¼ yd. *Ans.*

Here the first two terms are divided by 11; and the *second* being thus entirely cancelled, it is only necessary to *divide* the third term by 8, the cancel of the first.

Example 4.—If 7 yd. cost 28*s*., how much will £5, 3*s*. purchase?

s.		£	s.		yd.
~~28~~	:	5	3	::	~~7~~
4		20			
		4)103			
		25¾ yd. *Ans.*			

Here the first and last terms are divided by 7; and the *last* being entirely cancelled, the second is divided by 4, the cancel of the first.

Example 5.—If 7 yd. cost 28*s*., what will 26 yd. cost?

$$\begin{array}{ccccc} \textit{yd.} & & \textit{yd.} & & \textit{s.} \\ \cancel{7} & : & 26 & :: & \cancel{28} \\ & & 4 & & 4 \\ \hline & & 104 & = & £5,\ 4s.\ \textit{Ans.} \end{array}$$

Here the first and last terms are divided by 7; and the *first* being entirely cancelled, 26 is multiplied by 4, the cancel of the last.

Exercises.

1. If 14 lb. of rice cost 6*s*., how much will 56 lb. cost at the same rate? . . . . Ans. 24*s*.
2. If 72 yd. of broad cloth cost £64, what will be the price of 9 yd.? . . . . Ans. £8.
3. If 16*s*. buy 18 yd. of cotton twist, how much cotton will 48*s*. buy? . . . . Ans. 54 yd.
4. How much wine may I buy for £396, when 68 gallons cost £51? . . . . Ans. 528 gal.
5. If 84 lb. of soap cost 49*s*., how much will 36 lb. come to at the same rate? . . . . Ans. 21*s*.
6. Bought 27 yd. for £11, how much will £44 buy? Ans. 108 yd.
7. How many acres of land may be rented for £63, if the rent of 364 acres be £546? . . . . . . Ans. 42 ac.
8. If 54 workmen can do a piece of work in 28 days, how many men could have done the same in 18 days? . Ans. 84 men.
9. If 39 men can perform a piece of work in 275 days, in what time would 143 men have performed it? . . Ans. 75 days.
10. How many boys could be maintained on £5220 for a year, when the annual maintenance of 16 boys cost £464? Ans. 180 boys.
11. How much will a man's wages amount to in 146 days, when his wages for 365 days are £35? . . . Ans. £14.
12. If 24 yd. of tartan cost £6, what will be the price of 764 yd. at the same rate? . . . . . . Ans. £191.
13. If 17 yd. of Orleans cloth cost £1, 9*s*. 7½*d*., how much would 85 yd. cost at the same rate? . . . Ans. £7, 8*s*. 1½*d*.
14. If 37 yd. 2 qr. of linen cost £4, 17*s*. 3*d*., how much should I pay for 98 yd.? . . . . Ans. £12, 14*s*. 1¾*d*. $\frac{1}{25}$.
15. What ought I to pay for 6 cwt. 1 qr. 6 lb. of tea, when 31 lb. cost £7, 15*s*.? . . . . . Ans. £176, 10*s*.
16. If 95 yd. of cloth cost £106, 18*s*. 5¼*d*., how much will 5 yd. cost at the same rate? . . . Ans. £5, 12*s*. 6½*d*. $\frac{7}{19}$.
17. If 47 yd. of velvet cost £34, 10*s*. 3¾*d*., what will 117 yd. cost? . . . . Ans. £85, 18*s*. 5¼*d*.
18. If 41 hogsheads of wine cost £287, 2*s*. 6¾*d*., what will be the price of 33 hogsheads at the same rate? Ans. £231, 2*s*. 0¾*d*.
19. How much sugar may be bought for £45, 11*s*. 3*d*., when the cost of 6 cwt. 1 qr. 12 lb. is £18, 4*s*. 6*d*.? Ans. 15 cwt. 3 qr. 16 lb.

20. If 14 lb. of iron cost 3*s.* 7½*d.*, how much iron may I purchase for £21, 4*s.* 1½*d.*? . . . . . Ans. 14 cwt. 2 qr. 14 lb.

21. What is the price of 8 pieces of scarlet, each 57 yd., when 76 yd. cost £72, 3*s.* 4¼*d.*? . . . Ans. £162, 7*s.* 6½*d.* ¼.

22. What is the price of 5 boxes of tea, each containing 37 lb., at £4, 13*s.* 9¾*d.* for 17 lb.? . . . Ans. £51, 0*s.* 10¾*d.* $\frac{4}{17}$.

23. How much will 41 yd. 2 qr. amount to at £22, 17*s.* 6*d.* for 25 English ells? . . . . . . Ans. £30, 7*s.* 6½*d.* $\frac{22}{25}$.

24. If the rent of 82 ac. 3 ro. 20 po. be £241, 17*s.* 2½*d.*, how much will the rent of 10 ac. 1 ro. amount to at the same rate?

Ans. £29, 18*s.* 3$\frac{0}{4}$*d.* $\frac{444}{663}$.

25. What is the price of 56 cwt. 3 qr. 14 lb. of sugar at 15*s.* 4½*d.* for 18 lb.? . . . . Ans. £272, 1*s.* 0½*d.*

26. What is the value of 126 cwt. of oatmeal, when 21 cwt. cost £19, 5*s.* 3¾*d.*? . . . . . Ans. £115, 11*s.* 10½*d.*

27. What is the price of 7 cwt. 3 qr. 18 lb. of sugar at 4*s.* 4½*d.* for 11 lb.? . . . . . . Ans. £17, 12*s.* 4½*d.* $\frac{8}{11}$.

28. What is the price of 15 cwt. 27 lb. of tea at £3, 7*s.* 4½*d.* for 7 lb.? . . . . Ans. £821, 9*s.* 10½*d.*

29. What is the price of 3 cwt. 16 lb. of iron at 7*s.* 1*d.* for 17 lb.?

Ans. £7, 6*s.* 8*d.*

30. If 18 men can perform a piece of work in 28 days, how many men could do it in a fourth part of the time? Ans. 72 men.

31. A servant's wages for 65 days amount to £3, 18*s.*, what will be the amount of his wages for a year? . Ans. £21, 18*s.*

32. If 7 bushels of wheat weigh 448 lb., what will be the weight of 3 pecks? . . . . Ans. 48 lb.

33. If I pay £10, 4*s.* 9*d.* of income-tax, being at the rate of 7*d.* in the pound, what is my annual income? . . Ans. £351.

34. Bought 420 gallons of oil for £76, 10*s.* 7½*d.*, what was the price of 25 gallons at the same rate? . Ans. £4, 11*s.* 1¼*d.* $\frac{3}{14}$.

35. If a person can perform a journey in 6 days, riding 9 hours each day, how long will it take him to perform the same journey if he rides 12 hours a day? . . . . . Ans. 4½ days.

36. A vessel has provisions for 15 days, but being obliged, by certain circumstances, to continue at sea for 20 days, to what quantity must the daily ration of 20 lb. be reduced to make the provision last during that time? . . . . Ans. 15 lb.

37. If 21 barrels of 32 gallons hold a certain quantity of goods, how many barrels of 42 gallons will hold the same quantity?

Ans. 16 barrels.

38. If a man walk 7 miles in 2 hours 10 min., how many miles will he walk, at the same rate, in 4 hours? . Ans. 12$\frac{12}{13}$ miles.

39. Last year 30 men reaped a farm in 27 days, how many must be engaged to reap it this year in 10 days? Ans. 81 men.

40. If I lend a friend £275 for nine months, how long must he lend me £1925 to return the favour?

Ans. 1 mo. 1 week 1 day, or 1$\frac{2}{7}$ mo.

41. When 19 gallons of whisky cost £8, 1*s.* 6*d.*, what quantity may I purchase for £243, 2*s.*? . . . . Ans. 572 gallons.

42. How much wheat may I purchase for £996, 18*s.* $1\frac{1}{2}$*d.*, when 68 qr. cost £185, 14*s.* 6*d.*? . . . . . Ans. 365 qr.

43. If 4 tons 6 cwt. of railway bars cost £39, 17*s.* $5\frac{1}{4}$*d.*, how much must be paid for 1723 tons 1 qr. at the same rate?

Ans. £15976, 13*s.* $8\frac{0}{4}$*d.* $\frac{53}{344}$.

44. If 27 oxen are grazed in a field for 112 days, how many oxen could have been grazed equally well in the same field for 48 days? . . . . Ans. 63 oxen.

45. If the soldiers in a besieged garrison have provisions sufficient for 5 months at the rate of 20 oz. per man a day, how long will they be able to hold out when each man's allowance is reduced to 12 oz. a day? . . . Ans. $8\frac{1}{3}$ months.

46. The rents of a parish amount to £1750, and a poor-rate is wanted of £98, 8*s.* 9*d.* What must be the assessment per pound?

Ans. 1*s.* $1\frac{1}{2}$*d.*

47. How much coffee may be bought for £12, 18*s.* $3\frac{1}{2}$*d.* when 2 lb. cost 3*s.* $4\frac{1}{4}$*d.*? . . . . Ans. 1 cwt. 1 qr. 14 lb. $\frac{2}{181}$.

48. If 327 yd. of linen cost £43, 8*s.* 9*d.*, how much will 237 yd. 2 qr. cost at the same rate? . . . Ans. £31, 10*s.* $11\frac{1}{2}$*d.* $\frac{76}{109}$.

49. At the rate of £15, 7*s.* $9\frac{1}{2}$*d.* for 23 gallons of wine, how much may be purchased for £210, 7*s.*? Ans. $314\frac{2746}{7387}$ gallons.

50. If the sixpenny loaf weigh 3 lb. 2 oz. when wheat is at 50*s.* per qr., how much ought it to weigh when wheat is selling at 56*s.* per qr.? . . . . Ans. 2 lb. $12\frac{9}{14}$ oz.

51. If the quarter loaf cost $8\frac{1}{2}$*d.* when wheat is at 60*s.* per qr., what ought to be its price when wheat is 49*s.* per qr.?

Ans. $6\frac{3}{4}$*d.* $\frac{23}{30}$.

52. A bankrupt's effects are valued at £983, 12*s.* 4*d.*, and his debts are £1728, 12*s.* $7\frac{1}{4}$*d.*, how much will his creditors receive per pound? . . . . Ans. 11*s.* $4\frac{1}{2}$*d.* $\frac{70284}{276681}$.

53. If 52 cwt. 18 lb. of beef cost £158, 1*s.* $8\frac{1}{2}$*d.*, how much may be bought for £82, 7*s.* $2\frac{1}{2}$*d.*? . . . . Ans. 27 cwt. 17 lb.

54. If a clerk has a salary of £72, 18*s.* a year, commencing on the 1st of January, how much will he have to receive on leaving his situation on the 25th of September? Ans. £53, 10*s.* $6\frac{1}{4}$*d.* $\frac{187}{365}$.

55. Sold 14 yd. 2 qr. 1 nail for £10, 15*s.* 4*d.* from a piece of cloth which at first contained 28 yd. 1 qr., what is the value of the remainder at the same rate? . Ans. £10, 2*s.* $4\frac{1}{2}$*d.* $\frac{222}{233}$.

56. What is the price of 56 packs of wool, each 114 stones, at £13, 17*s.* $2\frac{3}{4}$*d.* for 28 stones? . . . . Ans. £3160, 8*s.* 3*d.*

57. If 57 masons build a house in 108 days, in what time could 64 masons have done it? . . . . . Ans. $96\frac{3}{16}$ days.

58. If a cistern of 230 gallons has a pipe which discharges 5 gallons in a minute, and another has a pipe which discharges 6 gallons in a minute, and if both cisterns are emptied in the same time, how many gallons does this last cistern contain?

Ans. 276 gal.

59. If the carriage of 14 cwt. 0 qr. 28 lb. for 65 miles comes to a certain sum of money, what weight may I have carried 87 miles for the same sum? . . . . Ans. 24 cwt. 3 qr. 28 lb.

THE FOLLOWING METHOD of working accounts in proportion will contribute much to the extension of the pupil's ideas of arithmetic. He ought to be required to solve several of the foregoing exercises in a manner similar to that employed in the following examples:

Example 1.—What is the value of 184 yd. of cloth when 16 yd. cost £5, 11*s*. 9*d*.?

$$£5,\ 11s.\ 9d. = 1841d.,\ \therefore \text{ the price of 1 yd.} = \frac{1841d.}{16}$$

$$\therefore \text{the price of 184 yd.} = \frac{\overset{d.}{1841} \times 184}{16} = \frac{\overset{d.}{1841} \times 23}{2} = \overset{d.}{15421\tfrac{1}{2}} = \overset{£\ \ s.\ \ d.}{64\ \ 5\ \ 1\tfrac{1}{2}}$$

In this example, £5, 11*s*. 9*d*. is changed to pence; the result, 1841*d*., is therefore the price of 16 yd. in pence; hence the price of 1 yd. must be equal to $\frac{\overset{d.}{1841}}{16}$, and the value of 184 yd. must be equal to 184 times this last fraction; and therefore equal to $\frac{\overset{d.}{1841} \times 184}{16}$: cancelling 184 and 16 by 8, we obtain $\frac{\overset{d.}{1841} \times 23}{2}$; performing the indicated operations, we get 15421½*d*., which changed to pounds, is equal to £64, 5*s*. 1½*d*.

Example 2.—If 6 compositors set up a work in 24 days, in what time could 9 compositors have done it?

6 compositors set up the work in 24 days.

$\therefore$ 1 " " " $\overset{\text{days.}}{24} \times 6$

$\therefore$ 9 " " " $\frac{\overset{\text{days.}}{24} \times 6}{9} = \overset{\text{days.}}{8} \times 2 = 16$ days.

In this example, 6 compositors set up the work in 24 days, therefore, it is obvious that 1 compositor would take 6 times as many days; that is, 1 compositor takes $\overset{\text{days.}}{24} \times 6$. Now, it is also obvious that 9 compositors would just take the *ninth* part of the time that 1 compositor would; therefore $\frac{\overset{\text{days.}}{24} \times 6}{9}$ is the time required. Cancelling, we find the answer to be 16 days.

COMPOUND PROPORTION.

Compound Proportion is the Rule employed instead of Simple Proportion, when more than three terms (usually five) are given, to find the unknown quantity.

The question, If 12 persons earn £30 in 25 days, how much will 18 persons earn, at the same rate, in 56 days? is a case of Compound Proportion.

The principle of the rule is the same as in Simple Proportion.

In questions of Compound Proportion, one of the given numbers is always of the same kind with the number required, and the others, taken by pairs, are also of the same kind. Every question in Compound Proportion may be resolved into as many questions in Simple Proportion as there are pairs of terms; but, instead of working the simple questions separately, it is better to combine them into one. The answer will be found by the following rule, which depends on these principles, as will afterwards appear.

RULES.

I. Rule for Stating.—1. Write down for the *third* term that number which is of the same kind as the answer required.

2. For the first and second terms, take two numbers of the same kind, and state them as in Simple Proportion, placing the greater and the less, first and second, or the *reverse*, as the case requires.

3. Take other two numbers of the same kind with each other, and state them in like manner, placing them directly under the preceding two; and so on with any other pairs, till all the numbers are stated.

II. Rule for Working.—1. Reduce the numbers of the first and second terms to the same denomination, and if the third term is compound, reduce it to a simple number, as in Simple Proportion.

2. Multiply the numbers of the first term together: then those of the second term; thus reducing the different numbers of these two terms into a single quantity for each.

3. Proceed with the three terms thus found, as in Simple Proportion.

Note.—The terms may be cancelled, when practicable, on the *same principle as in* Simple Proportion.

Example 1.—If I give 16 men £45 for 28 days' work, what must I give, at the same rate, to 20 men for 35 days' work?

$$\begin{array}{llll} 16 \text{ men} : & 20 \text{ men} :: & \pounds 45 & \\ \underline{28 \text{ days}} & \underline{35 \text{ days}} & & \\ 128 & 700 & & \\ \underline{32} & \underline{\quad 45} & \pounds \quad s. \quad d. & \\ 448\,)\,31500\,(& & 70 \quad 6 \quad 3 & Ans. \end{array}$$

Here money being the answer required, £45 is written for the third term: then 16 men and 20 men being terms of the same kind, 16 and 20 are written as the first and second, according to the rule in Simple Proportion: 28 days and 35 days being also of the same kind, 28 and 35 are placed respectively below 16 and 20. The 16 is then multiplied by 28, making 448 for the first term: the 20 is next multiplied by 35, making 700 for the second: the three terms, 448, 700, and 45, are then proceeded with as in Simple Proportion.

The terms after being stated according to the Rule, may also be treated as follows:

Draw a line, then write all the numbers on the *left*-hand column as factors under the line, and all the rest as factors above it; cancel when possible; the value of the resulting fraction will be the answer required.

Example 2.—If 10 months' provisions for 180 boys cost £3760, what will be the cost of 12 months' provisions for 96 boys?

$$\begin{array}{ccccc} \text{mo.} & & \text{mo.} & & \pounds \\ 10 & : & 12 & :: & 3760 \\ \text{boys.} & & \text{boys.} & & \\ 180 & : & 96 & & \end{array}$$

$$\frac{\overset{1}{\cancel{12}} \times \overset{32}{\cancel{96}} \times 376\cancel{0}\ \pounds}{1\cancel{0} \times \underset{\underset{5}{\cancel{15}}}{\cancel{180}}} = \frac{\pounds 12032}{5} \qquad \begin{array}{r|r|r} \pounds & s. & d. \\ 5)12032 & 0 & 0 \\ \hline 2406 & 8 & 0 \end{array}$$

Pupil. In stating this question, £3760 is put for the third term, being of the same kind with the number required—namely, money. I next take 10 months and 12 months, and say, for 10 months the provisions cost £3760, for 12 months the provisions cost *more;* therefore, 12, the *greater* number, is placed for the *second*, and ten for the *first* term; again, I take 180 boys and 96 boys, and say, the provisions for 180 boys cost £3760, the provisions for 96 boys cost *less;* therefore, 96, the *less* number, is placed as the consequent under the 12, and 180 for the antecedent under 10. I then draw a line, and place the numbers 10 and 180 under it, and the others, 12, 96, and 3760, above, in the form of a fraction; I then cancel 12, and 180 by 12, and place the quotient 1 above the 12, and the quotient 15 under the 180, and dash the numbers 12 and 180. I next cancel 10 and 3760 by cutting off the cipher from each. I then cancel 96 and 15 by 3, dash them, and write down the quotients 32 and 5. I now multiply together 32 and 376, the numbers not dashed above the line, and place the product 12032, which is pounds, above a new line, and indicate its division by 5, by writing 5, the only number under the line not dashed, below it; then performing the division, I find the answer to be £2406, 8s.

Example 3.—If the carriage of 54 cwt. 2 qr. 7 lb. for 46 miles be £1, 15*s*., what distance may 23 cwt. 1 qr. 15 lb. be carried for £2, 5*s*. 6*d*.?

cwt.	qr.	lb.	lb.	lb.		lb.		miles
54	2	7	= 6111	2619	:	6111	::	46
23	1	15	= 2619					
£	*s*.	*d*.		*d*.		*d*.		
1	15	0	= 420*d*.	420	:	546		
2	5	6	= 546*d*.					

$$\frac{\overset{679}{\cancel{6111}} \times \overset{13}{\cancel{546}} \times \overset{23}{\cancel{46}}}{\underset{291}{\cancel{2619}} \times \underset{\underset{5}{\cancel{10}}}{\cancel{420}}} = \frac{203021}{1455} \text{ miles}$$

$$1455)203021(139\tfrac{8}{15} \text{ miles } Ans.$$

$$\begin{array}{r}
1455 \\ \hline
5752 \\
4365 \\ \hline
13871 \\
13095 \\ \hline
\end{array}$$

$$97\left|\frac{776}{1455}\right| = \frac{8}{15}$$

In this example, the compound numbers, 54 cwt. 2 qr. 7 lb., and 23 cwt. 1 qr. 15 lb., are changed to the same simple denomination, namely, pounds, and the results stated as before: also, the compound numbers, £1, 15*s*. and £2, 5*s*. 6*d*., are treated in the same way. The working then proceeds as in the last example.

If the terms of one proportion be multiplied by the corresponding terms of another proportion, the products will be in proportion.

Thus, $\quad 6 : 9 :: 8 : 12$

$\quad 4 : 11 :: 16 : 44$, are two proportions;

then will $\quad 6 \times 4 : 9 \times 11 :: 8 \times 16 : 12 \times 44$.

For from the first proportion $\frac{6}{9} = \frac{8}{12}$, and from the second $\frac{4}{11} = \frac{16}{44}$;

Therefore, $$\frac{6 \times 4}{9 \times 11} = \frac{8 \times 16}{12 \times 44};$$

that is, $\quad 6 \times 4 : 9 \times 11 :: 8 \times 16 : 12 \times 44$.

This last proportion is said to be *compounded* of the other two; *and hence* the Rule of COMPOUND PROPORTION.

We now proceed to illustrate the Rule by the following example:

If 12 men build 18 roods of a wall in 30 days of 9 hours each, how many days of 8 hours each would 18 men take to build a similar wall 52 roods long?

This question is resolvable into the three following simple ones:

1. If 12 men build a certain length of a wall in 30 days, how many days would 18 men take to build the same length of wall? For a moment, let the answer to this question be denoted by a days.

2. If 18 men build 18 roods of a wall in a days, how many days would they take to build a wall 52 roods long? Let the answer to this be denoted by b days.

3. If 18 men build 52 roods of a wall in b days of 9 hours each, how many days of 8 hours each would they take to perform the same piece of work? The answer to this is the number of days required; denote it by c.

The statement of the first question will stand thus, $\overset{\text{men.}}{18} : \overset{\text{men.}}{12} :: \overset{\text{days.}}{30} : \overset{\text{days.}}{a}$

" " second " " $\overset{\text{ro.}}{18} : \overset{\text{ro.}}{52} :: a : b$

" " third " " $\overset{\text{hr.}}{8} : \overset{\text{hr.}}{9} :: b : c$

Then, regarding the terms of the several proportions as abstract numbers, and compounding them together, we obtain

$$18 \times 18 \times 8 : 12 \times 52 \times 9 :: 30 \times a \times b : a \times b \times c;$$

and striking out the common factors a and b from the terms of the second ratio, we have

$$18 \times 18 \times 8 : 12 \times 52 \times 9 :: \overset{\text{days.}}{30} : \overset{\text{days.}}{c}$$

Hence $\dfrac{12 \times 52 \times 9 \times 30}{18 \times 18 \times 8} = c$, the number of days required.

The names being now given to the indicated operations, furnish the rule.

Exercises.

1. If 75 men cut down 45 acres of corn in 4 days, how many acres will 108 men, working equally well, cut down in 25 days? Ans. 405 acres.

2. If 16 persons can be maintained for 60 days on £84, how much money would be required to support, in similar circumstances, 96 men for 365 days? . . . . Ans. £3066.

3. If 18 compositors can set up 24 sheets in 8 days, how many sheets could 45 compositors set up in 14 days? . Ans. 105.

4. If 25 horses consume 8 bushels of oats in 3 days, how many bushels would 42 horses consume in 15 days, at the same rate of living? . . . . Ans. $67\frac{1}{5}$ bushels.

5. If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of $12\frac{1}{2}$ sheets? . . . . Ans. 125 reams.

6. If 84 men mow 72 acres of grass in 15 days, how many acres will 96 men mow in 12 days? . Ans. 65 ac. 3 ro. $12\frac{4}{7}$ po.

7. If 27 men build 54 roods of a garden-wall in 26 days, how many roods will 32 men, working equally well, build in 39 days? Ans. 96 roods.

8. If £100 gain £5 of interest in 12 months, how much interest will £750 gain in 42 months? . . . . Ans. £131, 5*s*.

9. If the freight of a ship of 130 tons for 3 months be £81, what would the freight of a ship of 216 tons for 7 months amount to? . . . . Ans. £314, 0*s*. $7\frac{1}{4}$*d*. $\frac{7}{13}$.

10. If £350 gain £7, 17*s*. 6*d*. in 219 days, how much interest will £100 gain in 365 days? . . . . . Ans. £$3\frac{3}{4}$.

11. If 10 horses plough 18 acres of land in 7 days, how many horses would be required to plough 171 acres in 35 days, at the same rate of working? . . . . . . Ans. 19 horses.

12. If a barrel of beer serve a family of 8 persons for 13 days, how many barrels will be required to serve equally well a family of 24 persons for a year? . . . . . Ans. $84\frac{3}{13}$ barrels.

13. If 236 men eat 160 qr. of wheat in 108 days, how many quarters will 76 men eat in one year and 67 days? Ans. $206\frac{6}{59}$.

14. If a man can travel 360 miles in 12 days of 8 hours each, how many miles, at the same rate of walking, will he travel in 60 days of 6 hours each? . . . . . . Ans. 1350 miles.

15. If 24 men can perform a piece of work in 16 days of 12 hours each, in what time will 20 men accomplish a piece of similar work 4 times as large, when the days are only eight hours long? . . . . Ans. $115\frac{1}{5}$ days.

16. If 320,000 bricks, 9 inches long, 5 inches broad, and $2\frac{1}{4}$ inches thick, are required for the construction of a magazine, how many bricks, 12 inches long, 6 inches broad, and 3 inches thick, would be required for the same purpose? Ans. 150,000 bricks.

17. If 90 men can dig a trench 200 yd. long, 3 yd. broad, and 2 yd. deep, in 6 days of 10 hours each, in how many days of 12 hours each will 50 men dig a trench 320 yd. long, 4 yd. broad, and $2\frac{1}{2}$ yd. deep? . . . . Ans. 24 days.

18. If 18 men eat 15*s*. worth of bread in 3 days, when wheat is selling at 54*s*. per qr., what value of bread will 54 men eat in 27 days, when wheat is selling at 50*s*. per qr.? . Ans. £18, 15*s*.

19. If the carriage of 2 qr. 7 lb. for 54 miles is 8*s*. $7\frac{1}{2}$*d*., how far may 1 cwt. 3 qr. 14 lb. be carried for £1, 7*s*. 6*d*. at the same rate of charge? . . . . Ans. $51\frac{15}{23}$ miles.

20. A road-contractor engaged to finish $2\frac{1}{2}$ miles of road in 84 days; but after employing 60 men for 54 days, he found that they had only finished 880 yards; how many additional men must he engage, so that the work may be finished within the *prescribed time?* . . . . Ans. 372 men.

SIMPLE PROPORTION

IN FRACTIONS.

In Proportion of Fractions, the questions are *stated* as in Integers; but the several terms are reduced, multiplied, and divided by the Rules for the reduction, multiplication, and division of Fractions.

The work may also be performed by converting the first and second terms to a common denominator; the fractions will then be proportional to the numerators of the new fractions. These new numerators may then be used instead of the original fractions.

Example.—If $4\frac{7}{8}$ cwt. of camphor cost £$29\frac{5}{6}$, how much will $1\frac{2}{3}$ cwt. cost at the same rate?

$$
\begin{array}{ccccc}
 & & & & £ \\
4\frac{7}{8} & : & 1\frac{2}{3} & :: & 29\frac{5}{6} \\
8 & & 3 & & 6 \\
\hline
\frac{39}{8} & & \frac{5}{3} & & \frac{179}{6}
\end{array}
$$

$$
\therefore \frac{5}{3} \times \frac{179}{\underset{3}{\cancel{6}}} \times \frac{\overset{4}{\cancel{8}}}{39} = \frac{£3580}{351} = £10\ 3s.\ 11\tfrac{3}{4}d.\ \tfrac{53}{117}\ \textit{Ans.}
$$

Pupil. In this example, I put £$29\frac{5}{6}$ for the third term; and as the answer will obviously be less than this, I put $1\frac{2}{3}$ for the second, and $4\frac{7}{8}$ for the first term. I now change the several mixed numbers to their fractional forms, and then multiply the second term, $\frac{5}{3}$, by the third, $\frac{179}{6}$, and divide the product by the first, $\frac{39}{8}$; or, what amounts to the same thing, I multiply by its reciprocal, $\frac{8}{39}$. I next cancel 8 and 6 by 2, and write the resulting quotients, 4 and 3, above and below them; and then multiply those numbers that are not cancelled, and find the numerator of the answer to be 3580, and the denominator 351. This fraction is that of a pound, because the third term is pounds; this, when valued, gives £10, 3s. $11\frac{3}{4}$d. $\frac{53}{117}$ for the answer required.

Exercises.

1. If $\frac{3}{4}$ yd. cost £$\frac{7}{18}$, what will be the value of $\frac{9}{14}$ yd. at the same rate? . . . . Ans. 6*s*. 8*d*.
2. If $14\frac{3}{7}$ yd. cost £$2\frac{5}{48}$, what will $5\frac{3}{5}$ yd. cost at the same rate? Ans. 16*s*. 4*d*.
3. What is the value of $5\frac{3}{7}$ acres of land, when $1\frac{7}{12}$ acres cost £59, 6*s*. 6*d*.? . . . . Ans. £203, 8*s*.
4. What is the price of $5\frac{3}{7}$ yd. of velvet at 7*s*. 6*d*. for $\frac{3}{4}$ yd.? Ans. £2, 14*s*. $3\frac{1}{4}$*d*. $\frac{5}{7}$.
5. If $5\frac{4}{9}$*s*. pay for $1\frac{5}{18}$ lb. of tea, how much tea may be bought for £$7\frac{7}{12}$? . . . . . . Ans. 1 qr. $7\frac{25}{42}$ lb.
6. If $\frac{1}{16}$ of a ship is worth £475, 10*s*., what is the value of $\frac{8}{9}$ of $\frac{23}{24}$ of her at the same rate? . Ans. £6480, 17*s*. $9\frac{1}{4}$*d*. $\frac{1}{3}$.
7. How much will 8 cwt. 3 qr. of sugar cost, when 2*s*. $7\frac{1}{2}$*d*. is the price of $4\frac{3}{8}$ lb.? . . . . . . Ans. £29, 8*s*.
8. What cost 3 chests of tea, each 3 cwt. 14 lb. at 5*s*. 7*d*. for $1\frac{1}{8}$ lb.? . . . . . . Ans. £260, 11*s*. $1\frac{1}{4}$*d*. $\frac{1}{3}$.
9. How much wine may be bought for £27, 15*s*. 6*d*., if $6\frac{3}{17}$ gal. cost £$7\frac{1}{5}$? . . . . Ans. $23\frac{1349}{1632}$ gal.
10. If $13\frac{1}{2}$ ounces avoirdupois cost £$1\frac{4}{5}$, what will $18\frac{6}{7}$ lb. cost at the same rate? . . . . . Ans. £40, 4*s*. $6\frac{3}{4}$*d*. $\frac{3}{7}$.
11. A silver cup weighs 1 lb. $7\frac{1}{3}$ oz., find its value at the rate of 7*s*. $10\frac{1}{2}$*d*. for 1 oz. $2\frac{1}{2}$ dwt. . . . . Ans. £6, 15*s*. 4*d*.
12. What is the price of $24\frac{3}{8}$ English ells, when $5\frac{5}{9}$ yd. cost £7, 8*s*. $11\frac{1}{2}$*d*.? . . . . . Ans. £40, 16*s*. $11\frac{1}{4}$*d*. $\frac{9}{32}$.
13. I gave £$2\frac{5}{8}$ for $1\frac{3}{8}$ cwt. of lead, how much might I have purchased for £$7\frac{7}{11}$ at the same rate? . . Ans. 4 cwt.
14. If $5\frac{4}{9}$ cwt. of barilla cost £16, 4*s*. $9\frac{1}{4}$*d*., how much would $17\frac{1}{7}$ lb. cost at the same rate? . . Ans. 9*s*. $1\frac{1}{2}$*d*. $\frac{177}{686}$.
15. If a railway train, running at the rate of $27\frac{1}{2}$ miles an hour, traverse a distance in $2\frac{5}{12}$ hours; in what time will another running $22\frac{3}{4}$ miles an hour traverse the same distance? . . . . Ans. $2\frac{503}{546}$ hours.
16. For every $4\frac{3}{8}$ miles that A walks, B goes $5\frac{4}{9}$ miles. How long will B take to traverse a distance walked by A in $7\frac{7}{9}$ hours? . . . . Ans. $6\frac{1}{4}$ hours.

DECIMAL FRACTIONS.

Decimal Fractions are those which are written decimally in the same way as integers—the only difference being that a *point* is placed before the figures for distinction. They express tenths or combinations of tenths: thus—·3, three-tenths; ·47, forty-seven hundredths. The term is derived from the Latin word *decem*, signifying ten.

In *Decimal* Fractions, instead of both numerator and denominator being expressed, as in *Vulgar* Fractions, the *numerator* only is written down, and the *denominator* is always understood to be either 10, 100, 1000, or some other combination of tenths, according to the number of figures in the numerator.

If the numerator consists of 1 figure, tenths are meant, and the understood denominator is 10; if of 2 figures, hundredths are meant, and the denominator understood is 100, and so on; the denominator being always 1, with as many nothings annexed as there are figures in the numerator: for example—$\frac{9}{10}$ are written decimally ·9; or $\frac{99}{100}$ as ·99. An integer with a fraction, as $3\frac{9}{10}$, is written thus—3·9; the *point* [·] marking the separation between them.

As in integers, a figure *increases* in value ten times by every removal to the *left* of the units' place, so in decimals, a figure *decreases* in value ten times by every removal to the *right* of the units' place.

The first figure to the *right* of the point, or, as it is termed, the first *place* of decimals, expresses so many *tenths;* the second figure to the right, so many *hundredths;* the third figure, so many *thousandths;* and so on, according to the following table:

·1	=	$\frac{1}{10}$	or 1-tenth.	·1	=	1-10th.
·01	=	$\frac{1}{100}$	" 1-hundredth.	·12	=	12-100ths.
·001	=	$\frac{1}{1000}$	" 1-thousandth.	·123	=	123-1000ths.
·0001	=	$\frac{1}{10\cdot000}$	" 1-ten-thousand.	·1234	=	1234-10,000ths.
·00001	=	$\frac{1}{100,000}$	" 1-hund.-thous.	·12345	=	12345-100,000ths.
·000001	=	$\frac{1}{1,000,000}$	" 1-millionth.	·123456	=	123456-1,000,000ths.

Decimals are read from left to right, as in whole numbers, beginning at the first figure after the point, and naming them according to the number of parts expressed by the figures; thus —5·375, is read five, and three hundred and seventy-five *thousandths.*

The number 78·512 is read Seventy-eight, and five hundred and twelve thousandths; or more generally, Seventy-eight, *decimal* five, one, two, the local values of the figures being always understood.

Since 78·512 = $\frac{78512}{1000}$, it follows generally that a number, whether consisting partly of *integers* and partly of *decimals*, or entirely of *decimals*, is equal to a fraction having the original number considered as a whole number for its numerator, and for its denominator one with as many ciphers annexed to it as there are decimal places; hence the rules for operating on fractions may be applied to decimals.

The *annexing* of nothings to the *right* of decimals does not alter or increase their value: thus ·5 and ·500 express the same value, because ·5 bears the same proportion to 10, that ·500 does to 1000, each being equal to one-half ($\frac{1}{2}$).

It should be carefully borne in mind, however, that the *prefixing* of nothings to the *left* of decimals decreases their value tenfold for every nothing prefixed: thus ·1 ($\frac{1}{10}$) becomes ·01 ($\frac{1}{100}$) by prefixing *one* nothing.

Since the *local* values of figures in whole numbers and decimals are determined by the same number—namely, *ten*, all operations on decimals are performed in the same way as the corresponding operations on whole numbers. The only peculiarity lies in the pointing off of the decimals.

Exercises.

1. Express 8·37, ·625, ·0029, 3·1416, 783·17, ·000003, 162·05, 100·001, ·063729, and 17·947, as decimal fractions.

2. Express $\frac{182}{10}$, $\frac{7854}{1000}$, $\frac{271828}{100000}$, $\frac{13}{1000000}$, $\frac{31037}{1000}$, $\frac{789}{100000}$, $\frac{9}{100000}$, $\frac{33}{1000000}$, and $\frac{825}{100000}$, as decimals.

The pupil should place the decimal point at the *top* or *middle* of the figure, as in the foregoing examples, and not at the *bottom*, as it then indicates multiplication.

REDUCTION OF DECIMALS.

I. To convert a Vulgar Fraction into a Decimal.

Rule—1. Annex as many nothings to the numerator of the fraction as will form a number greater than the denominator, and then divide this number by the denominator.

2. If there is any remainder, annex to it more nothings, and then divide as before; and so on till there is no remainder, or till as many decimals have been got as may be thought necessary.

The answer must contain as many decimals as there have

been nothings annexed to the numerator; if there are not as many after dividing, *prefix* the requisite number of nothings to the quotient.

Example 1.—Convert $\frac{7}{256}$ to a decimal.

```
256)700(·02734375
    512
    ----
    1880
    1792
    ----
     880
     768
     ----
     1120
     1024
     ----
       960
       768
       ----
       1920
       1792
       ----
        1280
        1280
```

Pupil. In this example I require to add two ciphers to the numerator, 7, before I obtain a number greater than the denominator, 256; I then divide 700 by 256, and obtain the quotient figure 2. From the quotient I point off *two* decimal places; but as there is but one, I supply the deficiency by writing down a cipher on the left, and in this way I get ·02. Again, as there is a remainder, 188, I annex a cipher to it, and continue the division, as in whole numbers, until there is no remainder. Hence the decimal required is ·02734375.

Example 2.—Convert $\frac{5}{6}$ to a decimal.

```
6)5000
  ----
  ·833
```

In this example, each remainder after the first is 2, and hence the same quotient figure 3 is repeated continually.

Example 3.—Convert $\frac{7}{22}$ to a decimal.

```
22)70(·31818
   66
   --
    40
    22
    ---
    180
    176
    ---
      40
      22
      ---
      180
      176
      ---
        4
```

In this example, after the first three figures, ·318 of the quotient, are obtained, we get a remainder 4, which is equal to the first remainder, and hence the same figures 18 will be continually repeated in the quotient.

THE REASON OF THE RULE may be shewn in the following way:

$$\frac{7}{256} = \frac{700000000}{25600000000} = \frac{\frac{700000000}{256}}{\frac{25600000000}{256}} = \frac{2734375}{100000000} = ·02734375.$$

It is only some Vulgar Fractions that admit of being exactly expressed by decimals, as in Example 1, where, on dividing, the fraction terminates exactly in the decimal; which is hence called a *terminate* or *finite* decimal.

There are other fractions, as in Examples 2 and 3, which cannot be exactly expressed by decimals, as they do not terminate exactly; however, by carrying the division to several places of decimals, as 8333, &c., the difference between the decimal and the exact fraction becomes too trifling to be appreciable; such decimals are called *interminate.*

The Term, Recurring Decimal, is applied to those interminate decimals in which the same figure or figures are continually repeated; they are called *Repeating*, or, *Circulating*, according to the number of figures repeated. The part repeated is called the *period.*

A *Repeating* decimal is where the *same* figure is repeated, and is indicated by a dot placed over the recurring figure; thus, ·833, &c., is written as $\cdot 8\dot{3}$.

A *Circulating* decimal is where two or more figures are repeated, and is indicated by a dot over the first and last recurring figures; thus, ·31818, &c., is written as $\cdot 3\dot{1}\dot{8}$, and ·73925925, &c., as $\cdot 73\dot{9}2\dot{5}$.

When we use the first three or four places of a decimal consisting of a large number of figures, instead of the decimal itself, the decimal thus abridged is sometimes called an *approximate* decimal.

Thus, $\frac{1}{3}$ = ·3333, &c.; $\frac{1}{7}$ = ·14285714, &c.; $\frac{7}{256}$ = ·027343, &c.; the decimals, ·3333, ·14285714, and ·027343, are *approximate* decimals. The smallness of the error that we introduce by using approximate decimals, instead of the fractions from which they are derived, depends upon the number of decimal places we employ: thus, if we use ·14 instead of $\frac{1}{7}$, the error is less than $\frac{1}{100}$, since the true value is more than $\frac{14}{100}$, but less than $\frac{15}{100}$; if we use ·142, the error is less than $\frac{1}{1000}$; and so on.

In approximate decimals, the figure we stop at is generally increased by 1, if the figure in the next decimal place is 5 or upwards: and to denote that the approximate decimal is less than the complete decimal, we sometimes annex +; but to denote that it is greater, we annex —.

Thus, in approximating to 3·1415926, if we retain 5 places of decimals, we write 3·14159+, denoting that the complete decimal is greater; but if we retain 4 places only, we write 3·1416—, denoting that the complete decimal is somewhat less.

Exercises.—Reduce the following fractions to decimals:

1. $\frac{1}{2}$,	Ans. ·5.	5. $\frac{3}{4}$,	Ans. ·75.
2. $\frac{1}{4}$,	" ·25.	6. $\frac{12}{25}$,	" ·48.
3. $\frac{5}{8}$,	" ·625.	7. $\frac{7}{625}$,	" ·0112.
4. $\frac{7}{32}$,	" ·21875.	8. $\frac{1}{400}$,	" ·0025.

9. $\frac{3}{14}$,	Ans.	$\cdot 2\dot{1}4285\dot{7}$.	13. $\frac{3}{28}$,	Ans.	$\cdot 10\dot{7}1428\dot{5}$.
10. $\frac{5}{27}$,	"	$\cdot\dot{1}8\dot{5}$.	14. $\frac{6}{2175}$,	"	$\cdot 0027585567$—
11. $\frac{2}{9}$,	"	$\cdot\dot{2}$.	15. $\frac{4}{13}$,	"	$\cdot\dot{3}0769\dot{2}$.
12. $\frac{45}{52}$,	"	$\cdot 86\dot{5}38846\dot{1}$.	16. $\frac{7}{1001}$,	"	$\cdot\dot{0}0699\dot{3}$.

II. To convert a *terminate* Decimal into a Vulgar Fraction.

Rule.—Write the given decimal as the numerator, and for the denominator, write 1, and as many nothings as there are figures in the decimal: then reduce the fraction thus obtained to its lowest terms.

Example.—Reduce ·625 to a vulgar fraction.

$$\frac{625}{1000} = \frac{5}{8} \textit{ Answer.}$$

Here the numerator is 625, and as there are three figures in the decimal, the denominator is 1000.

The Reason of the Rule is obvious from what is stated at page 136.

Exercises.—Reduce the following decimals to vulgar fractions:

17. ·125, ·824, ·925, and ·72875, . Ans. $\frac{1}{8}$, $\frac{103}{125}$, $\frac{37}{40}$, and $\frac{583}{800}$.
18. ·21875, ·00112, ·14875, and ·7125, Ans. $\frac{7}{32}$, $\frac{7}{6250}$, $\frac{119}{800}$, and $\frac{57}{80}$.
19. ·015625, ·08125, ·2125, and ·1175, Ans. $\frac{1}{64}$, $\frac{13}{160}$, $\frac{17}{80}$, and $\frac{47}{400}$.

III. To convert a compound number (as 2*s.* 6*d.*) to the Decimal of a higher denomination.

Rule—1. Convert the given sum, when compound, to its lowest denomination; convert also *one* of the specified higher denomination to the same denomination as the other.

2. Annex as many nothings to the former as will make it greater than the latter; then divide the one by the other, continuing to annex nothings and to divide till there is no remainder, or as far as the division is wished to be carried.

There must be as many decimals in the answer as there have been ciphers annexed.

Example 1.—Convert 2*s.* 6*d.* to the decimal of a pound.

```
   2s. 6d.
   12
240)30000
    £·125  Ans.
```

Here 2*s.* 6*d.* is reduced to its lowest denomination, pence = 30, and one of the specified higher denomination, pounds, is also reduced to pence = 240; nothings are then annexed to 30, and the dividend divided by 240. As three nothings have been annexed, there are three decimals in the answer.

Example 2.—Convert 14*s.* 7¾*d.* to the decimal of a pound.

Since 14*s.* 7¾*d.* = 703*f.*, and £1 = 960*f.*, therefore 14*s.* 7¾*d.* = £$\frac{703}{960}$; and this fraction, changed to a decimal, becomes £·732291$\dot{6}$.

$$\begin{array}{r} 4\,)\;3{\cdot}00 \\ 12\,)\;7{\cdot}75000 \\ 20\,)14{\cdot}6458\dot{3} \\ \pounds{\cdot}732291\dot{6} \end{array}$$

The same result is obtained by proceeding as in the margin. Change 3*f.* to the decimal of a penny by dividing by 4; then prefix the 7 to the decimal ·75, and change the mixed decimal to the decimal of a shilling by dividing by 12; and lastly, prefix the 14*s.*, and change the result to the decimal of a pound by dividing by 20. This method is most easily learned from an example, and is generally more convenient than that given in the Rule.

Exercises.

20. Convert 12*s.* 6*d.*, 5*s.* 4*d.*, and 6*s.* 3½*d.* to the decimal of a pound, ... Ans. £·625, £·2$\dot{6}$, £·31458$\dot{3}$.
21. Convert 17*s.* 5¾*d.*, 18*s.* 7½*d.*, and 13*s.* 6*d.* to the decimal of a pound, . . . Ans. £·873958$\dot{3}$, £·93125, £·675.
22. Convert 3*s.* 10¼*d.*, 19*s.* 9*d.*, and 16*s.* 8*d.* to the decimal of a pound, ... Ans. £·192708$\dot{3}$, £·9875, £·8$\dot{3}$.
23. Convert 4*d.*, 6*d.*, and 8*d.* to the decimal of a pound, Ans. £·01$\dot{6}$, £·025, £·0$\dot{3}$.
24. Convert 1*s.* 2*d.*, 3*s.* 5¼*d.*, and 6*s.* 2¾*d.* to the decimal of a guinea, . . Ans. ·0$\dot{5}$g., ·164$\dot{6}$8253$\dot{9}$g., ·2966$\dot{2}$6984$\dot{1}$g.
25. Convert 2 oz. 14 dwt. 12 grains to the decimal of a lb. Ans. ·22708$\dot{3}$ lb.
26. " 2 qr. 17 lb. to the decimal of a cwt. Ans. ·6517$\dot{8}$5714$\dot{2}$ cwt.
27. " 3 cwt. 3 qr. 8 lb. to the decimal of a ton, Ans. ·1910$\dot{7}$1428$\dot{5}$ ton.
28. " 15 lb. to the decimal of a cwt. Ans. ·1339$\dot{2}$8571$\dot{4}$ cwt.
29. " 1 ro. 10 po. to the decimal of an acre, Ans. ·3125 ac.
30. " 3 ro. 27 po. to the decimal of an acre, Ans. ·91875 ac.
31. " 365 days 5 ho. 48 min. 51 sec. to the decimal of a day, ... Ans. 365·2422569$\dot{4}$ day.

To convert Shillings, Pence, and Farthings into the decimals of a pound, the following is a convenient rule in practice:

Reckon *half* the number of the shillings as the first decimal; *thus*, consider 12*s.* as ·6; if the number of shillings is odd (as 13*s.*),

call the odd shilling 50, to which add the number of farthings in the pence and farthings, increased by *one*, if they are 24 or upwards; this sum will be the second and third figures of the decimal. This decimal will never differ from the true decimal by so much as a unit in the third figure, and will therefore never differ from the given sum by so much as one farthing.

Example.—Convert 13*s.* 6½*d.* into the decimals of £1. Ans. £·677.

Here the half of 13*s.* is 6*s.*, and 1 over: the 6 is placed as the first decimal; then the odd 1*s.* = 50, and 6½*d.* = 26 + 1, make 77, which are placed as the second and third decimals.

THE REASON of the above process may be shewn thus: When the number of shillings is *even*, the half of that number will obviously consist of one figure, and express so many *tenths* of a pound; and as 1 farthing is the 960th part of a pound, and 960, increased by a 24th part of itself, becomes 1000, any number of farthings increased by a 24th part will express so many *thousandth* parts of a pound. When the number of shillings is *odd*, there will be a remainder of 1*s.*; and as 1*s.* = £$\frac{1}{20}$ = £$\frac{50}{1000}$, therefore, the last obtained sum must be increased by 50, when there is a remainder of 1*s.*, to obtain the thousandth parts required.

Exercises.

32. 7*s.* 6*d.* = ·375	36. 17*s.* 9*d.* = ·887	40. 19*s.* 1*d.* = ·954
33. 3*s.* 4*d.* = ·166	37. 12*s.* 7*d.* = ·629	41. 1*s.* 10*d.* = ·091
34. 4*s.* 6*d.* = ·225	38. 14*s.* 3*d.* = ·712	42. 0*s.* 7½*d.* = ·031
35. 18*s.* 4*d.* = ·916	39. 15*s.* 6*d.* = ·775	43. 0*s.* 8¼*d.* = ·034

IV. TO FIND THE VALUE OF A DECIMAL OF A GIVEN DENOMINATION.

RULE—1. Consider the decimal as so many of the given denomination, and then reduce it to the next lower denomination.

2. Point off from the *right* of this lower denomination as many figures as there are in the given decimal, and the figures that remain at the *left*, if any, express how many of *this* denomination the decimal contains.

3. Reduce the figures *pointed off* to the next lower denomination; then point off as before, and so on, to the lowest denomination required.

The figures that remain at the *left*, after each pointing off, with any remainder at the last, express the value of the given decimal.

Example 1.—Find the value of £·375.

£375
20
7·500
12
6·000

Here £·375 is reckoned as £375, and reduced to shillings, 7500. We then point off three figures from the right, as the decimal contains three figures, leaving 7*s.*; then reducing to pence the figures pointed off, 500, making 6000, we again point off three figures, leaving 6*d.* The answer is thus 7*s.* 6*d.*

Example 2.—What is the value of ·71285 cwt.?

cwt.
·71285
4
2·85140 qr.
28
68112
170280
23·8392 lb.
16
50352
83920
13·4272 oz.
16
25632
42720
6·8352 dr.

Pupil. In this example, I multiply the given decimal, ·71285, by 4, the number of quarters in a cwt., and from the product, 285140, I point off five decimal places, the number of places in the given decimal; hence ·71285 cwt. is equal to 2·8514 qr. I omit the cipher, because ciphers on the right of a decimal do not alter its value (p. 136.) I next multiply ·8514 by 28, and point off four decimal places; the result is 23·8392 lb. I then multiply the decimal ·8392 by 16, and point off as before; the result is 13·4272 oz. The decimal ·4272 I multiply by 16, and point off as before: the result is 6·8352 dr. Therefore, ·71285 cwt. = 2 qr. 23 lb. 13 oz. 6·8352 dr.

Reason of the Rule.—Since ·71285 cwt. = $\frac{71285}{100000}$ cwt. (p. 136); the value of the decimal ·71285 cwt. must be equal to the value of the fraction $\frac{71285}{100000}$ cwt., which may be found by Rule IX. p. 89, the pointing off the decimals being nothing else than dividing by the denominator (p. 149).

Exercises.

Find the value of the following:

44. £·6375; £·78125, ... Ans. 12*s.* 9*d.*; 15*s.* 7½*d.*
45. £1·928125; £·15625, . Ans. £1, 18*s.* 6¾*d.*; 3*s.* 1½*d.*
46. £·61925; £2·345675, Ans. 12*s.* 4½*d.* + ·48; £2, 6*s.* 10¾*d.* + ·848.
47. £·7853 mile, . . . Ans. 1382 yd. 0 ft. 4·608 in.
48. 1·475 ton, ... Ans. 1 ton 9 cwt. 2 qr.
49. 3·7962 cwt. . . Ans. 3 cwt. 3 qr. 5 lb. 2·7904 oz.
50. ·69573 day, . . . Ans. 16 ho. 41 min. 51·072 sec.
51. 2·73125 ac. . . . . . Ans. 2 ac. 2 ro. 37 po.
52. £276·38275, ... Ans. £276, 7*s.* 7¾*d.* + ·44.

53. ·9125 guinea, ... Ans. 19*s.* 1¾*d.* + ·8.
54. ·78125 lb. Troy, ... Ans. 8 oz. 15 dwt. 12 gr.
55. ·4525*s.* . . . . Ans. 5¼*d.* + ·72.
56. ·683575 yd. . . . . . . Ans. 2 ft. 0·6087 in.
57. ·242256 day, . . . Ans. 5 ho. 48 min. 50·9184 sec.

THE DECIMALS OF £1 may be conveniently valued by the following Rule, three decimal places being taken.

Double the *first* decimal figure for the shillings, but if the *second* figure be 5 or upwards, take 5 from it, and make the shillings 1 more; the remaining figures, diminished by 1, if they be 25 or upwards, are the farthings in the required sum.

Thus, to value £·864, double 8, the first decimal figure, and add 1, because the second figure is greater than 5; this gives 7*s.*; the 64, less 5 taken from its first figure, is 14, which, being farthings, gives 3½*d.* The value of £·864 is therefore 7*s.* 3½*d.*

This Rule will give the answer *nearly* correct; it will never be more than the fraction of a farthing too much or too little.

THE REASON OF THE RULE is shewn as follows: The first decimal figure is tenth parts of a pound, but the tenth part of a pound is 2*s.*, hence the double of the first decimal is shillings; the second decimal place is hundredth parts of a pound, therefore 5 of them make one-twentieth part of a pound, and are therefore one shilling; the remainder of the second decimal, with the third annexed, are thousandth parts of a pound; but 1000, diminished by a twenty-fifth part of itself, is 960, which is the farthings in a pound, hence the second and third decimals diminished by one for every 25, leave the remaining farthings.

Exercises.

58. £·825,	Ans. £0	16	6	64. £ ·182,	Ans. £0	3	7¾
59. £·375,	" 0	7	6	65. £ ·003,	" 0	0	0¾
60. £·750,	" 0	15	0	66. £ ·085,	" 0	1	8¼
61. £·924,	" 0	18	5¾	67. £3·199,	" 3	3	11¾
62. £·731,	" 0	14	7¼	68. £7·869,	" 7	17	4½
63. £·693,	" 0	13	10¼	69. £3·497,	" 3	9	11¼

REDUCTION OF *RECURRING* DECIMALS.

We have already seen that there are some fractions which cannot be accurately expressed by decimals; and that, in such cases, the same figures are continually repeated in some certain order, giving rise to what was called a *recurring* decimal.

When the decimal consists entirely of figures which recur, it is called a *pure* recurring decimal; but when it consists of a non-recurring and a recurring part, it is called a *mixed* recurring decimal. Thus, $\cdot\dot{7}13\dot{9}$, $\cdot\dot{8}$, $\cdot\dot{4}2857\dot{1}$ are *pure* recurring decimals; and $\cdot 39\dot{2}$, $\cdot 57\dot{6}8\dot{4}$ are *mixed* recurring decimals.

I. To convert a *pure* recurring Decimal to a Vulgar Fraction.

Rule.—Write down the *period* for the numerator of the fraction, and as many nines as there are figures in the period, for the denominator. The resulting fraction may then be changed to its lowest terms.

Example 1.—Required the vulgar fraction which is equal to $\cdot\dot{6}$.

$$\cdot\dot{6} = \tfrac{6}{9} = \tfrac{2}{3} \textit{ Ans.}$$

In this example the period consists of a single figure—namely, 6; this 6 is written as the numerator, with *one* 9 for the denominator of the fraction. The fraction is therefore $\frac{6}{9} = \frac{2}{3}$.

Example 2.—Convert $\cdot\dot{3}6\dot{9}$ to a vulgar fraction in its lowest terms.

$$\cdot\dot{3}6\dot{9} = \tfrac{369}{999} = \tfrac{41}{111} \textit{ Ans.}$$

In this example the period consists of *three* figures—namely, 369: 369 is therefore written as the numerator of a fraction, and *three* nines for the denominator. The given recurring decimal is then equal to the fraction $\frac{369}{999} = \frac{41}{111}$.

Reason of the Rule.—In Example 2, let v represent the value of the recurring decimal $\cdot\dot{3}6\dot{9}$;

$$\begin{array}{llcl}
\text{then} & 1000 \text{ times } v & = & 369\cdot369369 \text{ \&c.} \\
\text{but} & \quad\ \ 1 \quad\ \text{''} \quad v & = & \quad\ \cdot369369 \text{ \&c.} \\
\therefore \text{by subtraction} & \ \ 999 \quad \text{''} \quad v & = & 369; \\
\text{and hence} & \quad . \quad . \quad . \quad v & = & \tfrac{369}{999},
\end{array}$$

which is just what the rule prescribes. Similar reasoning will apply in every other case.

Exercises.

1. Change $\cdot\dot{7}$, $\cdot\dot{3}$, and $\cdot\dot{9}\dot{0}$ to vulgar fractions, . Ans. $\frac{7}{9}$, $\frac{1}{3}$, $\frac{10}{11}$.
2. " $\cdot\dot{7}0\dot{2}$, $\cdot\dot{8}5714\dot{2}$, and $\cdot\dot{8}14\dot{5}$ to vulgar fractions, Ans. $\frac{26}{37}$, $\frac{6}{7}$, $\frac{905}{1111}$.

II. To convert a *mixed* recurring Decimal to a Vulgar Fraction.

Rule—1. Subtract the *non*-recurring figures from the decimal, and write the remainder for the numerator.

2. For the denominator, write as many *nines* as there are *recurring* figures, and annex to them as many *nothings* as there are *non*-recurring figures: the resulting fraction may then be reduced to its lowest terms.

Example.—Convert $7\dot{2}3\dot{6}$ to a vulgar fraction.

$$\cdot 7\dot{2}3\dot{6} \text{ less } 72 = \frac{7164}{9900} = \frac{199}{275}$$

Here we subtract the non-recurring figures, 72, from the decimal, leaving 7164 for the numerator, and then write two nines and two nothings for the denominator: the resulting fraction is then reduced to its lowest terms.

Reason of the Rule.—In the example, let v represent the value of the mixed recurring decimal; or let $v = \cdot 72363636$, &c.

Then, since the non-recurring part consists of two figures, we multiply by 100;

$\therefore$ 100 times $v = 72\cdot 3636$, &c., a pure recurring decimal.

And since the recurring part consists of two figures, if we multiply this last product by 100,

we have	10000 times	$v = 7236\cdot 3636$, &c.
But	100 „	$v = 72\cdot 3636$, &c.
$\therefore$ by subtraction,	9900 „	$v = 7164 = 7236 - 72.$
and hence . . .		$v = \frac{7164}{9900}.$

Similar reasoning may be applied in every other case.

Exercises.

3. Express $\cdot 541\dot{6}$, $\cdot 1\dot{9}\dot{6}$, and $\cdot 15\dot{7}4\dot{0}$, as fractions in their lowest terms, ... Ans. $\frac{13}{24}$, $\frac{13}{66}$, $\frac{17}{108}$.

4. Express $\cdot 793\dot{5}$, $\cdot 79\dot{3}\dot{5}$, $\cdot 7\dot{9}3\dot{5}$, and $\cdot\dot{7}93\dot{5}$, as fractions in their lowest terms, ... Ans. $\frac{3571}{4500}$, $\frac{1964}{2475}$, $\frac{3964}{4995}$, $\frac{2645}{3333}$.

5. Express $32\cdot 71\dot{5}$, $7\cdot\dot{9}3\dot{6}$, $8\cdot 9\dot{3}\dot{6}$, and $21\cdot 9\dot{0}4\dot{5}$, as fractions in their lowest terms, . . . Ans. $\frac{7361}{225}$, $\frac{881}{111}$, $\frac{983}{110}$, $\frac{12157}{555}$.

ADDITION OF DECIMALS.

RULE—1. Place the given numbers under each other, so that all their decimal points may stand in the same vertical column.

2. Add them together as in Simple Addition.

3. Place the decimal point in the sum, immediately under the points of the numbers added. The sum thus pointed will be the answer required.

When any of the numbers contain *Recurring* Decimals, these are to be carried out to as many places as may be judged necessary, and then added according to the rule.

It will be afterwards shewn (pp. 154–157) how to proceed with the Addition of Recurring Decimals, and also with Subtraction, Multiplication, and Division, when *perfect* accuracy is required.

THE REASON OF THE RULE is manifest from what is stated under Simple Addition, that it is only numbers of the same denomination that can be added together.

Example.—Add together 15·063, ·002857, 308·62, 769·3276, 58·739127, and ·69325.

```
  15·063
    ·002857
 308·62
 769·3276
  58·739127
    ·69325
────────────
1152·445834 Ans.
```

Here the sums are arranged according to the Rule, and added together as in Simple Addition.

Exercises.

1. Find the sum of 19·023, 1·7854, 736·93072, 15·391, ·08365, ·713926, 8327·591, and ·00086, . . . Ans. 9101·519556.

2. Add together 31·01, 162·718, ·037, 8·6195, 387·21682, 16·3102, ·38279, ·00615, and 27·382, . . Ans. 633·68246.

3. Required the sum of $71{\cdot}\dot{3}$, $16{\cdot}2\dot{8}\dot{4}$, ·7395, $162{\cdot}7\dot{3}5\dot{4}$, $18{\cdot}\dot{2}\dot{9}$, $1{\cdot}\dot{6}$, and $3{\cdot}9\dot{7}$, true to seven places of decimals, Ans. 275·0304909.

In this example the decimals should be carried out to eight, or even nine places of decimals, in order to secure perfect accuracy in the seventh decimal place.

4. What is the sum of ${\cdot}7\dot{8}\dot{3}$, $16{\cdot}\dot{2}9\dot{5}$, ${\cdot}\dot{8}23\dot{4}$, and ${\cdot}\dot{9}$, true to six places of decimals? . . . . . . Ans. 18·902616.

5. Take the fractions $\frac{7}{8}$, $\frac{3}{11}$, $\frac{1}{29}$, $\frac{7}{25}$, $\frac{13}{17}$, and $\frac{19}{400}$; change them to decimals, and find their sum true to six places of decimals, Ans. 2·274415.

6. Take the 7th, 8th, and 9th exercises in the addition of compound numbers; convert the shillings, pence, and farthings to the decimals of a pound, and find their sums true to five places of decimals, . . Ans. 448·04479; 390·58645; 321·72291.

7. Add together 12·07, 16·7083, 154, ·154, $78·\dot{3}06\dot{8}$, and $·\dot{1}4285\dot{7}$, so that the sum may be true to five places of decimals,

Ans. 261·38196.

SUBTRACTION OF DECIMALS.

Rule.—Write the less number below the greater, with the points directly under one another, as in Addition; then proceed as in Simple Subtraction. The decimal point is placed in the answer below the other points.

When there are not as many figures in the upper as in the under line, *nothings* are supposed to be annexed to the former.

The Reason of the Rule is manifest.

Example.—Take 37 392685 from 113·283.

113·283
37·392685
75·890315 *Ans.*

In this example the numbers are arranged as in the margin, and ciphers are conceived to be added to the greater, which do not alter its value, so that the number of decimals in both numbers may be the same. The difference is then found as in Simple Subtraction.

Exercises.

1. Find the difference between 316·281 and 30·379624,

Ans. 285·901376.

2. Take 3·16847 from 11, ... Ans. 7·83153.

3. Find the difference between $78·\dot{3}\dot{1}$ and $19·\dot{6}8\dot{4}$, true to six places of decimals, ... Ans. 58·628446.

4. Change the two fractions $\frac{2}{7}$ and $\frac{6}{23}$ to decimals, and then find their difference true to seven places of decimals, Ans. ·0248447.

5. Take the 5th, 6th, and 9th exercises in Compound Subtraction, change the compound numbers to decimals of a pound, and find the difference true to five places of decimals,

Ans. 535·66041; 92·93229; 487·84895.

MULTIPLICATION OF DECIMALS.

RULE—1. Multiply the factors together as in Sim
Multiplication, without attending to the points.

2. Point off from the product as many decimals as are co
tained in *both* factors; if the product does not contain as man
prefix nothings to make up the required number.

Examples.—Multiply 3·061 by 2·5, and ·2312 by ·021.

(1.)
```
 3·061
    25
 -----
 15305
 6122
 ------
 7·6525 Ans.
```

In No. 1, there being four decimals in the two quantities, four decimals are pointed off in the answer.

In No. 2, there being seven decimals in the two quantities, and only five in the product, two nothings must be prefixed to make up the number.

(2.)
```
    ·2312
     ·021
 --------
     2312
    4624
 --------
 ·0048552 A
```

Example 3.—Multiply 783·150926 by 2·37954.

```
       783·150926
          2·37954
   --------------
       3132603704
      3915754630
     7048358334
    5482056482
   2349452778
  1566301852
 ----------------
 1863·53895445404
```

In this example, as there are six decin places in the multiplicand, and five in t multiplier, eleven decimal places are pointed from the product, being the number of decin places in the two factors. The produ 1863·53895445404, is the answer required.

THE REASON for pointing off the decimals will appear expressing the factors as fractions: thus, in example 1—

$$783·150926 \times 2·37954 = \frac{783150926}{1000000} \times \frac{237954}{100000} = \frac{186353895445404}{10000000000}$$
$$= 1863·53895445404;$$

hence it is plain that the number of decimal places in the produ is equal to the number of ciphers in the denominators of the t factors; that is, to the number of decimal places in both factor

Exercises.

1. Multiply 274·93857 by ·0283, . Ans. 7·780761531.
2. 〃 3·1415967 by 3·795, . . . 〃 11·92235947
3. 〃 ·0759638 by ·00684, . . . 〃 ·00051959239
4. Find the continued product of ·7384, ·6197, ·023, and 1·6, Ans. ·01683918246

5. What is the continued product of ·00259, 3·817, ·00615, and 21·39? . . . . . Ans. ·001300492417455.
6. Multiply 3·84062 by 70000, 2300, ·0032, Ans. 268843·4; 8833·426; ·012289984.

To MULTIPLY by 10, 100, or 1 with any other number of nothings annexed, it is only necessary to remove the decimal point as many places to the *right*, as there are nothings in the multiplier, thus—46·78 multiplied by 10, becomes 467·8.

From the nature of the notation, this is very obvious. Thus:

$$3{\cdot}765 \times 10 = 37{\cdot}65, \text{ for } 3{\cdot}765 \times 10 = \frac{3765 \times 10}{1000} = \frac{3765}{100} = 37{\cdot}65;$$

$$\text{and } 3{\cdot}765 \times 100 = 376{\cdot}5, \text{ for } 3{\cdot}765 \times 100 = \frac{3765 \times 100}{1000} = \frac{3765}{10} = 376{\cdot}5.$$

Also, similarly, $3{\cdot}765 \times 100000 = 376500$.

Exercises.—7. Multiply 61·273 by 10, 100, 1000, 100000.

WHEN THE NUMBER OF DECIMAL PLACES IN THE FACTORS IS LARGE, or when the factors are numerous, the work becomes exceedingly laborious. As in practice it is generally sufficient to have the result correct to six or seven decimal places, or even less, we may shorten our labour by proceeding according to the following Rule:

1. Commencing at the decimal point of the multiplicand, count off as many decimal figures as are required in the product, annexing ciphers when necessary to complete the number of places sought.

2. Invert the multiplier, and place its unit's figure directly under the last figure counted off in the multiplicand.

3. Multiply by the several figures of the multiplier, rejecting all those figures of the multiplicand that are to the right of the multiplying figure, but retaining the *carryings* from them, and set down the several partial products, so that their right-hand figures may stand in the same vertical column.

4. Add the several partial products together, and from the sum point off the number of decimal places required to be retained, prefixing ciphers when necessary.

Note.—In carrying from the rejected figures, it is sufficient to go back one or two places, and to carry 1 from 5 to 14, 2 from 15 to 24, and so on.

In thus contracting the work, the last figure in the product may not be correct; it is therefore better to calculate for *one figure* more than is necessary to be retained.

Example 1.—Multiply ·01784625 by 23·781267, reserving only five decimal places in the product.

```
·0178 4,625
 2187·3 2   Multiplier inverted.
 -------
  35693
   5354
   1249
    143
      2
 -------
 ·42441
```

Pupil. In this example five places are to be retained in the product, I therefore count off from the multiplicand five places of decimals, commencing at the point—namely, ·01784. The multiplier inverted is 2187·32; the units, 3, I place directly under 4, the last figure counted off, and arrange the other figures as in the margin. I then say, 2 times 6 are 12, and 1 carried makes 13; I carry 1, because the rejected figures 25, multiplied by 2, give 5 for the figure to be rejected. The rest of the multiplication goes on in the usual way. In multiplying by 3, I say 3 times 6 are 18, carry 2; 3 times 4 are 12, and 2 are 14—4 and carry 1; and so on with the other figures of the multiplier. The product is ·42441.

·0178	4625	·01784625	
	23·7812	23·7812	
	3569250	·35692	50
1	784625	·05353	875
142	77000	·01249	2375
1249	2375	·00142	77000
5353	875	·00001	784625
35692	50	·00000	3569250
·42440	5240500	·42440	5240500

The reason of the operation is obvious from the account worked out and arranged as in the usual way. The amount of labour *saved* is shewn by means of vertical lines drawn through the accounts, the part rejected lying to the right of these lines. The product in this way is ·424405, which, if we stop at the fifth decimal place, we make ·42441, the result by the contracted method. The reason is best seen from the work on the right of the page, superfluous ciphers being rejected, and the figures to the right of the vertical line, which also shews the saving of labour by using this method.

Example 2.—Multiply $3{\cdot}\dot{2}87\dot{5}$ by $23{\cdot}\dot{9}6\dot{1}$, so as to retain only six places of decimals in the product.

```
 3·28758 7,587
 9169169·3 2
 -----------
   65751752
    9862763
    2958829
     197255
       3288
       2959
        197
          3
          3
 -----------
  78·777049
```

Here the multiplicand is carried out to nine places of decimals, and the multiplier, inverted, written as before, carrying the recurring decimal, 961, one place beyond the left-hand figure in the multiplicand. The work then proceeds exactly as in Example 1.

Exercises.

8. Multiply 71·032751 by 2·6719238, so as to retain five places of decimals in the product, ... Ans. 189·79409.

9. Multiply ·03281674 by 234·781, reserving six places of decimals in the product, ... Ans. 7·704747.

10. Multiply ·453 by ·01694, so as to retain four places of decimals in the product, ... Ans. ·0076.

11. Multiply $23{\cdot}\dot{6}\dot{9}$ by $\cdot 132\dot{8}$, so as to retain six places of decimals in the result, ... Ans. 3·149065.

12. Multiply $4{\cdot}68\dot{3}$ by $14{\cdot}\dot{2}9\dot{3}$, so as to retain three places of decimals in the product, ... Ans. 66·940.

13. Multiply $1{\cdot}8\dot{2}35\dot{7}$ by $\cdot 078\dot{5}$, reserving six places of decimals in the product, ... Ans. ·143294.

DIVISION OF DECIMALS.

RULE—1. Divide as in Simple Division, without attending to the points.

2. Point off as many decimals in the answer, as the dividend contains *more* than the divisor. If the quotient has not as many figures as will allow of this, *prefix* as many nothings as will make up the number.

3. When the dividend has not as many decimals as the divisor, *before* dividing, annex as many nothings to the dividend as will make the decimals in both equal.

When there is a remainder after dividing, the division may be carried further by annexing either nothings or the figures of a recurring decimal, as the case may be, to the dividend, which, of course, must be taken into account in pointing off the decimals in the answer.

Example 1.—Divide 74·23973 by 6·12, . Ans. 12·1306748.

```
6·12)74·23973(12·130
     612
     ----
     1303
     1224
     ----
      799
      612
     ----
      1877
      1836
      ----
       413
```

Pupil. In this example, since there are two decimal places in the divisor, and five in the dividend, there are 5 − 2 = three decimals in the quotient, when all the figures of the dividend have been brought down, I therefore point off three decimal places in the quotient, which becomes 12·130. As there is a remainder, more decimal places may be obtained by annexing nothings, and continuing the division.

Example 2.—Divide 3·36 by ·105.

```
·105)3·360(32·
     3 15
     ----
      210
      210
```

Before dividing, a nothing is here annexed to the dividend, to make the decimals in the dividend and divisor equal: and being thus equal, there are no decimals in the answer.

Example 3.—Divide ·336 by 21·

```
21·)·336(·016
     21
     ---
     126
     126
```

Here the quotient is 16, but as the dividend has three decimals, and the divisor none, the answer ought to have three decimals; a nothing, therefore, requires to be prefixed to the quotient, to make up the number.

Example 4.—Divide 51·2 by 613·912, . . Ans. ·083399575.

```
613·912)51·20000(·083
          4911296
          -------
          2087040
          1841736
          -------
           245304
```

In this example there are three decimal places in the divisor, and only one in the dividend, hence *two* ciphers are annexed to make up the deficiency. As the divisor is still greater than the dividend, the division is effected by annexing more nothings.

When the divisor is large, instead of annexing a nothing, or a figure of a recurring decimal, to each remainder, a figure may be cut off from the divisor, *carrying* from the rejected figures of the divisor, as in Note p. 149.

Example 1.—Divide 7·691 by 3·9164, so that the quotient may contain four places of decimals.

```
3,·9,1,6,4)7·6910 (1·9638        3·9164)7·6910 (1·96379
           39164                        39164|
           -----                        -----|
           37746                        37746|0
           35248                        35247|6
           -----                        -----|--
            2498                         2498|40
            2350                         2349|84
            ----                         ----|---
             148                          148|560
             117                          117|492
             ---                          ---|----
              31                           31|0680
              31                           27|4148
                                           --|-----
                                            3|65320
                                            3|52476
```

The numbers being prepared as in the former examples, and the integral part of the quotient obtained, instead of bringing down a cipher, a figure is cut off from the right of the divisor. The next quotient figure is 9; 3916 being multiplied by this, and 4 carried from the rejected figure 4, the product, 35248, is subtracted, and the remainder is 2498. Cutting off the 6 from the divisor, the next figure, 6, of the quotient is found. Cutting off successively 1 and 9 from the divisor, the next two figures, 3 and 8 of the quotient, are found. The quotient is then very nearly 1·9638. The reason of the contracted process is very obvious, from the above account wrought out on the right at full length, *and the figures* rejected on the right of the vertical line.

Example 4.—Divide $396\cdot\dot{2}9\dot{3}$ by $41\cdot\dot{9}6\dot{7}$, so as to have six decimal places in the quotient.

```
41,9,6,7,9,6,7)396293939(9·442771
               377711703
                18582236
                16787187
                 1795049
                 1678719
                  116330
                   83936
                   32394
                   29378
                    3016
                    2938
                      78
                      42
                      36
```

The divisor and dividend are extended by repeating the figures of the respective recurring decimals. The operation is then carried on in the same way as in the contracted process in Example 3. It is necessary to observe, that when the divisor does not contain so many figures as those required in the quotient, the division must be carried on entire, till the figures required in the quotient be one less than the figures of the divisor, before we begin to cut off figures from the divisor. In this example, for instance, there is one place of whole numbers (this is obvious from dividing 396, the integral part of the dividend, by 41, the integral part of the divisor), and six places of decimals, so that the divisor must contain at least seven figures: to secure greater accuracy, a figure more is taken in the annexed example.

Reason of the Rule.—The removal of the decimal points in the divisor and dividend to the same number of places to the right, is just multiplying the divisor and dividend by the same number; and this operation does not affect the value of the quotient. The divisor being now a whole number, the number of decimal places in the quotient must be equal to the number of decimal places in the dividend, since the dividend is the product of the divisor and quotient.

Exercises.

1. Divide 4·98152 by ·073, and 7·3, . Ans. 68·24 and ·6824.
2. " 546·145811575 by 6·8531, and ·0068531, Ans. 79·69325, and 79693·25.
3. " 78·5 by ·01953125, & 195·3125, Ans. 4019·2, and ·40192.
4. " 16·7235 by 98·762$\dot{9}$, ... Ans. ·1693297.
5. " $71\cdot23\dot{7}$ by ·069184, ... Ans. 1029·6857.
6. " $9\cdot0\dot{1}3\dot{2}$ by ·271834, ... Ans. 33·157.
7. " $17\cdot\dot{3}9\dot{1}$ by $26\cdot\dot{5}$, so as to have six decimal places in the quotient, ... Ans. ·654905.
8. " $41\cdot\dot{8}37\dot{4}$ by $\cdot\dot{9}\dot{3}$, so as to have five decimal places in the quotient, ... Ans. 44·53667.

To Divide by 10, 100, or 1 with any other number of nothings annexed, it is only necessary to remove the decimal point as many places to the *left*, as there are nothings in the divisor; thus—124·5 divided by 100, becomes 1·245. Also,

$$3\cdot765 \div 10 = \cdot3765, \text{ for } 3\cdot765 \div 10 = \frac{3765}{1000\times10} = \frac{3765}{10000} = \cdot3765.$$

Similarly, 3·765 ÷ 1000 = ·003765; &c.

Exercises.—Divide 7·2754 by 10, 100, 10000, 1000000.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF *RECURRING* DECIMALS.

In arithmetical calculations in which Recurring Decimals are employed, we have already shewn how the results may be obtained to any degree of accuracy we please.

But when the *correct* results are required, work by the following Rules.

ADDITION.

Rule.—Extend the *recurring* parts beyond the longest finite part, till the periods be similar,* then operate as in Simple Addition; but observe to carry to the right-hand column, the same figure that would be carried if the decimals were extended one place further.

* The periods are made similar, by carrying them out till the number of figures beyond the longest finite part in any of them, be equal to the least common multiple of the number of places in each of the given periods.

Example.—Find the exact sum of $45{\cdot}3\dot{4}\dot{5}$, $13{\cdot}845\dot{2}3\dot{7}$, $76{\cdot}88\dot{1}47\dot{6}$, and $4{\cdot}93\dot{3}0769\dot{2}$.

Here the number of places in the periods are 2, 3, 4, and 6, the least common multiple of which is 12, and the longest finite part is that of the second, which has 3 finite figures; the decimals must therefore be carried to 15 decimal places, the first three places will be finite, and the next 12 will be a recurring period. The work will stand thus:

$$
\begin{array}{r}
45{\cdot}345\dot{4}5454545454\dot{5} \\
13{\cdot}845237237237237 \\
76{\cdot}881476147614761 \\
4{\cdot}933076923076923 \\
\hline
140{\cdot}955\dot{2}4485338834\dot{6}7
\end{array}
$$

The Reason of the Rule will appear, by observing that the period of 12 figures in the first line is made up of 6 complete periods of $\dot{4}\dot{5}$; in the second line, of 4 complete periods of $\dot{2}3\dot{7}$; in the third, of 3 complete periods of $\dot{4}76\dot{1}$; and the fourth, of 2 complete periods of $\dot{0}7692\dot{3}$, so that the next period of 12 figures would commence with all the periods the same as in the fourth decimal place, and go on for other 12 figures before they again coincided.

Exercises.

1. Find the sum of $\cdot7\dot{3}\dot{4}$, $\cdot\dot{7}9\dot{6}$, $\cdot2\dot{3}$, and $\cdot7\dot{8}5\dot{4}$, . Ans. $2{\cdot}5\dot{4}9959\dot{0}$.
2. " " $4{\cdot}\dot{3}$, $6{\cdot}\dot{4}\dot{5}$, $3{\cdot}\dot{9}\dot{0}$, and $5{\cdot}7\dot{3}5\dot{2}$, Ans. $20{\cdot}4\dot{3}2204\dot{9}$.
3. " " $\cdot\dot{7}85\dot{4}$, $3{\cdot}\dot{1}41\dot{6}$, $\cdot28\dot{5}\dot{4}$, $3{\cdot}\dot{9}0\dot{8}$, and $8{\cdot}71\dot{7}$, Ans. $16{\cdot}83\dot{9}2339414121\dot{5}$.

SUBTRACTION.

Rule.—Extend the *recurring* parts beyond the longest finite part, till the periods be similar, then operate as in Simple Subtraction, but observe to carry *one* to the first figure of the lower line, if its next figure would be greater than that of the upper line, were each extended one place further.

Example.—Find the difference of $64{\cdot}385\dot{2}7\dot{6}$ and $51{\cdot}684\dot{3}$.

Here the number of figures in the recurring parts are 4 and 1, the least common multiple of which is 4, and the longest finite part is that of the second, which has three finite figures; they must therefore be carried to $3+4=7$ decimal places, and the work will stand as follows:

From	$64{\cdot}385\dot{2}76\dot{5}$
Take	$51{\cdot}6843333$
Remainder	$12{\cdot}700\dot{9}43\dot{1}$

Since the next figure in the upper line would be 2, and that in the lower 3, there would be one to carry to 3, the first figure of the lower line: hence we must say, 4 from 5 and 1 remains.

The Reason of the Rule is similar to that given for Addition.

Exercises.

1. Find the difference between $\cdot7\dot{3}25\dot{6}$ and $\cdot6\dot{1}\dot{9}$, . Ans. $\cdot1\dot{1}33\dot{7}$.
2. " " " $4{\cdot}35\dot{6}\dot{3}$ & $1{\cdot}4\dot{7}8\dot{5}$, Ans. $2{\cdot}87\dot{7}7850\dot{5}$.
3. " " " $\cdot75\dot{3}6\dot{1}$ and $\cdot437\dot{1}41\dot{6}$,

Ans. $\cdot3164\dot{7}199945219\dot{7}$.

MULTIPLICATION.

I. When *one* of the factors is a Recurring Decimal.

Rule.—Make the recurring decimal the multiplier, and for the recurring part substitute the equivalent fraction; then multiply as in mixed numbers (p. 92), and point off the decimals from the product as before.

Example.—Multiply $37{\cdot}4675$ by $34{\cdot}7\dot{4}\dot{5} = 34{\cdot}7\tfrac{5}{11}$.

$$\begin{array}{r}
37{\cdot}4675 \\
\underline{34{\cdot}7\tfrac{5}{11}} \\
11)\underline{1873375} \\
170306\tfrac{9}{11} \\
2622725 \\
1498700 \\
\underline{1124025} \\
1301{\cdot}82531\tfrac{9}{11} \\
\text{or } 1301{\cdot}82531\dot{8}\dot{1}
\end{array}$$

The last form of the answer is obtained by changing $\tfrac{9}{11}$ into $\cdot\dot{8}\dot{1}$, but as there were 5 decimals in the product before, there are now 7.

Exercises.

1. Multiply $76{\cdot}345$ by $74{\cdot}8\dot{3}$, and $49{\cdot}3865$ by $4{\cdot}78\dot{3}$, Ans. $5713{\cdot}1508\dot{3}$ and $236{\cdot}2320\dot{9}1\dot{6}$.

2. " $\cdot49725$ by $73{\cdot}\dot{6}\dot{3}$, and $5964{\cdot}5$ by $\cdot72\dot{4}\dot{5}$, Ans. $36{\cdot}61568\dot{1}\dot{8}$ and $4321{\cdot}551\dot{3}\dot{6}$.

3. " $3{\cdot}8765$ by $8{\cdot}76\dot{0}\dot{9}$, and $296{\cdot}25$ by $\cdot99\dot{5}\dot{4}$, Ans. $33{\cdot}961664\dot{0}\dot{9}$ and $294{\cdot}9034\dot{0}\dot{9}$.

II. When *both* factors are Recurring Decimals.

Rule—1. Convert the *recurring* part of each to an equivalent vulgar fraction (p. 144), and then convert each factor to a single fraction (p. 84).

2. Multiply the numerators together for a dividend, and the denominators together for a divisor, and the quotient will be the true product.

Example 1.—Find the product of $73{\cdot}\dot{3}5\dot{1}$ and $42{\cdot}5\dot{6}\dot{3}$.

$$\text{Here } 73{\cdot}\dot{3}5\dot{1} = 73\frac{351}{999} = \frac{73278}{999},$$

$$\text{and } 42{\cdot}5\dot{6}\dot{3} = 42\frac{563-5}{990} = 42\frac{558}{990} = \frac{42138}{990}.$$

$$73{\cdot}\dot{3}5\dot{1} \times 42{\cdot}5\dot{6}\dot{3} = \frac{73278}{999} \times \frac{42138}{990} = \frac{3087788364}{989010} = \overset{\textit{Ans.}}{3122{\cdot}1\dot{0}02457\dot{7}}.$$

Example 2.—Find the product of $\cdot\dot{3}5\dot{1}$ and $\cdot7\dot{3}\dot{6}$.

$$\text{Here } \cdot\dot{3}5\dot{1} = \frac{351}{999} = \frac{13}{37}, \text{ and } \cdot7\dot{3}\dot{6} = \frac{736-7}{990} = \frac{729}{990} = \frac{81}{110};$$

$$\cdot7\dot{3}\dot{6} \times \cdot\dot{3}5\dot{1} = \frac{81}{110} \times \frac{13}{37} = \frac{1053}{4070} \text{ (p. 92)} = \cdot2\dot{5}8722\dot{3}. \quad \textit{Ans.}$$

Exercises.

4. What is the product of $7{\cdot}\dot{3}$ and $1{\cdot}\dot{9}\dot{2}$? . . Ans. $14{\cdot}\dot{1}4\dot{8}$.

5. Find the product of $17{\cdot}\dot{3}\dot{6}$ by $5{\cdot}72\dot{4}$, . Ans. $99{\cdot}39\dot{7}\dot{1}$.

6. " " $27{\cdot}5\dot{4}\dot{5}$ by $8{\cdot}734\dot{6}$, Ans. $240{\cdot}600\dot{3}\dot{6}$.

DIVISION.

WHEN THE DIVISOR, OR DIVIDEND, OR BOTH RECUR.

RULE—1. Convert the *recurring* part or parts, to equivalent vulgar fractions (p. 144), and then convert each to a single fraction (p. 84).

2. Invert the divisor, and multiply the numerators together for a dividend, and the denominators together for a divisor, then divide, and the quotient thus obtained will be the true quotient.

Example 1.—Divide $68\cdot3\dot{4}\dot{5}$ by $8\cdot41$.

$$\text{Here } 68\cdot345 = 68\cdot\frac{345-3}{990} = 68\frac{342}{990} = \frac{67662}{990},$$

$$\text{and } 8\cdot41 = \frac{841}{100}. \quad \text{Hence } 68\cdot3\dot{4}\dot{5} \div 8\cdot41 = \frac{67662}{990} \div \frac{841}{100}$$

$$= \frac{67662}{990} \times \frac{100}{841} = \frac{676620}{83259} = 8\cdot126689\frac{549}{83259}. \quad \textit{Ans.}$$

Example 2.—Divide $41\cdot7\dot{1}\dot{8}$ by $34\cdot74\dot{3}$.

$$\text{Here } 41\cdot7\dot{1}\dot{8} = 41\cdot\frac{718-7}{990} = 41\frac{711}{990} = \frac{41301}{990},$$

$$\text{and } 34\cdot74\dot{3} = 34\frac{743-74}{900} = 34\frac{669}{900} = \frac{31269}{900};$$

$$\text{hence } 41\cdot7\dot{1}\dot{8} \div 43\cdot74\dot{3} = \frac{41301}{990} \div \frac{31269}{900} = \frac{41301}{990} \times \frac{900}{31269}$$

$$= \frac{41301}{110} \times \frac{100}{31269} = \frac{413010}{343959} = 1\cdot200753\frac{198873}{343959}. \quad \textit{Ans.}$$

Example 3.—Find the quotient of $\cdot\dot{3}5\dot{1}$ divided by $\cdot7\dot{3}\dot{6}$.

$$\frac{\cdot\dot{3}5\dot{1}}{\cdot7\dot{3}\dot{6}} = \text{(p. 94)}\ \frac{13}{37} \times \frac{110}{81} = \frac{1430}{2997} = \cdot\dot{4}7714381\dot{0}. \quad \textit{Ans.}$$

Exercises.

1. Divide $\cdot7\dot{3}2\dot{6}$ by $9\cdot\dot{5}\dot{4}$, . Ans. $\cdot07\dot{6}751990085323418\dot{6}5$.
2. " $8\cdot42\dot{6}\dot{3}$ by $16\cdot85\dot{2}\dot{7}$, ... Ans. $\cdot5$.
3. " $\cdot26\dot{2}\dot{7}$ by $1\cdot92\dot{6}$, ... Ans. $\cdot\dot{1}8\dot{6}$.

Miscellaneous Exercises in Decimals.

1. A heavy body falling freely near the surface of the earth passes over 16·1 feet in one second of time, and the space passed over in any time is 16·1 feet, multiplied by the square of the time in seconds. How far would a body fall in 12, 14, or $20\frac{1}{2}$ seconds?
Ans, 2318·4 ft., 3155·6 ft., 6766·025 ft.

2. The area of a circle whose radius is one is 3·14159, and the area increases as the square of the radius; what is the area of a circle whose radius is 20·75 feet? . Ans. 1352·650844375 sq. ft.

3. The number which Lord Napier assumed had its logarithm 1 is expressed by the series $2 + \frac{1}{2} + \frac{1}{2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{2 \times 3 \times 4 \times 5} +$ &c.; the new factor introduced into the denominator at each step being one greater than the previous. Find the decimal that corresponds to the series,
Ans. 2·718281828459, &c.

4. When the diameter of a circle is 113, its circumference is 355 very nearly; find in decimals the circumference of a circle whose diameter is one, and the diameter of a circle whose circumference is one, . . . Ans. 3·14159 +, ·31831 −.

5. A cubic foot of gold weighs 19258 oz., and a cubic foot of silver weighs 10474 oz.; if the weight of silver be taken as the unit, what decimal will express the weight of gold; and if the weight of gold be taken as the unit, what decimal will express the weight of silver? . . . Ans. 1·83864 and ·543878.

6. The circumference of a circle, whose diameter is 1, can be calculated from the formula $16 \times \left\{\frac{1}{5} - \frac{1}{3 \times 5^3} + \frac{1}{5 \times 5^5} - \frac{1}{7 \times 5^7} + \frac{1}{9 \times 5^9} - \text{\&c.}\right\} - 4 \times \left\{\frac{1}{239} - \frac{1}{3 \times 239^3} + \frac{1}{5 \times 239^5} - \text{\&c.}\right\}$; find it to 5 decimals, ... Ans. 3·14159.

COMMERCIAL ALLOWANCES.

IN BUYING AND SELLING goods by weight, there are usually certain allowances or deductions made for the weight of the boxes, packages, &c., which contain the goods.

Gross weight, is the total weight of the goods, and the boxes or casks, &c., in which they are packed.

Tare, is the weight of the boxes, &c., containing the goods, which is to be deducted from the gross, and is at so much a box, or cwt., &c.

Net weight, is what remains after the tare has been deducted from the gross.

Draft, is a deduction made on some goods, to allow for the loss of weight in selling by retail; it is deducted from the gross weight.

Several other allowances were formerly made, but they are nearly out of use.

The following example will shew the mode of deducting the tare from a quantity of goods:

Example 1.—What is the net weight of 5 tierces of coffee; the first weighing 5 cwt. 2 qr. 17 lb., and tare, 2 qr. 11 lb.; the second, 6 cwt. 1 qr. 23 lb., and tare, 3 qr. 1 lb.; the third, 5 cwt. 3 qr. 15 lb., and tare, 2 qr. 27 lb.; the fourth, 6 cwt. 3 qr. 19 lb., and tare, 3 qr. 20 lb.; and the fifth, 6 cwt. 2 qr. 4 lb., and tare, 3 qr. 21 lb.?

	Gross weight.			Tare.		
	cwt.	*qr.*	*lb.*	*cwt.*	*qr.*	*lb.*
1.	5	2	17	0	2	11
2.	6	1	23	0	3	1
3.	5	3	15	0	2	27
4.	6	3	19	0	3	20
5.	6	2	4	0	3	21
	31	1	22	3	3	24
Subtract tare,	3	3	24			
Net weight,	27	1	26			

Example 2.—What is the net weight of 9 chests of tea, weighing 6 cwt. 2 qr. 15 lb., draft, 2 lb. per chest, and tare, 18 lb. per cwt.?

	cwt.	qr.	lb.	
lb.	6	2	15	gross.
2 × 9 =	0	0	18	draft.
	6	1	25	
16 lb. = $\frac{1}{7}$ of 1 cwt.	0	3	20	
2 lb. = $\frac{1}{8}$ of 16 lb.	0	0	13	
	1	0	5	tare.
	5	1	20	net weight.

In this example the draft is first deducted: the tare is calculated on the remainder, as in Practice, and found to be 1 cwt. 0 qr. 5 lb.; this, deducted from 6 cwt. 1 qr. 25 lb., leaves 5 cwt. 1 qr. 20 lb. for the net weight.

NOTE.—In the calculation of the tare, when a fraction of a lb. occurs, it is rejected when under $\frac{1}{2}$ lb.; but when greater than $\frac{1}{2}$ lb., the tare is increased by 1 lb. If greater accuracy is required, the calculation may be carried out to one or two decimal places, or the resulting fractions may be taken and then summed.

Exercises.

1. Required the net weight of 7 bales of Russian flax, the first weighing 3 cwt. 1 qr. 12 lb., and tare, 1 qr. 9 lb.; the second, 4 cwt. 2 qr. 13 lb., and tare, 1 qr. 15 lb.; the third, 4 cwt. 3 qr., and tare, 2 qr. 1 lb.; the fourth, 4 cwt. 3 qr. 21 lb., and tare, 2 qr. 17 lb.; the fifth, 5 cwt. 2 qr. 13 lb., and tare, 3 qr. 7 lb.; the sixth, 5 cwt. 2 qr. 27 lb., and tare, 3 qr. 12 lb.; and the seventh, 6 cwt., and tare, 3 qr. 20 lb. . . . Ans. 30 cwt. 2 qr. 5 lb.

2. What is the net weight of 4 bales of wool, the first of which weighs 3 cwt. 3 qr. 12 lb.; the second, 3 cwt. 2 qr. 23 lb.; the third, 3 cwt. 3 qr. 6 lb.; and the fourth, 4 cwt. 16 lb., allowing for draft 2 lb. per bale, and for tare 4 lb. per cwt.?—also, what is the value of the wool at 1*s.* 9$\frac{1}{4}$*d.* per lb.? Ans. Net weight, 14 cwt. 3 qr. 15 lb.; value, £149, 6*s.* 8$\frac{1}{4}$*d.*

3. What is the net weight of 3 barrels of figs weighing 6 cwt. 2 qr. 4 lb., the tare allowed per barrel being 29 lb.?
Ans. 5 cwt. 3 qr. 1 lb.

4. What is the net weight of 7 hogsheads of sugar, each weighing 12 cwt. 1 qr. 25 lb., draft, 2 lb. per hhd., and tare, 1 cwt. 2 qr. 27 lb. per hhd.? . . . . . . Ans. 75 cwt.

5. Find the net weight of 5 bags of rice, weighing as follows: First, 1 cwt. 23 lb.; second, 1 cwt. 1 qr. 11 lb.; third, 1 cwt. 1 qr. 22 lb.; fourth, 1 cwt. 2 qr.; and fifth, 1 cwt. 2 qr. 17 lb.; tare, 20 lb. per cwt., and draft, 2 lb. per bag: and its value at 18*s.* 7$\frac{1}{2}$*d.* per cwt. . . Ans. 5 cwt. 3 qr. 6 lb.; £5, 8*s.* 1*d.* nearly.

PERCENTAGES.

Percentages are calculations in Interest, Discount, Commission, Insurance, &c., at given rates per 100. The term is derived from the Latin words *per*, signifying *by*, and *centum* (contracted into cent.), signifying *hundred*, and means *by the hundred*. Thus, 5 per cent. means 5 for every hundred.

TABLE OF PERCENTAGES IN MONEY.

$\frac{1}{8}$	per cent.	=	$\frac{3}{10}d.$	per £1	5	per cent.	=	1/	per £1
$\frac{1}{4}$	"	=	$\frac{3}{5}d.$	"	$7\frac{1}{2}$	"	=	1/6	"
$\frac{1}{2}$	"	=	$1\frac{1}{5}d.$	"	10	"	=	2/	"
1	"	=	$2\frac{2}{5}d.$	"	$12\frac{1}{2}$	"	=	2/6	"
2	"	=	$4\frac{4}{5}d.$	"	25	"	=	5/	"
$2\frac{1}{2}$	"	=	$6d.$	"	$33\frac{1}{3}$	"	=	6/8	"
3	"	=	$7\frac{1}{5}d.$	"	50	"	=	10/	"
4	"	=	$9\frac{3}{5}d.$	"	Cent. per cent.		=	20/ or double.	

The calculations termed Interest, Discount, Commission, Insurance, &c., are questions of percentage. These, on account of their frequent occurrence in all mercantile transactions, are considered under their different heads; but a few examples are also given here to shew the general nature of percentages, and the mode of working them.

Questions of percentage, whether in regard to money or quantities of any other kind, are virtually wrought by the rules of **Proportion**: the word *cent.* being expressed as 100, in stating the questions.

Example 1.—A gas company reduces the price of its gas from 8*s*. 6*d*. to 7*s*. 6*d*. per 1000 cubic feet; what is the reduction per cent.?

$$
\begin{array}{rrcrcr}
s. & d. & & £ & & s. \\
8 & 6 & : & 100 & :: & 1 \\
12 & & & 20 & & \\
\hline
102 & & & 2000 & & \\
 & & & 12 & & \\
 & & & 102\overline{)24000} & & \\
 & & & 20\overline{)235\ \ 3\frac{1}{2}} & & \\
 & & \textit{Ans.} & £11\ \ 15\ \ \ 8\frac{1}{2} & \text{per cent.} &
\end{array}
$$

Here we say, as 8*s*. 6*d*. is to £100, so is 1*s*. (the difference between the two rates) to the percentage sought.

Example 2.—A gentleman wishes to buy an estate which produces a rental of £700 per annum; what sum must he pay,

in order that the £700 will be equal to 3½ per cent. on his outlay?

```
£  s.        £          £
3  10   :   700   ::   100
20           20
70         14000
             100
      7,0)140000,0
   Ans.   £20,000
```

Here we say, as £3, 10*s*., the percentage, is to £700, the rent, so is £100 to the required sum.

Example 3.—A gentleman paid £176 of income-tax. The tax was at the rate of 3 per cent. on his income; what was the amount of his income?

```
£      £        £
3  :  176  ::  100
      100
  3 )17600
Ans. £5866  13  4
```

Here we say, as £3, the tax on £100, is to £176, the tax on the income, so is £100 to the income required.

Exercises.

1. What is the amount of a tax of 6 per cent. on a rental of £24? Ans. £1, 8*s*. 9½*d*. ⅖.
2. If the charge for conveyance in a stage-coach be lowered from £2, 2*s*. to £1, 15*s*., what reduction is that per cent.? Ans. £16, 13*s*. 4*d*.
3. If a house yield an annual rental of £60, what should I pay for it, in order to clear £4½ per cent. on the outlay? Ans. £1333, 6*s*. 8*d*.
4. What should I give for the same house to clear £6 per cent.? Ans. £1000.
5. A person does business to the extent of £2000 per annum, and pays a rent of £82; what percentage is this rental on the £2000? . . . . Ans. $4\frac{1}{10}$ per cent.
6. If a person pays a tax of £8, at 3 per cent. on his income, what is his income? . . . . . Ans. £266, 13*s*. 4*d*.
7. The receipts of a railway company in 1841 were £997, and in 1842, £1232; what was the increase per cent.? Ans. £23, 11*s*. 4¾*d*. $\frac{881}{997}$.
8. In 1841, wool was exported to the value of £555,620, and in 1842, the value of what was exported was £510,965; what was the decrease per cent.? . Ans. $8\frac{79}{2137}$ per cent.
9. Bought paper at 15*s*. a ream, and sold it at 16*s*.: what was gained per cent.? . . . . . Ans. £6, 13*s*. 4*d*.
10. Sold cloth at 18*s*. a yard, and lost 5 per cent. upon it; what did it cost? . . . . Ans. 18*s*. 11¼*d*. $\frac{9}{19}$ a yard.
11. What must I pay to insure property to the amount of £4000, at the rate of 1½ per cent.? . . . Ans. £60.

SIMPLE INTEREST.

INTEREST is the sum paid for the loan or use of money, by the person who borrows it to the person who lends it.

The sum lent is called the *Principal*, and the allowance for lending it, the *Interest*.

Interest is calculated at the rate of so much per cent. for a year, and the words *per annum*, meaning *by the year*, are either expressed or understood.

The *Rate* is the interest of £100 for one year. Thus, '5 per cent. per annum' means interest at the *rate* of £5 for the loan of £100 for a year.

A common rate of interest on ordinary transactions is £5 per cent., but it varies from 2 to 5 per cent.

It is termed *Simple* Interest when charged on the principal only; and *Compound*, when the interest for a given time is added to the principal, and then interest charged on the amount of both.

I. To find the Interest on a given sum for *one* year.

RULE.—Multiply the principal by the rate per cent., and divide the product by 100.

Note.—This is also the Rule by which any given per cent. is calculated, when no particular period is specified, as in the question, 'How much is 4 per cent. on £150?' Thus, £150 × 4 ÷ 100 = £6.

Example.—What is the interest on £325, 10*s.* for one year, at $2\frac{1}{2}$ per cent.?

£	*s.*
325	10
	$2\frac{1}{2}$
651	0
162	15
8,13	15
	20
	2,75
	12
	9,00

Here, after multiplying the principal by $2\frac{1}{2}$, the rate per cent., we divide the product by 100, as in Compound Division, Rule III., p. 68.

Interest £8 2 9

REASON OF THE RULE.—The Rules for finding Interest are virtually the same as those of Simple and Compound Proportion, according to the nature of the case.

INTEREST for *one* year is a case of Simple Proportion: for instance, the question, 'What is the interest on £40 for one year at 5 per cent.?' may be stated thus:

£		£		£
100	:	40	::	5

The question, as a case of Simple Proportion, may be expressed in this way—If the interest on £100 is £5, what will be the interest on £40?

INTEREST for more than one year, or for days, is a case of Compound Proportion: for instance, the question, 'What is the interest on £250 for 45 days, at 4 per cent.?' may be stated thus:

£		£		£
100	:	250	::	3
365 days.		45 days.		
36,500		11,250		
73,000				

The meaning is: If the interest of £100 for 365 days is £3, what will be the interest of £250 at the same rate for 45 days? hence it is a question in Compound Proportion. For convenience in working, the first term is doubled, making 73,000; and to put the second term on an equality, it would require to be also doubled; but it is more convenient, and has the same result, to double the third term, and thus multiply by twice the rate per cent. (See Rule for Days, p. 168.)

Exercises.—What is the interest on the following sums for a year?

1.	£185	0	0	at	2	per cent. . .	Ans.	£3	14	0
2.	240	10	0	"	$2\frac{1}{2}$	" . .	"	6	0	3
3.	175	6	8	"	3	" . .	"	5	5	$2\frac{1}{4}$ $\frac{3}{5}$
4.	310	9	8	"	$3\frac{1}{2}$	" . .	"	10	17	$4\frac{2}{4}$ $\frac{6}{25}$
5.	295	14	10	"	4	" . .	"	11	16	$7\frac{2}{4}$ $\frac{12}{25}$
6.	751	15	0	"	5	" . .	"	37	11	9

INTEREST FOR A YEAR, may also be readily calculated by taking the interest on £1, according to the percentage table, p. 161, and multiplying it by the number of pounds in the given sum.

Thus, to find the interest on £22, at $2\frac{1}{2}$ per cent., multiply 6*d.*, the interest on £1, by 22, and the answer is 11*s.*: or to find the interest on £37, at 5 per cent., multiply 1*s.*, the interest on £1, by 37, and the answer is 37*s.* = £1, 17*s.* When there are shillings or pence in the given sum, take the proportion of the interest for a pound.

INTEREST FOR A YEAR AT $2\frac{1}{2}$ PER CENT. on any number of *pounds*, may be readily found as follows:

Cut off the last figure of the principal, and divide the rest of the sum by 4; the quotient is the pounds of the answer: annex to any remainder, the figure that was cut off, and the half of this sum, reckoned as shillings—or if there is no remainder, the half of the figure that was cut off—is the shillings and pence of the answer.

This process is merely a short way of dividing by 40—as $2\frac{1}{2}$ is the fortieth part of 100.

Example.—What is the interest on £437, at $2\frac{1}{2}$ per cent.?

$$\begin{array}{r} \text{£} \\ 4\,)\underline{43{,}7} \\ \text{£}10\;\;18\;\;6 \end{array}$$

Here we cut off the 7, and on dividing 43 by 4, the answer is £10, and 3 over. Annex to the 3 the figure cut off, 7, making 37, counted as 37*s.*, and the half of this is 18*s.* 6*d.* The interest, therefore, is £10, 18*s.* 6*d.*

INTEREST FOR A YEAR AT 5 PER CENT. on any number of *pounds*, may be found as follows:

Cut off the last figure of the principal, and divide the rest of the sum by 2; the quotient is the pounds of the answer: if there is 1 over, annex to it the figure cut off, and this sum is the shillings of the answer; or if there is no remainder, the figure cut off is the shillings of the answer.

This process is merely a short way of dividing by 20—as 5 is the twentieth part of 100.

Example.—What is the interest on £276, at 5 per cent.?

$$\begin{array}{r} \text{£} \\ 2\,)\underline{27{,}6} \\ \text{£}13\;\;16 \end{array}$$

Here we cut off the 6, and on dividing by 2, the answer is £13, and 1 over: annex the 1 to the 6 cut off, making 16, which is the shillings of the answer.

II. To find the Interest for any number of Years.

RULE.—Multiply the principal by the number of years, and the product by the rate per cent.; then divide this last result by 100. The quotient will be the interest required.

Or, find the interest for *one* year by Rule I., and then multiply the amount by the number of years.

Example.—What is the interest of £729, 18*s.* 4*d.* for 5½ years, at 3¼ per cent.?

	£	*s.*	*d.*
	729	18	4
			5½
	3649	11	8
	364	19	2
	4014	10	10
			3¼
	12043	12	6
	1003	12	8½
1,00)	13047	5	2½
Interest,	£130	9	5¼ $\frac{7}{10}$

Or thus by decimals:

	£729·91$\dot{6}$ (p. 140.)
	5½
	3649583
	364958
	4014541
	3¼
	12043623
	1003635
1,00)	130,47·258
	130·472 = £130, 9*s.* 5½*d.* (p. 143).

The several steps are obvious. The pupil is recommended to work out the exercises both ways, as in the above example.

Note.—In the following exercises the exact answers are given; but it will generally be sufficient to have the answer true to the nearest farthing.

Exercises.

What is the interest on the following sums?

7. £847, 16*s.* 8*d.* for 2 years at 3 per cent.
Ans. £50, 17*s.* 4¾*d.* $\frac{1}{5}$.
8. £1256, 10*s.* 6½*d.* for 7 years at 2½ per cent.
Ans. £219, 17*s.* 10$\frac{0}{4}$*d.* $\frac{11}{20}$.
9. £732, 15*s.* 9¾*d.* for 5 years at 2¼ per cent.
Ans. £82, 8*s.* 9¼*d.* $\frac{31}{80}$.
10. £179, 11*s.* 7½*d.* for 4 years at 2¾ per cent.
Ans. £19, 15*s.* 0¾*d.* $\frac{39}{50}$.
11. £273, 19*s.* 10*d.* for 3½ years at 3 per cent.
Ans. £28, 15*s.* 4½*d.* $\frac{9}{25}$.
12. £2685, 18*s.* 6¼*d.* for 9¼ years at 3¼ per cent.
Ans. £807, 9*s.* 1½*d.* $\frac{409}{1600}$.
13. £1751, 9*s.* 2½*d.* for 11 years at 3⅞ per cent.
Ans. £746, 11*s.* 2¼*d.* $\frac{241}{400}$.
14. £987, 1*s.* 8¼*d.* for 5¾ years at 4½ per cent.
Ans. £255, 8*s.* 1¾*d.* $\frac{607}{800}$.
15. £567, 8*s.* 9¾*d.* for 6 years at 2⅞ per cent.
Ans. £97, 17*s.* 8$\frac{0}{4}$*d.* $\frac{67}{400}$.
16. £275, 19*s.* 2*d.* for 2½ years at 5 per cent.
Ans. £34, 9*s.* 10¾*d.*
17. £111, 11*s.* 1*d.* for 9 years at 4⅜ per cent.
Ans. £46, 3*s.* 8$\frac{0}{4}$*d.* $\frac{11}{125}$.

III. To find the Interest for Months.

Rule.—Multiply the principal by the number of months, and the product by the rate per cent.; then divide this last result by 1200. The quotient will be the interest required.

Example.—What is the interest of £374, 15*s.* 3*d.* for 7 months, at $3\frac{1}{3}$ per cent.?

$$
\begin{array}{rrrrl}
 & \pounds & s. & d. & \\
 & 374 & 15 & 3 & \\
 & & & 7 & \\
\hline
 & 2623 & 6 & 9 & \\
 & & & 3\frac{1}{3} & \\
\hline
 & 7870 & 0 & 3 & \\
 & 327 & 18 & 4\frac{2}{4} & \frac{1}{2} \\
\hline
1200 = \begin{cases} 12) \\ 1{,}00) \end{cases} & \begin{array}{r} 8197 \\ 6{,}83 \end{array} & \begin{array}{r} 18 \\ 3 \end{array} & \begin{array}{r} 7\frac{2}{4} \\ 2\frac{1}{4} \end{array} & \begin{array}{l} \frac{1}{2} \\ \frac{3}{8} \end{array} \\
 & 20 & & & \\
\hline
 & 16{,}63 & & & \\
 & 12 & & & \\
\hline
 & 7{,}58 & & & \\
 & 4 & & & \\
\hline
\end{array}
$$

$$\frac{2{,}34\frac{3}{8}}{100} = \frac{275}{800} = \frac{11}{32}$$

Interest = £6, 16*s.* $7\frac{1}{4}$*d.* $\frac{11}{32}$.

Or thus by decimals:

$$
\begin{array}{r}
\pounds \\
374{\cdot}762 \\
7 \\
\hline
2623334 \\
3\frac{1}{3} \\
\hline
7870002 \\
327916 \\
\hline
12{,}00)81{,}97{\cdot}918 \\
\hline
\pounds 6{\cdot}831 = \pounds 6,\ 16s.\ 7\tfrac{1}{2}d.
\end{array}
$$

This second method shews the advantage of proceeding by decimals: in the first, or exact method, 91 figures are used; in the second, only 51.

Exercises.

What is the interest on the following sums?

18. £187, 16*s.* 10*d.* for 5 months at $4\frac{1}{2}$ per cent.
Ans. £3, 10*s.* $5\frac{1}{4}$*d.* $\frac{3}{20}$.
19. £9325, 14*s.* 3*d.* for 1 year 2 months at 6 per cent.
Ans. £652, 15*s.* $11\frac{3}{4}$*d.* $\frac{22}{25}$.
20. £416, 9*s.* 7*d.* for $8\frac{1}{4}$ months at $2\frac{3}{4}$ per cent.
Ans. £8, 2*s.* $3\frac{2}{4}$*d.* $\frac{77}{480}$.
21. £172, 1*s.* 11*d.* for 1 year 10 months at $3\frac{3}{8}$ per cent.
Ans. £11, 8*s.* $8\frac{3}{4}$*d.* $\frac{867}{1200}$.
22. £395, 7*s.* 8*d.* for 2 years 5 months at 4 per cent.
Ans. £38, 4*s.* $4\frac{3}{4}$*d.* $\frac{43}{75}$.
23. £916, 3*s.* $6\frac{1}{4}$*d.* for $7\frac{1}{2}$ months at $5\frac{1}{2}$ per cent.
Ans. £31, 9*s.* $10\frac{1}{4}$*d.* $\frac{26}{32}$.
24. £872, 19*s.* 9*d.* for $3\frac{1}{4}$ months at $4\frac{1}{4}$ per cent.
Ans. £10, 0*s.* $11\frac{1}{2}$*d.* $\frac{811}{1600}$.

25. £541, 2*s*. 4*d*. for 1 year 5$\frac{1}{2}$ months at 6$\frac{3}{4}$ per cent.
Ans. £53, 5*s*. 3$\frac{3}{4}$*d*. $\frac{21}{40}$.

26. £193, 11*s*. 3*d*. for 3 years 11 months at 4$\frac{3}{8}$ per cent.
Ans. £33, 3*s*. 4$\frac{1}{4}$*d*. $\frac{1}{32}$.

27. £217, 5*s*. 2$\frac{3}{4}$*d*. for 11 months at 2$\frac{7}{8}$ per cent.
Ans. £5, 14*s*. 6$\frac{0}{4}$*d*. $\frac{6863}{9600}$.

28. £1000 for 1 year 3$\frac{1}{2}$ months at 5 per cent.
Ans. £64, 11*s*. 8*d*.

Interest for Months may also be found by calculating the interest for a year by Rule I., and then taking the $\frac{1}{2}$, $\frac{1}{3}$, or other aliquot part of the amount, according to the number of months.

Example.—What is the interest on £80 for three months, at 5 per cent.?

4)£4
 £1 *Ans*.

Here £4, the interest of £80 for 1 year, is divided by 4, as 3 months are $\frac{1}{4}$ of a year.

Interest at 5 per cent. for any number of months, may be conveniently found by reckoning the pounds of the given sum as so many pence, and then multiplying them by the number of months—interest at 5 per cent. being equal to 1*d*. a £ per month.

Thus, to find the interest on £24 for 5 months, reckon the £24 as 24*d*. = 2*s*.; then multiply by 5, and the answer is 10*s*. If there are shillings and pence in the given sum, add the proportion of the interest for £1.

IV. To find the Interest for Days.

Rule.—Multiply the principal by the number of days, and the product by *twice* the rate per cent.; then divide the result by 73,000: the quotient is the interest required.

Note.—Interest at 5 per cent. is found by multiplying the principal by the number of days, and dividing the product by 7300. This is merely an abridgment of the general rule.

Example 1.—What is the interest on £235, 10*s*. for 125 days, at 3 per cent.?

```
           £235  10
                125
         ----------
          29,437 10
                  6
73,000)176,625   0(£2  8  4½ Ans.
```

Here we multiply the principal by the number of days, 125, and the product by *twice* the rate per cent., 6, and then divide the last product by 73,000 for the answer.

Example 2.—What is the interest of £956, 15*s.* 9*d.* for 127 days, at 3½ per cent.?

	£	s.	d.	
	956	15	9	× 7
			12	
	11481	9	0	
			10	
	114814	10	0	
	6697	10	8	
	121512	0	8	
3½ × 2 =			7	
73,000)	850,584	1	9	
Interest,	£11	13	0¼ $\frac{13931}{18250}$	

Or thus by decimals:

£
956·787
127
6697509
11481444
121511949
7
73,000)850,583·643
Interest, £11·651 or £11 13 0¼

When decimals are employed, we obtain the same result as before, with considerably less labour.

The Division by 73,000 may be readily performed by the following rule, termed 'the third, tenth, and tenth rule:'

Rule.—Write below the *pounds* of the product * (the shillings and pence not being reckoned), ⅓ of itself, $\frac{1}{10}$ of the third, and $\frac{1}{10}$ of that tenth; and add the four lines together: then cancel the last two figures; reckon the next two figures as so many farthings—less 1 farthing for every 25; the double of the next figure as so many shillings; and the rest of the figures as so many pounds.

The whole forms the answer, and is nearly correct—there only requires a farthing to be subtracted for every £10 in the answer, to give nearly the exact interest.

* That is, the principal, multiplied by the number of days, and twice the rate per cent.

Example.—Divide £357,200 by 73,000.

		357,200
⅓	=	119,066
$\frac{1}{10}$ of ⅓	=	11,906
$\frac{1}{10}$ " $\frac{1}{10}$	=	1,190
		4893~~62~~
Ans.		£4 19 10½

Here, after adding 1-3d, 1-10th, and 1-10th to the sum, we cancel the last two figures of the total: we then reckon 93 as so many farthings—less 3, as there are 3 times 25 in 93, making 90, or 1*s.* 10½*d.*; the double of 9 is reckoned as so many shillings, or 18*s.*, and the other figure, 4, as 4 pounds. The answer is £4, 19*s.* 10½*d.*, which is about ½*d.* more than the exact sum.

The Division by 7300, in the case of 5 per cent. is performed in the same way as that by 73,000, only, instead of cancelling the last *two* figures, as above, cancel merely the *last* figure.

REASON OF THE RULE.—The reason of the rule for Days will be seen at once, on stating any account by Compound Proportion; for instance, Example 2 will stand as in the margin.

£		£	s.	d.		£
100	:	956	15	9	::	$3\frac{1}{2}$
days.		days.				
365	:	127				

$$\therefore \frac{£956\ 15\ 9 \times 127 \times 3\frac{1}{2}}{36500} = \text{Ans.}$$

$$\therefore \frac{£956\ 15\ 9 \times 127 \times 3\frac{1}{2} \times 2}{73000} = \text{Ans.}$$

The answer is equal to the principal, £956, 15*s.* 9*d.*, multiplied by the number of days, 127, and by the rate per cent., $3\frac{1}{2}$, and the continued product divided by 36500. Now, in order that the divisor may contain more ciphers, we multiply it by 2; this gives 73000 for the divisor; and that the fraction may retain its original value, we must multiply *one* of the factors in the numerator by 2. The rate per cent. being generally the smallest, its double is taken. Hence the rule.

With regard to the short method of dividing by 73000, it will be sufficient to observe, that since

$$\frac{1}{73000} = \cdot0000137, \text{ nearly} = \frac{1\cdot37}{100000}$$

therefore, when we are required to divide any number, as 850584, by 73000, we will obtain the same result by multiplying the number by 1·37, and then dividing the product by 100000. Now, in multiplying by 1·37, we let the given number stand for *once*, and the decimal ·37 we separate into parts, as in practice; namely, $\cdot33\frac{1}{3}$, $\cdot03\frac{1}{3}$, $\cdot00\frac{1}{3}$; thus:

	850584	= 1·	time the given number.
$\cdot33\frac{1}{3} = \frac{1}{3}$ of 1	283528	= $\cdot33\frac{1}{3}$	" "
$\cdot03\frac{1}{3} = \frac{1}{10}$ " $\cdot33\frac{1}{3}$	28352	= $\cdot03\frac{1}{3}$	" "
$\cdot00\frac{1}{3} = \frac{1}{10}$ " $\cdot03\frac{1}{3}$	2835	= $\cdot00\frac{1}{3}$	" "
∴	1165299	= 1·37	" "

Now, to divide 1165299 by 100000, we have merely to point off the five right-hand figures for decimals (p. 153). The reason of the correction is,

that $\frac{1\cdot37}{10000}$ is somewhat greater than the reciprocal of $\frac{1}{73000}$.

To find what the correction is, we may calculate the interest of £10000 for 73 days at 5 per cent. The true answer, we know, is 100. Working it out, however, by the rule, we find the interest to be 100·010, or nearly 10 farthings above the true answer; this is very nearly 1 farthing on every £10 of interest.

Exercises.

29. What is the interest of £781 for 47 days at 5 per cent.? Ans. £5, 0*s*. 6¾*d*. $\frac{81}{365}$.

30. What is the amount of £1127 for 213 days at 5 per cent.? Ans. £1159, 17*s*. 8$\frac{0}{4}$*d*. $\frac{128}{365}$.

31. What is the interest of £641, 12*s*. 6*d*. for 36 days at 4½ per cent.? . . . . . . Ans. £2, 16*s*. 11¼*d*. $\frac{1552}{1825}$.

32. What is the interest of £7138, 18*s*. 4*d*. for 158 days at 3¼ per cent.? . . . . . . Ans. £100, 8*s*. 8$\frac{0}{4}$*d*. $\frac{818}{1825}$.

33. Required the amount of £328, 16*s*. 11*d*. from January 7 to March 29 at 2⅝ per cent.* . Ans. £330, 15*s*. 2¾*d*. $\frac{1033}{73000}$.

34. Find the interest on £584, 11*s*. 3½*d*. from February 21 to August 17 at 3¾ per cent. . Ans. £10, 12*s*. 7¼*d*. $\frac{821}{14600}$.

35. What is the amount of £816, 17*s*. 6¾*d*. from June 14 to September 25 at 4⅝ per cent.? Ans. £826, 19*s*. 3$\frac{0}{4}$*d*. $\frac{32283}{58400}$.

36. What is the interest of £875, 14*s*. 6*d*. from August 10 to December 14 at 4¼ per cent.? Ans. £12, 16*s*. 11½*d*. $\frac{427}{9125}$.

37. What is the interest of £697, 8*s*. 5¼*d*. from June 17 to December 31 at 5 per cent.? . Ans. £18, 16*s*. 5$\frac{0}{4}$*d*. $\frac{1}{365}$.

38. What is the interest of £739 from March 5 to October 11 at 2½ per cent.? . . . . Ans. £11, 2*s*. 8½*d*. $\frac{14}{73}$.

39. What is the interest of £690 from July 28 to November 18 at 3⅞ per cent.? . . . Ans. £8, 12*s*. 10½*d*. $\frac{64}{365}$.

40. On January 1, 1844, A owed to B £173, 15*s*. How much will be due on May 15, supposing interest at 5 per cent. to be charged on the debt? . . Ans. £176, 19*s*. 3$\frac{0}{4}$*d*. $\frac{48}{73}$.

* NOTE.—In exercises 33 to 40, we require to calculate, in each case, the number of days from the one given period to the other. The following is the method of calculating the number of days—as, for instance, from January 7 to March 29—

Jan. 24	Here we take the remaining days in January *after* the 7th, then the days in February, and the days in March *up to* the 29th, and add them all together. The day reckoned *from* is not counted, but that *to* which we reckon, is counted.
Feb. 28	
Mar. 29	
Ans. 81 days.	

V. To calculate the Interest on sums or debts when partial payments are made.

A partial payment of a debt is made, when part of the principal is repaid after a certain time, leaving the balance at interest for a longer period.

Rule.—Multiply each sum or balance due, by the number of days that it lies at interest, and add together the different products; then multiply the sum-total by *twice* the rate per cent. and divide the result by 73,000.

Example.—A sum of £300 was borrowed on March 16; of which £50 was repaid on April 7, £100 on July 16, and the balance, including interest at 4 per cent., on October 11; how much will the last payment amount to?

		£	Days at interest.	Products.
Mar. 16.	Principal,	300 ×	22 =	6,600
April 7.	Paid,	50		
	Balance,	250 ×	100 =	25,000
July 16.	Paid,	100		
	Balance,	150 ×	87 =	13,050
				44,650
				8
			73,000)	357,200

	£	s.	d.
Interest due at Oct. 11,	£4	17	$10\frac{1}{4}$
Balance at July 16,	150	0	0
Ans.	£154	17	$10\frac{1}{4}$

Here £300 lies at interest from March 16 to April 7, that is, 22 days; we therefore multiply 300 by 22. On April 7, £50 was paid, and the balance, £250, lies at interest from April 7 to July 16, that is, 100 days; we therefore multiply 250 by 100. Again, on July 16, £100 was paid, and the balance, £150, lies at interest from July 16 to October 11, that is, 87 days; hence we multiply 150 by 87. We now add the products, and multiply their sum, 44650, by 8, twice the rate per cent., and then divide the product by 73,000; the interest is £4, 17*s.* $10\frac{1}{4}$*d.*, to which we add £150, the balance due at July 16; and the sum £154, 17*s.* $10\frac{1}{4}$*d.* is the amount of the last payment.

Exercises.

41. A bill of £1000 became due on June 17, of which £250 was paid Aug. 1, £300 on Oct. 11, £150 on Nov. 17, and the balance on Dec. 31; how much interest was then due at $4\frac{3}{4}$ per cent.? Ans. £16, 13*s.* $4\frac{3}{4}$*d.* $\frac{53}{73}$.

42. A person borrowed £500 on Feb. 2: he repaid $\frac{1}{4}$ of this sum on May 15, $\frac{1}{4}$ on Aug. 1, $\frac{1}{4}$ on Nov. 11, and the remaining $\frac{1}{4}$, together with the interest, on Dec. 31. Required the amount of the last payment, Ans. £140, 6*s.* $10\frac{2}{4}$*d.* $\frac{68}{73}$.

43. A gentleman borrowed £750 on Jan. 1 at $4\frac{1}{2}$ per cent., of which he is to repay £220 at Lady-day, £125 at Midsummer, £185 at Michaelmas, and the balance, together with the interest, on Christmas-day; how much will the last payment amount to? Ans. £240, 16*s.* $5\frac{1}{2}$*d.* $\frac{348}{365}$.

44. Required the interest, at $3\frac{3}{4}$ per cent., on £320 due on September 11, 1846, of which £116 was paid on November 23, £50 on December 21, £100 on January 19, 1847, £30 on February 22, and the balance on May 11, ... Ans. £3, 16*s.* $6\frac{1}{4}$*d.* $\frac{211}{365}$.

45. I lent £456 to a friend on March 14, and received as part payment £66 on April 30, £130 on July 11, £120 on August 15, £100 on October 19, and the balance on November 30; how much interest have I to receive, at this last date, at $3\frac{1}{2}$ per cent.?

Ans. £6, 13*s*. $0\frac{3}{4}$*d*. $\frac{1681}{1825}$.

46. A. received from the bank on March 1, £700, of which he paid £100 on April 1, £100 on May 1, and £100 on the 1st of each succeeding month until the whole amount was paid; how much was the last payment, including the interest at $4\frac{1}{2}$ per cent.?

Ans. £110, 11*s*. $8\frac{3}{4}$*d*. $\frac{2}{73}$.

VI. To calculate the Interest on Accounts-current.

An Account-current is an account in which is drawn out, in Dr. and Cr. (Debtor and Creditor) columns, a statement of the transactions that have taken place between two parties, during a given time.

The term *Dr.*, or *debtor*, is placed on the left, to shew that the correspondent is *debtor* for the sums on the left; and the term *Cr.*, or *creditor*, is placed on the right, to shew that the correspondent is *creditor* for the sums on the right. The word *To*, is used to denote Dr.: *By*, to denote Cr.

Example.—Required the interest at 4 per cent. on the following account-current to June 30.

Mr James Simpson, London, in Account-current with

Dr. Robert Duff, Liverpool. *Cr.*

			£	s.	d.				£	s.	d.
1859.						1859.					
Jan.	18	To Goods,	360	14	9	Jan.	1	By Goods,	102	17	0
Mar.	23	" do.	468	11	6	"	30	" Cash,	400	0	0
Apr.	17	" do.	124	10	0	Apr.	24	" Bill,	350	0	0
May	22	" do.	739	15	4	June	16	" Goods,	690	14	4
June	30	" Interest	3	8	$2\frac{1}{2}$	"	30	" Bal. fwd.	153	8	$5\frac{1}{2}$
			1696	19	$9\frac{1}{2}$				1696	19	$9\frac{1}{2}$
June	30	To Balance,	153	8	$5\frac{1}{2}$						

Liverpool, June 30, 1859.

Robert Duff.

This account is drawn out by R. Duff, and sent to J. Simpson on June 30. On the left or Dr. side are written all the sums that Simpson owes to Duff; and on the Cr. side the sums that Duff owes to Simpson. Interest at 4 per cent. is then calculated on the account, as below; and as Duff finds that Simpson is due him £3, 8*s*. $2\frac{1}{2}$*d*. of interest, he enters it on the Dr. side. He next adds the Dr. side, and finds the amount to be £1696, 19*s*. $9\frac{1}{2}$*d*.; then the Cr. side, which amounts to £1543, 11*s*. 4*d*. The difference between these, £153, 8*s*. $5\frac{1}{2}$*d*., is entered on the Cr. side, to balance the account, and then transferred to the Dr. side of a new account, shewing that J. Simpson is owing R. Duff £153, 8*s*. $5\frac{1}{2}$*d*.

INTEREST on the account is calculated as follows:

					Days.		Dr.	
Jan. 18,	To £360	14	9	×	163	. .	= 58800	
Mar. 23,	" 468	11	6	×	99	. .	= 46389	
Apr. 17,	" 124	10	0	×	74	. .	= 9213	
May 22,	" 739	15	4	×	39	. .	= 28851	146,253
							Cr.	
Jan. 1,	By £102	17	0	×	181	. .	= 18616	
" 30,	" 400	0	0	×	151	. .	= 60400	
Apr. 24,	" 350	0	0	×	67	.	= 23450	
June 16,	" 690	14	4	×	14	. .	= 9670	112,136

$$\begin{array}{r} 31,117 \\ 8 \\ 73,000)\overline{248,936} \\ \text{Interest, } £3\ \ 8\ \ 2\frac{1}{4} \end{array}$$

As the account is supposed to be settled up to June 30, the interest is calculated according to Rule V. page 172, on all the sums on both sides of the account, up to that date: as, for instance, on £360, 14*s.* 9*d.* from January 18 to June 30, or for 163 days; and so on. The products of the Dr. side are placed in one column, and of the Cr. side in another; each column is then added, and the smaller of the two sums deducted from the greater; the interest is then calculated on the difference; and the Dr. products in this case being the greater, the interest is entered on the Dr. side of the account-current.

In multiplying the sums by the number of days, the shillings and pence of the products have, for convenience, been left out.

Exercises.

47. Required the interest on the following account to December 31, at $4\frac{1}{2}$ per cent.:

Dr. Mr J. MACLAURIN in Account-curt. with W. FERRIE. *Cr.*

July 17,	To Goods, .	£450	July 30,	By Cash, .	£340
Aug. 15,	" Cash, . .	300	Sept. 17,	" do. . .	693
Sept. 4,	" do. .	721	Oct. 15,	" do. .	960
Nov 11,	" do. . .	875	Nov. 29,	" do. . .	123

Ans. Interest due to Ferrie, £5, 4*s.* $6\frac{1}{4}$*d.* $\frac{1737}{1825}$.

48. Jamieson and Son are indebted to Alexander Banks £452 on July 5: they grant him a bill for £165, payable on July 18; and another for £225, payable on August 1; they are due him for goods £347 on August 25, and £127 on September 11; they grant him a bill for £439, due on October 10; and on November 8 they send goods to the value of £716; on December 17 they receive from him £560. Required the account-current sent to Banks on December 31, allowing interest at 5 per cent. Ans. Jamieson and Son, owe Banks £1, 0*s.* $0\frac{1}{2}$*d.* $\frac{4}{5}\frac{5}{3}$ for interest; Banks is indebted to them £57, 19*s.* $11\frac{1}{4}$*d.* $\frac{4}{7}\frac{1}{3}$.

COMPOUND INTEREST is computed by adding to the principal, the interest due at any given time, as—at the end of a year; then reckoning interest on this new amount for a similar period, and again adding it as before; and so on.

Example.—What will £100 amount to in 3 years, at 5 *per ct.* compound interest?

	£100	0	0	
	5	0	0	
	105	0	0	1st year.
	5	5	0	
	110	5	0	2d year.
	5	10	6	
Ans.	£115	15	6	3d year.

Here we add to the principal, the interest for one year, £5; then to this amount, the interest for the second year, £5, 5*s.*; and to the last amount, the interest for the third year, £5, 10*s.* 6*d.* The total amount at the end of the third year is £115, 15*s.* 6*d.*—namely, principal, £100; and compound interest, £15, 15*s.* 6*d.*

Compound Interest may be calculated in this way when the time is only two or three years; but for longer dates, this would be a tedious process, and another method is employed. See Compound Interest, page 227, where the subject is treated of at length.

DISCOUNT.

Discount is a charge of so much per cent. made by bankers and others, for advancing money upon Bills, &c., before they are due. Discount is *deducted* from the given sum, and is thus the reverse of Interest.

A Bill is an agreement written on stamped paper, in which a debtor agrees to pay to his creditor on a certain day, a specified sum of money which he is owing to him.

The bill may at any period be *discounted* by a bill-broker or banker. The discounting of a bill consists in giving the money for it, less a certain sum for *interest.* Thus, if a bill for £100 for three months is discounted at 5 per cent., a charge equal to three months' interest is made by the discounter, and this is his profit for the loan of the money for that period.

The *net proceeds*, or, present worth of a bill is the net sum that is received for it, after deducting the discount.

According to a practice of old standing, bills are not presentable for payment till the third day after that which is specified for them to fall due. The three days allowed are called the *days of grace.* Thus, a bill drawn on the 5th of August, at three months, is not legally due till noon of the 8th of November.

DISCOUNT is also the term applied to the allowance or deduction frequently made at the settlement of accounts. Thus, a person who is owing an account of £100, on settling it, may receive an allowance of $2\frac{1}{2}$ per cent.; he would therefore pay only £97, 10*s*., the remaining £2, 10*s*. being allowed as discount.

When discount at so much per cent. is stated without any time being specified, as, 'discount 5 per cent. on £250,' the meaning is, that discount is to be reckoned at the rate of £5 for every £100 in the sum.

DISCOUNT IS CALCULATED IN THE SAME WAY AS INTEREST, whether for years, months, or days. When no particular time is specified, it is calculated by Rule I. of Interest.

Example.—What is the discount and net proceeds of a bill for £250, dated Aug. 1, due at 4 months after date, which was discounted on Sept. 23, at 4 per cent.?

	£	*s.*	*d.*
Bill,	250	0	0
Deduct discount for 72 days,	1	19	6
Net proceeds,	£248	0	6

Here the bill is payable on December 4, reckoning the *three days of grace*, that is in 72 days after September 23, the day on which it was discounted. The discount for 72 days is calculated as in Interest, Rule IV., and amounts to £1, 19*s*. 6*d*., which being deducted from the bill, leaves £248, 0*s*. 6*d*. as the net proceeds.

In discounting bills, any farthings in the answer are considered, by bankers, as a penny; thus, if the discount amounts to £1, 19*s*. $5\frac{1}{4}$*d*., it is reckoned as £1, 19*s*. 6*d*.

DISCOUNT AT 10 PER CENT. is calculated by merely taking $\frac{1}{10}$th of the given sum—that is, dividing it by 10: thus, discount on £370 at 10 per cent. is £37.

Exercises.

1. A bill dated January 1, at 3 months' date, for £739, 16*s*. 11*d*., was discounted on February 14. What was the discount and net proceeds? Ans. Discount, £4, 19*s*. $3\frac{3}{4}$*d*. $\frac{812}{1825}$; net proceeds, £734, 17*s*. $7\frac{0}{4}$*d*. $\frac{1013}{1825}$.

2. Required the discount and the net proceeds on the following bills at 4 per cent., which were discounted on April 4: one for £174, dated February 24, at 4 months; one for £1000, dated March 15, at 2 months. Ans. Discount, £6, 8*s*. $5\frac{1}{2}$*d*. $\frac{1322}{1825}$; net proceeds, £1167, 11*s*. $6\frac{1}{4}$*d*. $\frac{503}{1825}$.

3. The following bills were discounted on June 27: No. 20, for £360, dated April 14, at 5 months; No. 23, for £721, dated May 2, at 3 months; No. 31, for £875, 10*s*., dated May 15, at 2 months; and No. 32, for £691, 15*s*., dated June 3, at 4 months. What was the net proceeds, allowing interest at 4 per cent., and commission at $\frac{1}{8}$ per cent.? . . Ans. £2619, 0*s*. $4\frac{3}{4}$*d*. $\frac{327}{365}$.

DISCOUNT, AS *practically* understood, is calculated as above, by deducting the interest for the given time from the principal.

DISCOUNT, IN THE STRICTLY *correct* SENSE, is ascertained by calculating what is the sum which, with interest for the given time, will amount to the sum on which the *true* discount is to be taken.

The Interest on the smaller sum thus found, is equivalent to the TRUE DISCOUNT on the given sum; and the smaller sum is called the PRESENT WORTH of the other.

I. TO FIND THE *present worth* OF A DEBT DUE AT A CERTAIN TIME, THE RATE OF INTEREST BEING GIVEN.

RULE—1. Find, by Simple Proportion, what £100, with interest for the given time and rate, will amount to.

2. Then say, *as* the AMOUNT of £100, for the given time and rate, *is to* £100, *so is* the given debt to the present worth required.

II. TO FIND THE *true* DISCOUNT ON A DEBT.

RULE—1. Find as before what £100, with interest for the given time and rate, will amount to.

2. Then say, *as* the AMOUNT of the £100 *is to* the amount of the debt, *so is* the interest on £100 to the discount on the debt.

Or, Find the PRESENT WORTH of the debt by Rule I., and the difference between the present worth, and the amount of the debt is the *discount*.

The Reason of the rule is plain.

Example.—What sum of money will now discharge a debt of £3725, 11*s*. 9*d*., due 210 days hence, reckoning interest at 4 per cent.; and what is the true discount on the debt? . Answer: Present worth, £3641, 15*s*. $6\frac{1}{4}$*d*. $\frac{1265}{1867}$; Discount, £83, 16*s*. $2\frac{1}{2}$*d*. $\frac{602}{1867}$.

$365 : 210 :: £4 : £\frac{168}{73}$ = the interest of £100 for 210 days.

$\therefore £100 + £\frac{168}{73} = £100\frac{168}{73}$ = the amount of £100 for do.

$\therefore £100\frac{168}{73} : 100 ::$ £3725, 11*s*. 9*d*. : £3641, 15*s*. $6\frac{1}{4}$*d*. $\frac{1265}{1867}$ present worth.

Here, since 4 is the rate per cent., to find the interest for 210 days, we say, *as* 365 *is to* 210, *so is* £4 *to* £$\frac{168}{73}$; hence £$100\frac{168}{73}$ is equal to the *amount* of £100 for 210 days. Again, *as* £$100\frac{168}{73}$ *is to* 100, *so is* £3725, 11*s*. 9*d*. *to* the answer. Working out this by Simple Proportion, we obtain £3641, 15*s*. $6\frac{1}{4}$*d*. $\frac{1265}{1867}$ for the present worth required.

Again, $£100\frac{168}{73} : 3725\ 11\ 9 :: £\frac{168}{73} : £83\ 16\ 2\frac{1}{2}\frac{602}{1867}$ discount.

A SIMPLE RULE FOR CALCULATING THE TRUE DISCOUNT, and present worth of a sum of money, may be derived from the principle stated above, as follows:

RULE.—Multiply 100 by 365, if for days, or by 12, if for months, as the case may be. Then multiply the *rate* by the given days, or months; and add both the products together.

To find the PRESENT WORTH, say—*As* the sum of the products *is to* the *first* product, *so is* the debt to its present worth; then work out the question by Simple Proportion.

To find the DISCOUNT, say—*As* the sum of the products *is to* the *second* product, *so is* the debt to the discount.

The Example in the previous page will stand as under.

$100 \times 365 = 36500$

$4 \times 210 = \underline{840}$

		Present worth.
Sum . .	37340 : 36500 :: £3725 11 9 ;	£3641 15 $6\frac{1}{4}\,\frac{1265}{1867}$.
		Discount.
Again, .	37340 : 840 :: £3725 11 9 :	£83 16 $2\frac{1}{2}\,\frac{602}{1867}$.

Here $365 : 210 :: £4 : \frac{£4 \times 210}{365} =$ Interest of £100 for 210 days;

$\therefore £\left(100 + \frac{4 \times 210}{365}\right) : £100 :: £3725,\ 11s.\ 9d. :$ present worth.

If the first and second terms be multiplied by 365, which does not alter the ratio, it becomes $100 \times 365 + 4 \times 210 : 365 \times 100$ :: £3725, 11*s.* 9*d.* : present worth; but the first term is $100 \times 365 + 4 \times 210$; that is, 100 multiplied by the number of days in a year + the rate per cent. multiplied by the number of days for which the discount is to be given; and the second term is 100, multiplied by the number of days in a year. To find the discount, the second term of the proportion is 4×210, and the other terms the same as before.

THE REASON OF THE RULE is obvious from the above remarks.

Exercises.

4. What is the present worth of £456, 13*s.*, due 146 days hence, and the true discount on the debt? Ans. £447, 13*s.* $11\frac{0}{4}d.\ \frac{4}{17}$ present worth; £8, 19*s.* $0\frac{3}{4}d.\ \frac{13}{17}$ discount.

5. What sum of money will now pay a debt of £1000, due 2 years hence, interest at $3\frac{1}{2}$ per cent., and what is the true discount on the £1000? Ans. £934, 11*s.* $7\frac{0}{4}d.\ \frac{28}{107}$ present worth; £65, 8*s.* $4\frac{3}{4}d.\ \frac{79}{107}$ discount.

6. What is the present value of £461, 12*s.* 3*d.*, due 11 months hence, interest at $4\frac{3}{4}$ per cent., and the true discount on the debt? Ans. £442, 7*s.* $0\frac{1}{4}d.\ \frac{3487}{5009}$ present value; £19, 5*s.* $2\frac{1}{2}d.\ \frac{1522}{5009}$ discount.

7. I owe £738 in 23 days, £276 in 101 days, and £372 in 219 days. How much money will discharge these debts at present, interest being allowed at the rate of 4 per cent.?

Ans. £1372, 8*s.* $1\frac{0}{4}d.\ \frac{11814333}{10547931}$.

COMMISSION AND BROKERAGE.

Commission is a charge of so much per cent. made by an agent for buying or selling goods, &c., on account of another. The rate varies from 1 to 10 per cent.

Brokerage is a similar charge made by persons termed brokers, for assisting others in buying or selling goods, shares, &c. The rate is usually less than 1 per cent.

Rule.—**Commission** and **Brokerage** are calculated by multiplying the given sum by the rate per cent., and dividing the product by 100, as in Interest, Rule I.

When the rate is 1, 2, 3 per cent., &c., *pounds* are meant; and when the rate is ½, ¾, &c., these fractions of a pound are meant; when the rate is expressed in shillings and pence, take proportionate aliquot parts; thus, for 15*s.* take ¾ of £1.

The Reason of the Rule will be obvious by stating the accounts, as in Simple Proportion.

Example 1.—What is the commission on £735, 15*s.* 8*d.*, at 3½ per cent.?

	£	*s.*	*d.*
	735	15	8
			3½
	2207	7	0
	367	17	10
1,00)	25,75	4	10
	20		
	15,04		
	12		
	0,58		
	4		
	2,32		

$$\frac{2{,}32}{100} = \frac{8}{25}$$

Pupil. In this example, I multiply £735, 15*s.* 8*d.* by 3½; the product is £2575, 4*s.* 10*d.* This product I now divide by 100, and the quotient, £25, 15*s.* 0½*d.* $\frac{8}{25}$, is the commission required.

The answer may be readily found within a farthing, by annexing half the number of shillings to the pounds of the product, and pointing off three right-hand figures as decimals, then valuing the decimal. Thus, in the present example, half the number of shillings in the product is 2; this 2 annexed to the pounds, £2575, gives £25752. Pointing off the three right-hand figures, we have £25·752, which is equal to £25, 15*s.* 0½*d.*

Example 2.—What is the brokerage on £697, 13*s.* 9*d.* at 4*s.* 8*d.* per cent.?

	£	*s.*	*d.*
	697	13	9
4*s.* = ⅕ of £1	139	10	9
8*d.* = ⅙ " 4*s.*	23	5	1½
1,00)	1,62	15	10½
Ans.	£1	12	6½ $\frac{41}{50}$

or £1·628 = £1, 12*s.* 6¾*d.*

In this example, aliquot parts of £1 are taken for the 4*s.* 8*d.*; the result being divided by 100, gives the brokerage required—namely, £1, 12*s.* 6½*d.* $\frac{41}{50}$.

Exercises.—What is the commission on the following sums?

1. £325, 19*s.* 11*d.* at 5 per cent. . .	Ans.	£16	5	$11\frac{3}{4}$	$\frac{4}{5}$
2. £695, 10*s.* $11\frac{1}{4}d.$ at 3 per cent. .	"	20	17	$3\frac{3}{4}$	$\frac{39}{50}$
3. £384, 17*s.* 9*d.* at 2 per cent. . .	"	7	13	$11\frac{1}{4}$	$\frac{21}{25}$
4. £1234, 15*s.* $6\frac{3}{4}d.$ at $4\frac{1}{2}$ per cent. .	"	55	11	$3\frac{1}{2}$	$\frac{83}{200}$
5. £479, 1*s.* $8\frac{1}{4}d.$ at $3\frac{3}{4}$ per cent. .	"	17	19	$3\frac{3}{4}$	$\frac{3}{80}$
6. £673, 2*s.* 6*d.* at $5\frac{1}{4}$ per cent. .	"	35	6	$9\frac{1}{4}$	$\frac{1}{2}$
7. £7854, 14*s.* 3*d.* at $4\frac{7}{8}$ per cent. .	"	382	18	$4\frac{0}{4}$	$\frac{109}{200}$
8. £572, 4*s.* $6\frac{1}{4}d.$ at $12\frac{1}{2}$ per cent. .	"	71	10	$6\frac{3}{4}$	$\frac{1}{8}$

What is the brokerage on the following sums?

9. £439, 12*s.* 6*d.* at 3*s.* 4*d.* per cent. .	Ans.	£0	14	$7\frac{3}{4}$	$\frac{2}{5}$
10. £975, 10*s.* 3*d.* at 5*s.* 6*d.* per cent.	"	2	13	$7\frac{3}{4}$	$\frac{353}{1000}$
11. £1025, 15*s.* 4*d.* at $\frac{1}{8}$ per cent. .	"	1	5	$7\frac{1}{2}$	$\frac{23}{25}$
12. £731, 17*s.* $9\frac{1}{2}d.$ at $\frac{4}{5}$ per cent. .	"	5	17	$1\frac{0}{4}$	$\frac{114}{125}$
13. £456, 7*s.* 8*d.* at 7*s.* 6*d.* per cent. .	"	1	14	$2\frac{1}{2}$	$\frac{49}{50}$
14. £840, 9*s.* at $\frac{3}{4}$ per cent. . . .	"	6	6	$0\frac{3}{4}$	$\frac{6}{25}$
15. £5239, 1*s.* 4*d.* at 2*s.* 9*d.* per cent. .	"	7	4	$0\frac{3}{4}$	$\frac{71}{125}$

16. If a broker sells goods to the amount of £725, 14*s.*, what is his allowance at $\frac{3}{8}$ per cent.? . . Ans. £2, 14*s.* $5\frac{0}{4}d.$ $\frac{13}{25}$.

17. A broker procures sales for his employer to the amount of £1565, required his allowance at $\frac{1}{2}$ per cent. . Ans. £7, 16*s.* 6*d.*

18. An agent annually disposes of woollen stuffs to the amount of £820, 14*s.* 6*d.*; of cotton to the amount of £327, 15*s.* 4*d.*; of linen to the amount of £120, 19*s.*; of silk to the amount of £316, 14*s.* 9*d.*; and of other manufactures to the amount of £6432, 11*s.* 5*d.*; what is his annual income, supposing his commission to be at the rate of 4 per cent.? . . . . . Ans. £320, 15*s.*

19. An agent is allowed $5\frac{1}{2}$ per cent. for selling goods and guaranteeing the debts to his employers. His sales in a year amount to £23514, 16*s.* 9*d.*, his losses to £600, 11*s.* 3*d.*, and the necessary charges attending the business are £117, 4*s.* 4*d.*; what is his net annual income? . . Ans. £575, 10*s.* $8\frac{3}{4}d.$ $\frac{21}{50}$.

When an agent guarantees the debts to his employer, his commission is called a *Del-credere* commission.

20. An agent shipped for his employers goods to the value of £561, 4*s.* 10*d.*; the charges of package, porterage, &c., amounted to £3, 15*s.* 6*d.*; the shipping charges to £5, 10*s.* 8*d.* He is allowed $2\frac{1}{2}$ per cent. on the sum laid out; required the amount of the invoice, ... Ans. £584, 16*s.* $3\frac{1}{4}d.$ $\frac{1}{5}$.

See Invoice, pp. 74 and 184.

21. An agent sent his employer in Jamaica an account of the sales of 50 hhd. of sugar, amounting to £2750, 18*s.* 9*d.*; commission at $2\frac{1}{2}$ per cent., and brokerage $\frac{1}{2}$ per cent.; duty, freight, and other charges, £935, 7*s.* 6*d.*; what was the net proceeds due to his employer? . . . . . . Ans. £1733, 0*s.* $8\frac{1}{4}d.$

See Account Sales, p. 185.

INSURANCE.

INSURANCE is a contract by which certain persons or insurance-offices engage to make good to the party insuring, losses he may sustain of ships or their cargoes at sea, or of houses or goods by fire.

The parties who take upon themselves the risk, are called the *insurers*, or *underwriters;* and the person protected, the *insured.*

The sum paid to the insurers is called the *premium;* the stamped paper on which the contract is written, the *policy of insurance;* and the stamp-duty on the policy, the *policy-duty.* Besides the premium and duty, there is, in some cases, a commission charged.

Sums of money are also insured on persons' lives; an individual contracting to pay a certain premium annually during his life has a sum insured to be paid to his family at his decease.

I. To find the Premium on the Sum insured.

RULE.—Multiply the given sum by the rate per cent., and divide the product by 100, as in Commission.

When the rate is 1, 2, 3, per cent. &c., *pounds* are meant. When the rate is expressed in shillings and pence, take proportionate aliquot parts; thus, for 15*s.* take $\frac{3}{4}$ of £1, or for 8*s.* 4*d.* take $\frac{1}{3}$ of £1.

When the rate is expressed in guineas, calculate as if it were in pounds, and to the result add $\frac{1}{20}$, for the premium required.

In the following exercises, the stamp-duty on the policy, and the commission on the amount insured, are added to the premium, to give the whole expense of the insurance.

When the amount insured is not an exact number of £100, the policy-duty is calculated on the number of £100 next greater than the amount insured; thus, if the amount insured is £350, the policy-duty is calculated on £400.

Example.—What is the expense of insuring a cargo valued at £754, 14*s*. 7*d*., the premium being 3 guineas per cent., policy-duty 5*s*. per cent., and commission ½ per cent.?

	£	s.	d.
	754	14	7
			3
100)	2264	3	9
	£22	12	$10\frac{2}{4}\,\frac{20}{100}$

	£	s.	d.
	754	14	7
			½
100)	377	7	$3\frac{1}{2}$
	£3	15	$5\frac{1}{2}\,\frac{70}{100}$

		£	s.
Duty on £100	=	0	5
			8
〃 £800	=	2	0

		£	s.	d.
		22	12	$10\frac{2}{4}\,\frac{20}{100}$
Add $\frac{1}{20}$		1	2	$7\frac{1}{2}\,\frac{81}{100}$
Premium	=	23	15	$5\frac{3}{4}\,\frac{1}{100}$

		£	s.	d.	
Prem. on £754, 14*s*. 7*d*. at 3 guin. per cent.	=	23	15	$5\frac{3}{4}$	nearly.
Com. 〃 〃 〃 ½ 〃	=	3	15	$5\frac{3}{4}$	〃
Policy-duty on £800 〃 5*s*. 〃	=	2	0	0	
Whole expense, . .	=	29	10	$11\frac{1}{2}$	

Exercises.

Required the premium on the following sums.

1. £780 at 2½ guineas per cent. . . Ans. £20, 9*s*. 6*d*.
2. £1965 at 3¼ guineas per cent. . Ans. £67, 1*s*. $1\frac{1}{4}$*d*. $\frac{2}{5}$.
3. £873 at 4¾ guineas per cent. . Ans. £43, 10*s*. $9\frac{3}{4}$*d*. $\frac{6}{25}$.
4. £695 at 5 guineas per cent. . . . Ans. £36, 9*s*. 9*d*.

5. Find the expense of insuring household property to the amount of £650, the premium being 1*s*. 6*d*. per cent., and policy-duty 3*s*. per cent. . . . . . Ans. £1, 10*s*. 9*d*.

6. What is the expense of insuring £780 on goods from Leith to London at 1½ guineas per cent., policy-duty 2*s*. 6*d*. per cent., and commission ½ per cent.? . . . Ans. £17, 3*s*. $8\frac{1}{4}$*d*. $\frac{3}{5}$.

7. What is the whole expense of insuring £1821 on goods from Oporto to Leith, premium 5½ guineas per cent., policy-duty 5*s*. per cent., and commission ½ per cent.? Ans. £119, 0*s*. $4\frac{1}{4}$*d*. $\frac{1}{25}$.

8. What is the expense of insuring £2670 on goods from London to Jamaica at 6¼ guineas per cent., policy-duty 5*s*. 8*d*. per cent., and commission ½ per cent.? . Ans. £195, 13*s*. $1\frac{1}{2}$*d*.

9. What is the expense of insuring £3587 on a brewery at 2*s*. 6*d*. per cent., and policy-duty 3*s*. per cent.? Ans. £9, 17*s*. $8\frac{2}{4}$*d*. $\frac{3}{5}$.

II. To find how much must be insured, in order to cover a given Sum, besides paying all expenses of premium, &c.

A merchant sometimes insures not only the value of his property, but also the premium, duty, commission, and other charges; so that, in case of loss, he may be entitled to receive from the underwriters, or insurance-office, a sum equal to the value of the property and expenses of insurance. In this case, the property is said to be *covered*.

Rule.—Subtract the percentage to be paid for premium, duty, and commission, from £100; then state the case as a question in Simple Proportion, thus: '*As* the remainder—that is, £100 less the expenses—*is to* £100, *so is* the given sum to the amount to be insured.'

Example.—What sum must be insured to cover £2820 in case of loss, the premium being 5 guineas per cent., policy-duty 5*s.* per cent., and commission ½ per cent.?

Deduct from		£100
Premium,	105/	
Policy-duty,	5/	
Commission,	10/	6
		£94 : 100 :: 2820 : 3000 *Ans.*

Here, on deducting the premium, &c., on £100, from that sum, the remainder is £94; and, consequently, we must insure for £100, in order to cover £94; the question, therefore, is stated thus: 'As £94 is to £100, so is £2820 to the sum to be insured for.'

Proof.

Sum insured, ...		£3000
Premium on £3000 at 5 guin. per cent. =	£157 10	
Policy-duty " " 5*s.* " =	7 10	
Commission " " ½ " =	15 0	180
Net proceeds, ...		£2820

Exercises.

10. What sum must be insured to cover £3074, 16*s.* in case of loss, the premium being 3 guineas per cent., policy-duty 5*s.* 3*d.* per cent., and commission ½ per cent.? . . . Ans. £3200.

11. What sum must be insured to cover £750, premium 2⅛ guineas, and commission ¼ per cent.? . Ans. £774, 3*s.* 10¼*d.* $\frac{26}{31}$.

12. How much must be insured to cover £675, the premium being 4¼ guineas, and commission ¼ per cent.?

Ans. £710, 4*s.* 11¾*d.* $\frac{822}{7603}$.

13. How much must be insured to cover £1000, the whole expenses attending the insurance being £8, 7*s.* 6*d.* per cent.?

Ans. £1091, 8*s.* 1¾*d.* $\frac{718}{733}$.

14. What sum must be insured to cover £1250, the whole expenses attending the insurance being £5, 15*s.* per cent.?

Ans. £1326, 5*s.* 2¼*d.* $\frac{297}{377}$.

Note.—By the 'overtaker' in the above sale, is meant the additional barrel required for the coffee taken out of such of the tierces as have been opened on account of breakage or other damage. The freight is charged on the weight of the produce.

AN ACCOUNT-SALES is an account drawn out by a commission-agent, shewing the sales he has made of goods on account of another party. It contains a statement of the quantities sold, and the prices, also the charges for freight, commission, &c.

17. On the 6th February 1859, Henry Barclay & Co. of London shipped the following goods on board the *Rawkins*, J. Thomson, master, on account and risk of Messrs James Allan & Co. of Kingston, Jamaica: No. 1 to 6, 6 puncheons of strong calf-skin shoes, as per invoice, £278, 15*s.* 11*d.*; No. 7, a case of linen-tick assorted, as per invoice, £42; No. 8 to 16, 9 bales best tow Osnaburgs, as per invoice, £236, 5*s.*; No. 17 to 24, 8 cases white platillas, as per invoice, £328, 5*s.* 4*d.*; No. 25 to 38, 14 bales lint Osnaburgs, as per invoice, £367, 10*s.*; No. 39 to 41, 3 cases felt hats, as per invoice, £32, 2*s.* Entry, bond, and debenture, £4, 8*s.*; cartage, wharfage, and shipping charges, £7, 9*s.* 6*d.*; freight and bills of lading, £38, 10*s.* 6*d.*; insurance on £1500 at 2 per cent.; policy-duty, £3, 18*s.* 9*d.*; commission on £1335 at 5 per cent.; commission on the sum insured ½ per cent. What is the amount of the invoice? . . . . . Ans. £1443, 10*s.*

18. James Lowe of Dundee receives, on the 7th March 1859, by the *Brothers*, William Salmond, master, from Dublin, 9 bales of linen, to sell on account of Robert Hay of that place. On the 23d March he sells to Whyte & Co. No. 2, containing 20 pieces at 23*s.* 9*d.* per piece; No. 3, 15 pieces at 24*s.* 6*d.*; No. 5, 25 pieces at 26*s.* 9*d.*; No. 8, 17 pieces at 28*s.* 3*d.* On 1st April he sells to Gordon & Co. No. 1, 23 pieces at 30*s.* 6*d.*; No. 4, 25 pieces at 27*s.* 9*d.*; No. 6, 30 pieces at 30*s.* 6*d.*; No. 7, 15 pieces at 31*s.* 9*d.*; No. 9, 30 pieces at 32*s.* 6*d.* He pays for freight and land-waiters' fees, £7, 3*s.* 9*d.*; cartage, porterage, and wharfage, £1, 12*s.* 4*d.*; warehouse rent and insurance against fire, 7*s.* 3½*d.*; commission on the gross proceeds, 5 per cent. Find the net proceeds, and write out the account-sales, Ans. £264, 1*s.* 11¾*d.*

PROFIT AND LOSS.

PROFIT and Loss refers to calculations of the profits or losses of merchants in buying and selling goods.

The price at which a merchant buys his goods, is termed the *cost* price, and that at which he sells them, the *selling* price.

When they are sold for more than they cost, there is a *profit* on the transaction; and when sold for less, there is a *loss*.

The profit or loss is calculated on the *cost* price, and is usually stated at so much per cent.

The method of working questions of Profit and Loss, will be seen from the following examples. They are chiefly wrought by the rules of Practice and Simple Proportion.

I. TO ASCERTAIN THE TOTAL PROFIT OR LOSS IN SELLING A QUANTITY OF GOODS—THE RATES AT WHICH THEY WERE BOUGHT AND SOLD BEING GIVEN.

Example.—A merchant purchased 7 tons of iron rods, at £10, 17*s.* 9*d.* per ton, and sold them again at 16*s.* 4½*d.* per cwt.; how much did he gain on the whole?

Sold 7 tons = 140 cwt. at	£0	16	4½	=	£114	12	6
Bought 7 tons	10	17	9	=	76	4	3
				Total gain,	£38	8	3

The prices at which the goods were bought and sold, are calculated, and the amount of the one is then deducted from that of the other.

Here we find how much money was paid away by finding the cost of 7 tons, at £10, 17*s.* 9*d.* per ton, by Compound Multiplication. Then we find how much was received by valuing 140 cwt. (7 tons) at 16*s.* 4½*d.* per cwt. by Practice. Now, since the money received is *greater* than the money paid away, therefore the merchant *gains* £38, 8*s.* 3*d.*

Exercises.

1. A merchant purchased 345 cwt. Bengal rice at 16*s.* 11¾*d.* per cwt., and by the market rising, he was enabled to dispose of the whole at 19*s.* 5¼*d.* per cwt.; how much did he gain by the transaction? . . . . Ans. £42, 15*s.* 3¾*d.*

2. A person bought 137 cwt. of pearl sago at £1, 14*s.* 9¼*d.* per cwt., and sold it again at 4¼*d.* per lb.; whether did he gain or lose, and how much on the whole? . Ans. £33, 7*s.* 10½*d.* gain.

3. I sold 547 firkins of Irish butter at £2, 6*s.* 10½*d.* per firkin, and lost £210, 7*s.* 4*d.* on the sale; what was the prime cost of a firkin? . . . . Ans. £2, 14*s.* 6¾*d.* $\frac{108}{547}$.

4. I bought 2 butts of sherry, each containing 108 gallons, at 19*s.* 3½*d.* per gallon, but proving inferior, I was obliged to sell it at a loss of £11, 10*s.* 10*d.* on the whole; what was the selling price per gallon? . . . . . . Ans. 18*s.* 2½*d.* $\frac{19}{27}$.

5. A merchant bought $8\frac{1}{4}$ tons of lead at £17, 14*s.* 3*d.* per ton, and by selling he gained £9 on the whole; what was the selling price of a cwt.? . . . . . . Ans. 18*s.* $9\frac{1}{4}d.\frac{2}{85}$.

6. Bought 236 feet of wood at 3*s.* 10*d.* per foot, and sold it at 3*s.* $4\frac{1}{4}d.$; how much did I lose by the transaction?

Ans. £5, 13*s.* 1*d.*

7. A draper bought a quantity of superfine west of England cloth at 18*s.* $4\frac{1}{2}d.$ per yard, and by selling it at £1, 3*s.* 7*d.* he gained £42, 10*s.*; what quantity of cloth was purchased?

Ans. 163 yd. $3\frac{1}{3}$ nl.

8. A merchant purchased a quantity of Mauritius sugar at £2, 10*s.* per cwt., and by retailing it at $6\frac{1}{4}d.$ per lb., he gained £123 on the whole; what quantity did he sell?

Ans. 230 cwt. 2 qr. 14 lb.

9. Bought a quantity of shumac at £12, 11*s.* 6*d.* per ton.; but getting damaged, I was obliged to sell it at 11*s.* 8*d.* per cwt., and lost £178 on the whole; what quantity was sold?

Ans. 195 tons 19 cwt. 1 qr. $1\frac{87}{105}$ lb.

II. To ascertain the Profit or Loss *per cent.*—the cost and selling price being given.

Example.—If Manilla indigo be bought at 3*s.* $8\frac{1}{2}d.$ per lb., and sold at 4*s.* $3\frac{1}{4}d.$, what is the gain per cent.?

s.	*d.*		*d.*		£.	
3	$8\frac{1}{2}$	:	$6\frac{3}{4}$	::	100	: the gain per cent.
12			4			
44			27			
4			100			
178			)2700($15\frac{15}{89}$ *Ans.*			
			178			
			920			
			890			
			$\frac{30}{178}=\frac{15}{89}$.			

Here, since that which costs 3*s.* $8\frac{1}{2}d.$ is sold for 4*s.* $3\frac{1}{4}d.$, the gain is $6\frac{3}{4}d.$ The question then is this, if 3*s.* $8\frac{1}{2}d.$ laid out on goods gains $6\frac{3}{4}d.$, how much would be gained by laying out £100? This is clearly an account in Simple Proportion, and the statement of it is, *as* 3*s.* $8\frac{1}{2}d.$ of prime cost *is to* its gain, *so is* £100 of cost to its gain.

Note.—In questions of profit and loss, it must be remembered that the calculations are made on the *cost* price, and not on the *selling* price of the goods.

Exercises.

10. If American pork be bought for £3, 2*s.* per barrel, and sold at £3, 8*s.* $4\frac{1}{2}d.$, what is the gain per cent.? . . Ans. $10\frac{35}{124}$.

11. How much is gained per cent. by selling Muscatel raisins at £4, 11*s.* 3*d.* per cwt., which were purchased at £3, 17*s.* 9*d.*?

Ans. $17\frac{113}{311}$.

12. How much per cent. is $2\frac{1}{4}d.$ on every shilling of prime cost? . . . . Ans. $18\frac{3}{4}$.

13. Bought Malabar ginger at £1, 19*s.* per cwt., and sold it at £1, 12*s.* 8*d.*; how much was the loss per cent.? . Ans. 16$\frac{28}{117}$.

14. Bought 176 cwt. of Gouda cheese at £2, 5*s.* 4*d.* per cwt., paid for carriage £5, 7*s.* 3*d.*, and sold the whole at 6$\frac{3}{4}$*d.* per lb. What was my gain per cent.?—on the whole?

Ans. 37$\frac{12353}{97031}$ per cent., £150, 2*s.* 1*d.* on the whole?

15. A bankrupt pays his creditors, 8*s.* 9$\frac{3}{4}$*d.* per pound; at what rate per cent. was that? . . . . . Ans. £44, 1*s.* 3*d.*

III. To ascertain the *Selling* Price—the cost, and the profit or loss per cent. being given.

Example 1; *where there is a profit.*—I have a quantity of cochineal which cost me 5*s.* 3$\frac{1}{2}$*d.* per lb., and wish to know at what I must sell it per lb. to gain 16 per cent. Ans. 6*s.* 1$\frac{1}{2}$*d.* $\frac{16}{25}$.

£		£		*s.*	*d.*		*s.*	*d.*		
100	:	116	:	5	3$\frac{1}{2}$	:	6	1	$\frac{16}{25}$	*Ans.*

Since the gain is to be 16 per cent., £100 worth must be sold for £116. Hence this proportion, As £100 of prime cost is to £116, its selling price, so is 5*s.* 3$\frac{1}{2}$*d.* of prime cost, to its selling price. The question is wrought out in the usual way, and the answer found to be 6*s.* 1$\frac{1}{2}$*d.* $\frac{16}{25}$.

Example 2; *where there is a loss.*—If 6 per cent. is lost by selling goods at £2, 1*s.* 4*d.* per cwt., what was the prime cost of 1 cwt.?

£		£		£	*s.*	*d.*		£	*s.*	*d.*
94	:	100	::	2	1	4	:	2	3	11$\frac{1}{2}$ $\frac{30}{47}$

Here, if the goods had been sold for £94 (=100 − 6), the prime cost would have been £100. Hence the proportion in the margin.

Exercises.

16. A merchant, by selling sugar at £2, 19*s.* 7*d.* per cwt., loses 17 per cent.; what was the prime cost? Ans. £3, 11*s.* 9$\frac{1}{4}$*d.* $\frac{33}{83}$.

17. If iron, which was bought for £5, 3*s.* 4$\frac{1}{2}$*d.* per ton, be sold at such a rate as to gain 25 per cent., what was the selling price per ton? . . . . Ans. £6, 9*s.* 2$\frac{1}{2}$*d.* $\frac{1}{2}$.

18. A provision merchant sold Westphalia hams at £4, 3*s.* 10*d.* per cwt., and gained 13 per cent.; what did the hams cost him per cwt.? . . . . Ans. £3, 14*s.* 2$\frac{1}{4}$*d.* $\frac{7}{113}$.

19. A bookseller bought a copy of the *Encyclopædia Britannica*, for £22; at what price must he sell it so as to gain 30 per cent.? Ans. £28, 12*s.*

20. If 7$\frac{1}{2}$ per cent. be gained by selling butter at £4, 10*s.* 6*d.* per cwt., what was the prime cost per cwt.? Ans. £4, 4*s.* 2$\frac{0}{4}$*d.* $\frac{40}{43}$.

21. If 15$\frac{1}{4}$ per cent. be lost by selling Stockholm tar at 15*s.* 4*d.* per barrel, what was the prime cost?—and how much was lost on the sale of 456 barrels? . . Ans. Prime cost, 18*s.* 1$\frac{0}{4}$*d.* $\frac{148}{339}$; loss on 456 barrels, £62, 18*s.* 1$\frac{3}{4}$*d.* $\frac{8}{113}$.

22. A wood-merchant bought 129 loads of Quebec pine deals at 17*s.* 6¼*d.* per load; at what price must he sell each load so as to gain 24 per cent.? . . . . . Ans. £1, 1*s.* $9\frac{2}{4}$*d.* $\frac{2}{25}$.

23. A merchant sold a chest of tea containing 84 lb. for £25, 7*s.*, and gained at the rate of 20 per cent.; what was the prime cost per lb.? . . . . Ans. 5*s.* 0¼*d.* $\frac{3}{7}$.

24. If a load of African teak be bought for £11, 10*s.*, at what must it be sold to gain 18 per cent.? . Ans. £13, 11*s.* 4¾*d.* $\frac{1}{5}$.

25. How must wine, which cost 18*s.* 6*d.* per gallon, be sold so as to gain 22½ per cent.?—and how many gallons must be sold so as to gain £70, 4*s.* 8*d.*?

Ans. Selling price, £1, 2*s.* 7¾*d.* $\frac{4}{5}$; 337$\frac{457}{999}$ gallons.

IV. To ascertain the *Cost* Price—the selling price, and the profit or loss per cent. on the cost being given.

Example 1; *where there is a profit.*—What is the cost price of a yard of cloth, which I sold for 16*s.* 6*d.*, and thereby gained 10 per cent. on the cost?

£		£		*s.* *d.*		*s.*
110	:	100	::	16 6	:	15 *Ans.*

Goods sold for £110 will, at this rate, cost £100; therefore, to find what goods sold for 16*s.* 6*d.* will cost, state the question thus: 'If £110 cost £100, what will 16*s.* 6*d.* cost?'

Example 2; *where there is a loss.*—What is the cost price of a yard of cloth, which I sold for 18*s.*, and thereby lost at the rate of 10 per cent. on the cost?

£		£		*s.*		£
90	:	100	::	18	:	1 *Ans.*

Goods sold for £90 will, at this rate, cost £100; therefore, to find what goods sold for 18*s.* cost, state the question thus: 'If £90 cost £100, what will 18*s.* cost?'

Example 3.—By selling hops at 5 per cent. profit, I gained £69, 10*s.* 4*d.*; what did I pay for them? . Ans. £1390, 6*s.* 8*d.*

This question is nothing else than the following:

£		£ *s.* *d.*		£		£ *s.* *d.*
5	:	69 10 4	::	100	:	1390 6 8

By selling £100 (always regarded as prime cost) worth of hops I gain £5; what is the prime cost of the hops that I must sell to gain £69, 10*s.* 4*d.*? Hence the proportion in the margin.

Exercises.

26. By selling raisins at $4\frac{1}{2}$ per cent. profit, I gained £47, 14*s.*; what was the prime cost? . . . . . . Ans. £1060.

27. Bought nutmegs at 6*s.* 6*d.* per lb., and by selling them again at 6 per cent. profit, I gained altogether £32; what quantity did I sell? . . . . Ans. $1641\frac{1}{39}$ lb.

28. Sold 342 cwt. of sugar at 3 per cent. profit, and gained £26, 17*s.*; what was it bought at per cwt.?—sold at per cwt.?

Ans. Prime cost per cwt., £2, 12*s.* $4\frac{0}{4}$*d.* $\frac{16}{57}$; selling price per cwt., £2, 13*s.* $10\frac{3}{4}$*d.* $\frac{37}{57}$.

29. By selling 782 gallons of brandy at $3\frac{1}{2}$ per cent. loss, I lost altogether £37, 4*s.* 2*d.*; at what was it bought per gallon?—sold per gallon?

Ans. Prime cost, £1, 7*s.* $2\frac{1}{4}$*d.* $\frac{215}{2737}$; selling price, £1, 6*s.* $2\frac{3}{4}$*d.* $\frac{1087}{2737}$.

30. I sold soap at £2, 19*s.* per cwt., which was at 5 per cent. profit, and gained £73, 10*s.*; what quantity did I sell?

Ans. 523 cwt. $24\frac{40}{59}$ lb.

V. To ascertain what will be the Profit or Loss per cent. at a certain Selling Price—the Profit or Loss per cent. at another Selling Price being given.

Example 1; *where there is a profit.*—What will be the percentage gained by selling sugar at £44 a ton, if 5 per cent. is gained by selling it at £42 a ton?

£		£		£		£	
42	:	44	::	105	:	110	
Deduct, as the cost of £110,						100	
The remainder is the percentage required,						£10	viz. 10 per cent. gain.

Here the selling price, £42, which includes a gain of 5 per cent. on the cost, bears the same proportion to £44, the other selling price, that £105—namely, £100 of cost and 5 per cent. added—bears to £100 of cost, with the required percentage added: therefore, state the question thus: 'As £42 is to £44, so is £105 to £100 with the required percentage added.' The answer is £110, from which the cost, £100, is deducted, leaving £10 the required percentage.

Example 2; *where there is a loss.*—What will be the percentage lost by selling sugar at £38 a ton, if 10 per cent. is gained by selling it at £44 a ton?

£		£		£		£
44	:	38	::	110	:	95
The cost price of £95 is						100
The selling price deducted from the cost gives the required percentage, . .						£5 viz. 5 per cent. loss.

Here the selling price, £44, which includes a gain of 10 per cent., bears the same proportion to £38, the other selling price, that £110—namely, £100 of cost, and 10 per cent. of gain *added*—will bear to £100, with the required loss per cent. *deducted*; therefore, state the question thus: 'As £44 is to £38, so is £110 to £100, with the required percentage deducted.' The answer is £95, which is deducted from £100, the cost price, and the remainder is the percentage required.

Exercises.

31. If 15 per cent. be gained by selling hemp at £32, 15*s.* per ton, what is gained or lost per cent. by selling it at £30, 15*s.* 6*d.*?
Ans. £8, 1*s.* $3\frac{1}{2}d.$ $\frac{38}{131}$ gain.

32. By selling goods at 8*s.* I lost 16 per cent.; at what rate must I sell them to gain 7 per cent.? . . Ans. 10*s.* $2\frac{1}{4}d.$ $\frac{1}{7}$.

33. Sold a quantity of logwood at £4, 4*s.* 6*d.* per ton, by which I cleared 17 per cent.; but I afterwards raised the price to £4, 9*s.* per ton; what rate per cent. did I gain at this latter price? . . . . Ans. £23, 4*s.* $7\frac{1}{4}d.$ $\frac{7}{13}$.

34. If 8 per cent. be gained by selling 736 yards of linen for £125, 10*s.*; at what rate must the yard be sold so as to gain 16 per cent.? . . . . Ans. 3*s.* $7\frac{3}{4}d.$ $\frac{170}{207}$.

35. By selling molasses at £1, 10*s.* per cwt. I gained 14 per cent.; but the market falling, I was obliged to dispose of the remainder at £1, 2*s.* 6*d.* per cwt.: what was the gain or loss per cent. at this latter price? . . . . Ans. £14, 10*s.* loss.

Miscellaneous Exercises.

1. A merchant bought 7428 fleeces of wool at 12*s.* 3*d.* per fleece; and sold one-fourth at 13*s.* 9*d.*, one-fourth at 14*s.* 2*d.*, one-fourth at 14*s.* $7\frac{1}{2}d.$, and the remaining fourth at 15*s.* $10\frac{1}{4}d.$; what was his whole gain?—gain per cent.?
Ans. Whole gain, £872, 8*s.* $0\frac{3}{4}d.$; gain per cent., £19, 3*s.* $6\frac{0}{4}d.$ $\frac{8}{49}$.

2. If 8 per cent. be gained by selling 314 yards of cloth for £56, 17*s.* 6*d.*; at what must it be sold per yard so as to gain 26 per cent.? . . . . Ans. 4*s.* $2\frac{1}{2}d.$ $\frac{136}{137}$.

3. Bought American cheese at £2, 14*s.* per cwt., and sold it at £3, 1*s.* 6*d.*; what was my gain on 236 cwt.?—gain per cent.?
Ans. £88, 10*s.* on the whole; £13, 17*s.* $9\frac{1}{4}d.$ $\frac{1}{3}$ per cent.

4. A loss of 10 per cent. was sustained by selling Riga flax at £46 per ton; what was gained or lost per cent. by selling it at £50 per ton? . . . . . . Ans. £2, 3s. 5½d. $\frac{22}{23}$ loss.

5. Bought 718 cwt. of fine Mocha coffee at £5, 16s. 8d. per cwt., and sold one-half at 1s. 4d. per lb., and one-half at 1s. 6d. per lb.; what was the whole gain?—gain per cent.?

Ans. Whole gain, £1507, 16s.; gain per cent., £36.

6. Bought 80 casks of cod-oil, each containing 252 gallons, at £27, 10s. per cask; at what must I sell it per gallon to gain £200 on the whole? . . . . . . Ans. 2s. 4½d. $\frac{2}{7}$.

7. Bought a cask of rum, containing 252 gallons, at 17s. 4d. per gallon; but having lost 12 gallons by leakage, at what must I sell the remainder per gallon to gain 15 per cent. on the whole?

Ans. £1, 0s. $11\frac{0}{4}d.$ $\frac{16}{25}$.

8. Bought quills at 8s. a thousand; at what must I sell them a thousand to clear 20 per cent. and give 6 months' credit?

Ans. 9s. $10\frac{0}{4}d.$ $\frac{8}{25}$.

				s.		
100	:	120	::	8	:	the answer.
100	:	102½				

9. Bought 4914 lb. of mace at 3s. 4½d. per lb.; paid for various charges, £10, 17s. 9d.; sold 1726 lb. at 5s. 1d., 1271 lb. at 4s. 3½d., and the remainder at 2s. 9½d.: what was gained or lost on the whole? . . . . Ans. Gain, £138, 17s. 8d.

10. A merchant imported from Madeira 11 pipes of wine, which cost him 30 guineas per pipe, and which were bottled into 52 dozen each; bottles and other charges, 2s. 4d. per dozen. He sold the one half of it at 16s. 2d. per dozen, and the other half at 17s. per dozen. What did he gain or lose upon the whole?

Ans. Gain, £61, 1s.

11. A malster purchases 400 chaldrons of barley at £5, 10s. per chaldron; he pays for rent £25, for servants' wages £60, and duty at 2s. 7d. per bushel. Each chaldron of barley produces 40 bushels of malt; and he sells the malt at such a rate per bushel as to gain 20 per cent. on his outlay. Required the selling price per bushel, ... Ans. 6s. 6¼d. $\frac{8}{25}$.

12. A corn-merchant imported from Dantzic 1800 qr. of wheat, which cost 47s. 6d. per qr.; but having bonded it for 8 months, it lost 3 per cent. in measure. He paid for granary rent, £17; for duty, 4s. per qr.; and sold it at 64s. 3d. per qr. Required his gain or loss, reckoning interest at 5 per cent.

Ans. Gain, £825, 6s. 6d.

13. How must prunes which cost £1, 12s. per cwt. be sold to gain 25 per cent.? and what quantity must be sold at that rate to gain £100? . . . . Ans. £2; 250 cwt.

SHARES, STOCKS.

Shares and Stocks are terms applied to the capital of Joint-stock Companies, and to those large sums of money borrowed by Government, termed the National Debt.

The capital of joint-stock associations is raised among the partners, by shares fixed at a specified sum; the shares may be £10 each, £50 each, or any other sum; and some persons may hold ten shares, while others have fifty; and so on.

The original sum fixed upon for a share is called *par;* thus, if the shares be £10 each, then £10 is par. Should the undertaking prove lucrative, and yield a percentage larger than can be obtained from other investments, the shares come into brisk demand, and rise in value. A £10 share may bring £12, 10*s*., in which case it is said to be £2, 10*s*. *above* par. Sometimes £100 shares rise to be worth £160, and are then £60 above par, or selling at a premium of £60. In the same manner, when joint-stock undertakings fall in estimation, the shares either remain at or fall below par; thus, if a £10 share be worth only £9, it is said to be £1 *below* par.

Government Stocks are usually called *The Funds:* they take their special designations from the rate of interest paid on them; thus, the 3 per cents. mean that stock on which an annual dividend is paid of £3 per cent.; and so on. The usual practice, in buying and selling these, is to offer shares nominally of £100 par, at from £80 to £90, or upwards, each; and the market-price of these fluctuates according to the abundance or scarcity of money, and other circumstances.

The persons who negotiate the sale and purchase of stocks are termed *Brokers,* and the charge they make for their trouble is called *brokerage,* or *commission.*

Questions as to Stocks, &c., are wrought by Simple Proportion.

The method of working the questions connected with the various kind of stocks will be best learned from the following examples:

Example 1.—I sold £700 of the 3 per cents. at 91⅝ per cent.; what sum did I receive, the brokerage being ⅛ per cent.?

$$
\begin{array}{lcccrrrl}
 & & & & \pounds & s. & d. & \\
\cancel{100} & : & \cancel{700} & :: & 91 & 12 & 6 & \\
 & & & & & & 7 & \\
\hline
 & & & & 641 & 7 & 6 & = \text{value.} \\
2s.\ 6d. \times 7 = & & & & 0 & 17 & 6 & = \text{brokerage.} \\
\hline
 & & & & \pounds 640 & 10 & 0 & = \text{net sum received.}
\end{array}
$$

In this example, £100 worth of stock is sold for £91⅝, and the question is, how much will be received for £700 worth of stock? We obviously have the proportion above. The *true* value is found to be £641, 7*s.* 6*d.*; but as the broker that was employed to effect the sale charges 2*s.* 6*d.* per cent. = 17*s.* 6*d.*, the brokerage must be deducted from this sum, and the remainder, £640, 10*s.*, will be the *net* value.

Note.—In calculating the value of stocks, the sum paid for brokerage is *added* to the value of stock when bought, but *deducted* from it when sold. Brokerage, in the following exercises, is always understood at the rate of 1-8th per cent.

Example 2.—When the 3¼ per cents. were selling at 95⅜ per cent., a gentleman invested £859, 10*s.*; what quantity of stock would he purchase, brokerage being ⅛ per cent.?

£	*s.*		£	*s.*		£		£
95	10	:	859	10	::	100	:	900

Here 95⅜ + ⅛ = 95½, the sum paid for £100 worth of stock. The statement is obvious.

Example 3.—A person invested a certain sum in the 3 per cents. when they were selling at £94⅞; what rate per cent. had he for his money?

£		£		£		£
95	:	100	::	3	:	$3\frac{3}{19}$

Here, by paying £94⅞ + ⅛ = £95, he receives a yearly sum of £3; that is, £3 is the interest of £95, and the question now is, what is the interest of £100? Hence the proportion in the margin.

The general Rule for questions of this kind is, Multiply the original value of the share by the annual dividend, and divide the product by the selling price, increased by the brokerage.

Exercises.

1. How much would a person pay for £1500 in the 3 per cents. when the selling price is 91⅛ per cent.? . . Ans. £1368, 15*s.*

2. A gentleman sold £2100 of the 3¼ per cents. at 96¼ per cent. Required the net proceeds, . . . Ans. £2018, 12*s.* 6*d.*

3. What rate of interest does a person obtain by purchasing shares in the Glasgow and Ayr Railway at £72, the annual dividend being 7 per cent., and the original value of a share 50?
Ans. $4\frac{492}{577}$ per cent.

4. What income does a gentleman get by laying out £7733, 6*s.* 8*d.* in the purchase of 3 per cent. stock at 95⅞ per cent.?
Ans. £241, 13*s.* 4*d.*

5. A person bought £2500 of 3 per cent. stock at 91½, and afterwards sold out at par. How much did he gain by the transaction? . . . . Ans. £206, 5*s.*

6. One person vested £2598, 10*s.* in the 3 per cents. at $92\frac{5}{8}$ per cent., and another £2941, 2*s.* 6*d.* in the $3\frac{1}{4}$ per cents. at $94\frac{3}{4}$ per cent. What was the difference between their incomes from these sources?—and which stock was the most profitable investment?

Ans. Difference of their incomes, £16, 15*s.*; the $3\frac{1}{4}$ per cent. most profitable.

7. If I buy £8000 stock in the 3 per cents. at $93\frac{1}{2}$, including brokerage, how much must I pay for it; and what interest per cent. will I get for my money? Ans. £7480; £3, 4*s.* $2\frac{10}{187}$*d.* Interest.

8. How much $3\frac{1}{2}$ stock at 93 can I purchase for £2400; and what income should I derive from the purchase?

Ans. £$2580\frac{20}{31}$ Stock; £$90\frac{10}{31}$ Income.

9. How much 4 per cent. stock can be purchased by the transfer of £1500 stock from the 3 per cents. at 72 to the 4 per cents. at 90? . . . . Ans. £1200.

10. I possess £4590 stock in the $3\frac{1}{2}$ per cents. When this stock is at 93, I sell out, and then buy into the 3 per cents. at 85. Do I gain or lose, and how much per annum? Ans. I lose £9, 19*s.* $9\frac{1}{2}$*d.* $\frac{2}{5}$.

11. If I sell £5000 stock from the $3\frac{1}{4}$ per cents. at 98; allowing $\frac{1}{8}$ per cent. to the broker, how much ought I to receive? And I buy with the sum realised into the 3 per cents. at $95\frac{3}{4}$, allowing the same brokerage, what income shall I receive?

Ans. £4893, 15*s.* Sum to be received; £153, 2*s.* $6\frac{3}{4}$*d.* $\frac{699}{767}$ Income.

12. I bought an estate, which let for £600 a year, at 27 years' purchase. How much stock must I sell out of the 3 per cents. at 95 to pay for this estate; and how much more interest per cent. do I get by the estate than I got by the funds?

Ans. £$17052\frac{12}{19}$ Stock; Excess of interest, £$\frac{565}{1539}$ per cent.

PARTNERSHIP.

PARTNERSHIP is the rule for ascertaining the share of profit or loss belonging to each partner of a company, in proportion to his share of the joint capital. It is either Simple or Compound.

Simple Partnership is that in which the respective partners employ their capital for *equal* periods of time.

Compound Partnership is that in which the respective partners employ their capital for *unequal* periods of time.

I. SIMPLE PARTNERSHIP.

RULE.—Add together the different shares, and state and work the question as in Simple Proportion, for each partner, thus—'As the whole capital is to each partner's capital, so is the whole gain or loss to each partner's gain or loss.'

Example.—Four partners, A, B, C, and D, invested in trade respectively £820, £480, £800, and £1120, and their total profit in a year was £560; what is each partner's share of the profit? . Ans. A gets £65, 17*s.* $7\frac{3}{4}$*d.* $\frac{1}{17}$; B, £98, 16*s.* $5\frac{1}{2}$*d.* $\frac{10}{17}$; C, £164, 14*s.* $1\frac{1}{4}$*d.* $\frac{11}{17}$; D, £230, 11*s.* $9\frac{0}{4}$*d.* $\frac{12}{17}$.

A's share	.	= £320
B's 〃	.	= 480
C's 〃	.	= 800
D's 〃	.	= 1120
Total capital		= 2720

Here we add together the capital of all the partners, and then, to find A's share of profit, state the question thus: 'As the whole capital, £2720, is to A's capital, £320, so is the whole gain, £560, to A's share of the gain.' It is then calculated as in Simple Proportion. B and C's shares are found in the same way.

					£		£	*s.*	*d.*
2720	:	820	::	560	:	65	17	$7\frac{3}{4}\,\frac{1}{17}$
2720	:	480	::	560	:	98	16	$5\frac{1}{2}\,\frac{10}{17}$
2720	:	800	::	560	:	164	14	$1\frac{1}{4}\,\frac{11}{17}$
2720	:	1120	::	560	:	230	11	$9\frac{0}{4}\,\frac{12}{17}$
				Total profit,		£560	0	0

The work may be considerably shortened by dividing all the shares, 320, 480, 800, and 1120 by 160, and using the quotients, 2, 3, 5, and 7, in their stead. The pupil is recommended to attend to this. When the numbers in proportion to which the gain or loss is to be divided are small, the whole gain or loss may first be divided by their sum; then the quotient, multiplied by the several numbers, will give the partners' respective shares. Thus, using 2, 3, 5, and 7, their sum is 17; and £$\frac{560}{17}$ = £32, 18*s.* $9\frac{3}{4}$*d.* $\frac{9}{17}$: this, multiplied by 2, gives A's share; by 3, gives B's share, &c.

Proof.—Add all the shares of the gain or loss together, and the sum will be equal to the whole gain or loss when the calculations are correct.

Reason of the Rule.—It is obvious that each partner's share of the gain will be the same part of the whole gain, £560, that his share of the stock is of the whole stock, £2720, and the question is therefore an ordinary case of Simple Proportion.

$$\frac{\text{A's share}}{560} = \frac{320}{2720} = \frac{2}{17};\ \therefore \text{A's share} = \overset{\pounds}{560} \times \frac{2}{17} = \overset{\pounds}{\frac{560}{17}} \times 2$$

$$\frac{\text{B's share}}{560} = \frac{480}{2720} = \frac{3}{17};\quad \text{B's} \quad〃 = 560 \times \frac{3}{17} = \frac{560}{17} \times 3$$

$$\frac{\text{C's share}}{560} = \frac{800}{2720} = \frac{5}{17};\quad \text{C's} \quad〃 = 560 \times \frac{5}{17} = \frac{560}{17} \times 5$$

$$\frac{\text{D's share}}{560} = \frac{1120}{2720} = \frac{7}{17};\quad \text{D's} \quad〃 = 560 \times \frac{7}{17} = \frac{560}{17} \times 7$$

	£			£	*s.*	*d.*			£	*s.*	*d.*
Now	$\frac{560}{17}$	× 2	=	32	18	$9\frac{3}{4}\,\frac{9}{17}$	× 2	=	65	17	$7\frac{3}{4}\,\frac{1}{17}$
	$\frac{560}{17}$	× 3	=	32	18	$9\frac{3}{4}\,\frac{9}{17}$	× 3	=	98	16	$5\frac{1}{2}\,\frac{10}{17}$
	$\frac{560}{17}$	× 5	=	32	18	$9\frac{3}{4}\,\frac{9}{17}$	× 5	=	164	14	$1\frac{1}{4}\,\frac{11}{17}$
	$\frac{560}{17}$	× 7	=	32	18	$9\frac{3}{4}\,\frac{9}{17}$	× 7	=	230	11	$9\frac{0}{4}\,\frac{12}{17}$
								Proof,	£560	0	0

Instead of valuing the fraction £$\frac{560}{17}$ as above, we may convert it into a decimal (page 136) of five or six places; then this decimal may be multiplied by 2. The product, valued by the rule at page 143, will give A's share nearly, &c. The above operations, when conducted in this way, will stand thus—

£		£	s.	d.
$\frac{560}{17} \times 2 = 32{\cdot}941176 \times 2 =$	$65{\cdot}882 =$	65	17	$7\frac{3}{4}$
$\frac{560}{17} \times 3 = 32{\cdot}941176 \times 3 =$	$98{\cdot}823 =$	98	16	$5\frac{1}{4}$
$\frac{560}{17} \times 5 = 32{\cdot}941176 \times 5 =$	$164{\cdot}706 =$	164	14	$1\frac{1}{4}$
$\frac{560}{17} \times 7 = 32{\cdot}941176 \times 7 =$	$230{\cdot}588 =$	230	11	$9\frac{1}{4}$

Note.—By this rule a bankrupt's estate may be divided among his creditors. It may also be applied to the division of a number into parts, having given ratios to each other.

Exercises.

1. Three merchants, A, B, and C, form a joint capital, of which A contributes £700; B, £350; and C, £1000. At the end of a year, their gain is found to be £500. What is each partner's share of the profit? Ans. A's share, £170, 14*s.* $7\frac{1}{2}$*d.* $\frac{18}{41}$; B's, £85, 7*s.* $3\frac{3}{4}$*d.* $\frac{9}{41}$; C's, £243, 18*s.* $0\frac{1}{2}$*d.* $\frac{14}{41}$.

2. A, B, C, and D purchase a ship: A pays for 6 shares, B for 5, C for 3, and D for 4. They receive of net freight for a voyage to Jamaica £364, 17*s.* 6*d.* How much of this sum ought each to receive? Ans. A, £121, 12*s.* 6*d.*; B, £101, 7*s.* 1*d.*; C, £60, 16*s.* 3*d.*; D, £81, 1*s.* 8*d.*

3. A ship costs £3000: one underwriter insures on it £450; a second, £700; a third, £350; a fourth, £900; and the owner risks the remainder. During a storm, damages to the value of £375, 14*s.* 10*d.* were sustained. How much of this sum must each of the underwriters pay?—and how much will the owner lose? Ans. First pays £56, 7*s.* $2\frac{1}{2}$*d.* $\frac{4}{5}$; second, £87, 13*s.* $5\frac{1}{2}$*d.* $\frac{2}{15}$; third, £43, 16*s.* $8\frac{3}{4}$*d.* $\frac{1}{15}$; fourth, £112, 14*s.* $5\frac{1}{4}$*d.* $\frac{3}{5}$: owner loses £75, 2*s.* $11\frac{1}{2}$*d.* $\frac{2}{5}$.

4. A bankrupt owes to A, £115, 16*s.*; to B, £105; to C, £93, 11*s.* 5*d.*; to D, £71, 5*s.* 6*d.*; to E, £36, 14*s.* 7*d.*; to F, £15, 15*s.*; and to G, 11*s.* 6*d.* His money and the value of his goods amount to £329, 0*s.* 6*d.* What portion of this must each of the creditors receive? Ans. A receives £86, 17*s.*; B, £78, 15*s.*; C, £70, 3*s.* $6\frac{3}{4}$*d.*; D, £53, 9*s.* $1\frac{1}{2}$*d.*; E, £27, 10*s.* $11\frac{1}{4}$*d.*; F, £11, 16*s.* 3*d.*; G, 8*s.* $7\frac{1}{2}$*d.*

5. A piece of common, consisting of 1000 acres, was divided among four landlords, A, B, C, and D, in proportion to the rents of their several estates. The rent of A's estate was £2300; of B's, £1500; of C's, £1200; of D's, £970. What portion of the common was allotted to each? Ans. A gets 385 ac. 1 ro. $1\frac{323}{597}$ po.; B, 251 ac. 1 ro. $1\frac{1}{199}$ po.; C, 201 ac. 0 ro. $0\frac{160}{199}$ po.; D, 162 ac. 1 ro. $36\frac{388}{597}$ po.

6. Three merchants, L, M, and N, continue in trade for a year with a joint-stock of £3500. At the end of that time, L's share of the gain was £125; M's, £240; and N's, £135. What was each partner's stock? Ans. L's stock, £875; M's, £1680; N's, £945.

7. A, B, and C enter into partnership for a year with a joint-stock of £8900: A contributes £4000; B, £2140; and C, the remainder. At the end of the year their gain is found to be £1483, 6*s.* 8*d.* C managed the business, and was to have a salary of £445 for his trouble. What portion of the gain belongs to each partner? . Ans. A, £466, 13*s.* 4*d.*; B, £249, 13*s.* 4*d.*; C, £767.

8. Pure water is composed of two gases, oxygen and hydrogen, whose weights are to each other as 8 to 1. What weight of each is there in a gallon of water which weighs 10 lb.?

Ans. Oxygen, $8\frac{8}{9}$ lb.; hydrogen, $1\frac{1}{9}$ lb.

9. The amount of the legacies bequeathed in a will was £9360; but at the testator's death his effects were found to be only worth £8190. How much ought each of the legatees to receive per pound?—and how much would A receive, whose legacy was to have been £875? Ans. 17*s.* 6*d.* per pound: A received £765, 12*s.* 6*d.*

II. Compound Partnership.

Rule.—Multiply each share by the time that it has been employed in the business, and add together the products: then state and work the question for each partner, as in Simple Partnership; only use the *products* of the sums, instead of the sums themselves.

Example.—Three partners, A, B, C, invested the following sums in business: A, £250 for 6 months; B, £480 for 7 months; and C, £1000 for a whole year: the gain was £340. What was each partner's share of the profit?

	£	Months.		
A	250 ×	6	=	1500
B	480 ×	7	=	3360
C	1000 ×	12	=	12000
	Total products,			16860

			£	£	s.	d.
16860 :	1500 ::	340 :		30	4	$11\frac{3}{4}\frac{41}{281}$
16860 :	3360 ::	340 :		67	15	$1\frac{3}{4}\frac{183}{281}$
16860 :	12000 ::	340 :		241	19	$10\frac{1}{4}\frac{47}{281}$
	Total profit,			£340	0	0

Here each share is multiplied by the time it is employed, and the products are then added together. In order to find A's share, we say: 'As £16,860, the total products, is to £1500, the product of A's share, so is the total gain, £340, to A's share of the gain.' It is then calculated as in Simple Partnership. B and C's shares are found in the same way.

It may be remarked that C's share may be obtained by multiplying A's by 8, since the ratio of their shares is 1500 : 12000, which is equal to 1 : 8.

Reason of the Rule.—On referring to the Example, it will be manifest that A's stock of £250 for 6 months would just be the same as if he had employed a stock equal to 6 times £250 for 1 month; that B's of £480 for 7 months would be equal to a stock of 7 times £480 for 1 month; in like manner, C's of £1000 for 12 months the same as one 12 times £1000 for 1 month. Now, since the times are *equal*, it is obvious that their shares of the gain will be proportional to the products 250×6, 780×7, and 1000×12; and hence the application of Simple Partnership will determine their several portions.

Exercises.

10. A's stock of £340 was 4 months in trade; B's, of £510, was eight months; and C's, of £850, was 10 months; they gain £270, 13*s.* 6*d.*: what was each partner's share of the gain? Ans. A's, £26, 8*s.* $1\frac{3}{4}d.$ $\frac{1}{41}$; B's, £79, 4*s.* $5\frac{1}{4}d.$ $\frac{3}{41}$; C's, £165, 0*s.* $10\frac{3}{4}d.$ $\frac{37}{41}$.

11. A, B, and C had a joint-stock of £2400, of which A's part was £760, and continued in the trade for 5 months; B's was £870, and continued 8 months: the remainder was C's, and continued in the trade throughout a year. They lost £520, 18*s.* 10*d.* Required each partner's share of the loss. Ans. A's, £98, 19*s.* $6\frac{3}{4}d.$ $\frac{12}{25}$; B's, £181, 5*s.* $9\frac{2}{4}d.$ $\frac{24}{125}$; C's, £240, 13*s.* $6\frac{2}{4}d.$ $\frac{6}{125}$.

12. Two merchants, A and B, entered into partnership for 2 years: A contributed to the capital £960, and B, £1500. After 8 months of the time had elapsed, they admit C, with a capital of £720. On balancing their books at the end of the period, they found that their net gain amounted to £847, 15*s.* How must this gain be divided among them? Ans. A's, £276, 16*s.* $3\frac{3}{4}d.$ $\frac{33}{49}$; B's, £432, 10*s.* $6\frac{2}{4}d.$ $\frac{24}{49}$; C's, £138, 8*s.* $1\frac{3}{4}d.$ $\frac{41}{49}$.

13. Four graziers, A, B, C, D, rent a grass-field of 30 acres at £9, 10*s.* per acre; A puts in 40 oxen for 2 months; B, 30 oxen for 1 month; C, 35 oxen for 4 months; and D, 20 oxen for the rest of the year: what portion of the rent ought each to pay? Ans. A, £65, 2*s.* $10\frac{1}{4}d.$ $\frac{1}{7}$; B, £24, 8*s.* $6\frac{3}{4}d.$ $\frac{3}{7}$; C, £114; D, £81, 8*s.* $6\frac{3}{4}d.$ $\frac{3}{7}$.

14. A, B, and C enter into partnership for 2 years; A put in at first, £700, and after 8 months, £250 more; B put in £650, and after 15 months, he took out £300; C put in £850, and after 10 months, £400 more, but at the end of 18 months he withdrew £900. During their copartnership the gains amounted to £1684, 12*s.*; what was each man's share? Ans. A, £645, 5*s.* $11\frac{1}{2}d.$ $\frac{10}{181}$; B, £400, 4*s.* $2\frac{2}{4}d.$ $\frac{88}{181}$; C, £639, 1*s.* $10\frac{1}{4}d.$ $\frac{83}{181}$.

EQUATION OF PAYMENTS.

EQUATION is the rule for ascertaining the time at which two or more debts, due at as many different times by one person to another, may be paid at once, without loss to either party.

RULE—1. Write the different sums or debts below one another, then multiply each debt by the time that has to elapse before it is due, and place the products opposite the respective debts.

2. Add the debts in one sum for a divisor, and their products in another sum for a dividend: then divide the one by the other—that is, the sum of the products by the sum of the debts—and the quotient is the equated or average time required for paying the whole at once.

Example.—A gentleman owes £60, payable in 72 days; £85, in 128 days; £70, in 176 days; and £105, in 320 days. Required the average time at which the whole ought to be paid.

£		Days.		Products.
60	×	72	=	4320
85	×	128	=	10880
70	×	176	=	12320
105	×	320	=	33600
320				320)61120(191 days. *Ans.*

Here we multiply £60 by 72, the number of days before it is due; £85 by 128; £70 by 176; and £105 by 320. We then divide 61120, the sum of the products, by 320, the sum of the debts; and the quotient, 191, is the average number of days.

REASON OF THE RULE.—That the rule is *correct* will appear obvious by referring to the example. Thus, the interest of £60 for 72 days is equal to the interest of £1 for 4320 days; the interest of £85 for 128 days is equal to that of £1 for 10880 days, &c.: therefore, the sum of the interests of £60 for 72 days, of £85 for 128 days, of £70 for 176 days, and of £105 for 320 days, is equal to the interest of £1 for 61120 days. Now, £320, the amount of the debt, will produce as much interest in the $\frac{1}{320}$ part of this time; that is, in 61120 days $\div$ 320 = 191 days, the true equated or average time at *Simple Interest.* Hence the rule.

Note.—This rule is generally considered to be founded on erroneous principles, but it is as exact as any rule founded on the supposition of *Simple Interest* can be. The discussion of this point would require more room than the importance of the subject demands. The inquisitive student is referred to vol. i. p. 286 of the *Cambridge and Dublin Mathematical Journal*, for a sensible article on this rule.

Exercises.

1. A gentleman owes £56 in 40 days, £72 in 108 days, £106 in 175 days, £230 in 241 days, and £960 in 342 days. Required the average time at which the whole ought to be paid.

Ans. $289\frac{195}{356}$ days.

2. If a person owe £100 payable in 2 months, and £750 payable in 7 months; what is the just time for the payment of the two debts? . . . . Ans. $6\frac{7}{17}$ months.

3. What is the equated time for the payment of four debts—the first for £250, due in one year; the second of £560, payable in $1\frac{1}{4}$ years; the third for £490, due in two years; and the fourth for £1000, due in $3\frac{1}{2}$ years? . . Ans. $2\frac{83}{230}$ years.

4. A person has to pay £1750 as follows: £300 in 4 months, £125 in 5 months, £365 in 8 months, £400 in 10 months, and the rest in a year. What is the equated time for the payment of the whole? . . . . Ans. $8\frac{293}{350}$ months.

5. A gentleman has to pay to his banker £350 on February 2, £675 on May 15, £482 on August 1, and £1000 on November 11. What is the equated time at which the whole may be paid?

Ans. $174\frac{2392}{2507}$ days after February 2; that is, July 26.

6. I sold to Alexander Rutherford, Cork, goods payable as follows: hides to the value of £780, payable on Feb. 1; leather to the value of £610, payable on the 25th March; tallow to the value of £320, payable on May 15; find the mean time for the payment of the whole debt.

Ans. $37\frac{4}{5}$ days after Feb. 1; that is, on March 10.

7. One-fourth of a debt is due at Lady-day, one fourth at Midsummer, one-fourth at Michaelmas, and the balance at Christmas; at what time ought the whole to be discharged in one payment?

Ans. $138\frac{1}{2}$ days after March 25; that is, Aug. 10.

AVERAGE.

An average number is one that is intermediate between several other given numbers. Thus, if there are 4 numbers, 5, 6, 9, 8, their average is 7, because 4 numbers, each of which is 7, will amount to the same sum, namely, 28, as the four given numbers.

I. To Find the Average of several given Quantities.

Rule.—Add together the different quantities, and divide their amount by the *number* of the quantities; thus, if there are 3 different quantities, divide their amount by 3; and the quotient is the average.

Example.—What is the average of 8, 36, 14, 9, 43?

	8
	36
	14
	9
	43
5)	110
Average,	22

Here the different quantities are added together, and their sum is divided by 5, because there are 5 different quantities.

II. To find the Average price of Goods, &c., when there are different quantities and different prices.

Rule.—Multiply each quantity by its price; then add the quantities in one sum and the products in another, and divide the sum of the products by the sum of the quantities: the quotient is the average.

Example.—I have bought 2 yards of cloth, at 10*s.* each; 3 yards at 15*s.*; and 5 at 12*s.*; what is the average price?

2 × 10*s.* =	£1	0	0
3 × 15*s.* =	2	5	0
5 × 12*s.* =	3	0	0
10 10)	6	5	0
Average,		12	6

Here each quantity is multiplied by its price; and the sum of the products, £6, 5*s.*, is divided by 10, the sum of the quantities.

This rule applies to any other averages in which the quantities and the rates both vary.

Exercises.

1. The revenue of a public trust, during three years, was £44,261, 0*s.* 4*d.*, £47,471, 14*s.* 5*d.*, and £38,006, 5*s.* 11*d.*; what was the average yearly revenue? . Ans. £43,246, 6*s.* 10½*d.* ⅔.

2. The temperature, as indicated by the thermometer on the 1st day of November, was 49·10; on the 2d, it was 40·06; on the 3d, it was 39·00; on the 4th, it was 27·20; on the 5th, 28·; and on the 6th, 21·50; what was the average temperature of the six days? Ans. 34·14.

3. In a class of 6 boys at school, one was aged 14 years and 3 months; the second, 13 years 11 months; the third, 13 years 1 month; the fourth, 12 years; the fifth, 12 years 6 months; and the sixth, 11 years 9 months; what was the average age of the six boys? . . . . . Ans. 12 years 11 months.

4. The price of the 4 lb. loaf was in one shop 8*d.*; in a second shop, 7½*d.*; in a third, 7¼*d.*; and in a fourth, 6¾*d.*; what was the average price of the loaf in the four shops? . Ans. 7¼*d.* ½.

5. If a person purchase 12 articles, one of which costs 2*s.*; two, 4*s.* each; two, 5*s.* each; and seven, 8*s.* each; what did he pay on an average for each article? . . . . Ans. 6*s.* 4*d.*

DIVIDEND.

A **Dividend** is a division among the creditors, of the funds of a debtor, who finds himself unable to pay the debts which he has contracted.

On examining his *assets*—that is, the whole of his property and means—he discovers that he could settle with his creditors, provided each would accept a *dividend* on the amount of his account: this dividend is generally spoken of as at the rate of so much per pound. Supposing the debtor to be owing £1000, and only to possess assets to the value of £250, then he can pay only 5*s.* per pound, or 25 per cent. on his debts; if the creditors are satisfied, the debtor is relieved on making payment to this extent.

The Dividend is ascertained by dividing the total amount of the assets by the *number* of pounds that form the amount of the debts. As the assets are less than the debts, their amount requires to be converted into shillings or pence, as the case may be, to admit of the division.

Dividend is also the term applied to the profits, at a certain percentage on the amount of the shares, divided among the proprietors of joint-stock companies, &c.

Example.—A person is unable to pay his debts. He owes to A, £440; to B, £160; to C, £224—being in all £824. On examining his affairs, it is found that he possesses property only to the value of £226, 12*s.* What dividend per pound can he pay?

£	£ *s.*	
824)	226 12(	*Ans.* 5*s.* 6*d.* per £1.

Here the assets are divided by 824, the number of pounds forming the debts.

Exercises.

1. What dividend per pound will a person pay who is owing £1000, and whose assets amount to £800? . Ans. 16*s.* per £.

2. A person is owing to A, £300; B, £400, 10*s.*; C, £620, 15*s.*; D, £150, 15*s.*; and his whole assets amount to £184, what dividend will he be able to pay? . . . Ans. 2*s.* 6*d.* per £.

BARTER.

Barter is the exchanging of one kind of goods for another, in such a way that the value of the goods given away, may be equal to the value of those received.

No general rule can be given for the working of such questions; they must be treated according to the nature of each case. The following will serve as examples:

Example 1.—A and B barter as follows: A has 1385 yards of linen, at 2*s*. 7½*d*. per yard, for which B gives him £32, 7*s*. 6*d*. ready-money, and for the rest printed calicoes at 10½*d*. per yard. How many yards of calico did A receive?

A gives 1385 yards of linen, at 2*s*. 7½*d*.	=				£181	15	7½
B gives in money, . . .		£32	7	6			
〃 calicoes at 10½*d*. a yard, .		149	8	1½	181	15	7½

This sum of £149, 8*s*. 1½*d*. divided by 10½*d*., the price per yard, will give 3415 yards, the number required.

Example 2.—A merchant in Hull exported to his correspondent in Oporto 35 pieces of camlet, each 28 yards, at 5*s*. 2½*d*. per yard, and 80 pieces of serge, each 35 yards, at 3*s*. 3¼*d*. per yard. The correspondent was allowed 2½ per cent. commission on the gross amount, and was directed to expend the ⅜ of the net proceeds on port wine at £48, 16*s*. 3*d*. per pipe, and the remainder on raisins at £2, 5*s*. 7½*d*. per cwt. What quantities of each were imported?

					£	*s*.	*d*.
35 × 28 = 980 yd. at 5/2½ per yd.	=				255	4	2
80 × 35 = 2800 〃 3/3¼ 〃	=				457	18	4
					713	2	6
Commission at 2½ per cent.	=				17	16	6¾
Net proceeds, ...					695	5	11¼
⅜ of proceeds = value of the wine,		£260	14	8½ ⅞			
Remainder = value of the raisins,		434	11	2½ ⅛	695	5	11¼

Exercises.

1. How much coffee at £7, 9*s*. 6¾*d*. per cwt. should I get in exchange for 897 cwt. 42 lbs. of sugar at 6¾*d*. per lb.? Ans. 378 cwt.

2. How much tobacco at £5, 5*s*. per cwt. must be bartered for 6 cwt. 1 qr. 14 lb. of snuff at 4*s*. 6*d*. per lb.? Ans. 30 cwt. 2 qr. 11⅕ lb.

3. I exchanged 172 yards of black cloth at £1, 2*s*. 8*d*. per yard, for 688 pair of silk stockings; what was the price of the stockings per pair? . . . . Ans. 5*s*. 8*d*.

4. A delivers to B 31¼ yards of cloth at 5*s*. 6*d*., and 78 yards of cassimer at 7*s*. 8*d*., in barter for wool at 1*s*. 3*d*. per lb.; what quantity of wool does A receive? . . . Ans. 615$\frac{8}{10}$ lb.

Example.—Change the numbers $(234)_5$, $(5324)_7$, and $(1843)_{12}$, to the decimal scale.

$$
\begin{array}{r}
234 \\
5 \\
\hline
13 \\
5 \\
\hline
69 \ \textit{Ans.}
\end{array}
\qquad\qquad
\begin{array}{r}
5324 \\
7 \\
\hline
38 \\
7 \\
\hline
268 \\
7 \\
\hline
1880 \ \textit{Ans.}
\end{array}
\qquad\qquad
\begin{array}{r}
1843 \\
12 \\
\hline
20 \\
12 \\
\hline
244 \\
12 \\
\hline
2931 \ \textit{Ans.}
\end{array}
$$

Exercises.

3. Change the numbers $(31476)_9$, $(8196)_{12}$, $(2503)_6$, to the decimal scale, . . . Ans. $(20805)_{10}$, $(14082)_{10}$, $(615)_{10}$.

4. How are the numbers $(111022)_5$, $(607)_8$, represented in the decimal scale? . . . . . Ans. $(3887)_{10}$, $(391)_{10}$.

From what has been stated, it is obvious that, in performing Addition or Multiplication in any scale, the number resulting in any column must be divided by the base of the scale, the remainder set down, and the quotient carried to the next column.

N.B.—In Subtraction, when 1 is borrowed, it expresses as many units as there are units in the base in the place immediately to the right; and in Division, a remainder must be multiplied by the base, and the next figure taken in before the next division is performed. By attending to the preceding remarks, no difficulty will be found in performing the following exercises:

Exercises.

5. Change $(372)_{10}$, $(5834)_{10}$, $(7936)_{10}$, to the septenary scale, and find their sum in that scale, . . . Ans. $(56142)_7$.

6. Change $(39572)_{10}$, and $(16935)_{10}$, to the senary scale, and find their difference in that scale, . . . Ans. $(252445)_6$.

7. Change $(18394)_{10}$, and $(375)_{10}$, to the nonary scale, and find their product in that scale, . . . Ans. $(13872836)_9$.

8. Change $(314159)_{10}$, and $(712)_{10}$, to the quinary scale, divide the greater by the less, and find the quotient in that scale, ... Ans. $(3231\frac{1132}{10322})_5$.

9. Change $(76428)_{10}$, and $(374)_9$, to the duodecimal scale, and find their product in that scale, . . . Ans. $(7b27060)_{12}$.

10. Change $(47683)_{10}$, and $(2130)_4$, to the senary scale, and divide the former by the latter, . . . Ans. $(1225\frac{251}{420})_6$.

11. Change $(54230)_6$, and $(7213)_8$, to the quinary scale, and find their sum and difference,

Ans. Sum, $(324112)_5$; difference, $(104321)_5$.

INVOLUTION.

INVOLUTION is the term applied to the multiplication of a number one or more times by itself; thus, $2 \times 2 = 4$. The result is called a POWER of that number.

The *first* power of a number is the number itself before being multiplied; thus—	2
The *second* power, termed the SQUARE, is the number multiplied by itself; thus— . .	$2 \times 2 = 4$
The *third* power, termed the CUBE, is the number multiplied by itself, and the product again multiplied by it; thus—	$2 \times 2 \times 2 = 8$

First Power, .	1	2	3	4	5	6	7	8	9
Second " .	1	4	9	16	25	36	49	64	81
Third " .	1	8	27	64	125	216	343	512	729

And so on, with the higher powers.

The *Power* of a number is indicated by writing the number with a small figure above it, called the Index; thus, 2^2 means the second power of 2—namely, $2 \times 2 = 4$: 2^3 means the third power, $2 \times 2 \times 2 = 8$: 2^4 means the fourth power; and so on.

THE PRODUCT OF TWO POWERS of the same number is equal to that number raised to the power denoted by the sum of their indices.

Take any two powers of the same number, as 5^4 and 5^3; then $5^4 \times 5^3 = 5^{4+3} = 5^7$. For $5^4 \times 5^3 = 5.5.5.5 \times 5.5.5$ $= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$.

THE QUOTIENT OF ANY POWER of a number divided by a lower power of the same number, is equal to the number raised to the power denoted by the difference of their indices.

Take any two powers of 5, as 5^7 and 5^4, of which 5^4 is the lower; then $\frac{5^7}{5^4} = 5^{7-4} = 5^3$. For $\frac{5^7}{5^4} = \frac{5^{4+3}}{5^4} = \frac{5^4 \times 5^3}{5^4} = 5^3$ or 5^{7-4}.

ANY POWER OF A COMPOSITE NUMBER is equal to the product of the same powers of its factors.

Take any power of the composite number 35, as 35^3; then $35^3 = 7^3 \times 5^3$. For $35^3 = 35 \times 35 \times 35 = 7.5 \times 7.5 \times 7.5 = 7 \times 7 \times 7 \times 5 \times 5 \times 5 = 7^3 \times 5^3$.

Any power of a power of a number is equal to the number raised to that power denoted by the product of the index of the power of the given number, by the index of that power to which the given power is to be raised.

Take any power of any power of the number 5, as $(5^3)^2$; then $(5^3)^2 = 5^{3\times 2} = 5^6$. For $(5^3)^2 = 5^3 \times 5^3 = 5.5.5 \times 5.5.5 = 5^6 = 5^{3\times 2}$.

To raise a given number to any power.

Find the continued product of the given number, repeated as often as the index points out.

The process may sometimes be shortened by multiplying together powers already found.

A Vulgar Fraction is raised to any power by raising its terms to the given power; thus, the fourth power of $\frac{3}{4}$ is $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$.

Example 1.—What is the fifth power of 57?

Here, by multiplying 57 by 57, we get 3249 for the second power; and 3249, multiplied by 57, gives 185193 for the third power. Now the third power, 185193, multiplied by the second power, 3249, gives 601692057 for the fifth power.

Example 2.—What is the sixth power of $\frac{7}{12}$?

Here the sixth power of 7 is 117649, and the sixth power of 12 is 2985984; therefore, $\frac{117649}{2985984}$ is the answer required.

Example 3.—Required the fourth power of 1·035, true to five places of decimals.

```
1·071225 = 2d power.
522170·1 = 2d  "   inverted.
 1071225
   74986
    1071
     214
      21
       5
1·147522 = 4th power.
```

The second power of 1·035 is found to be 1·071225; and this, multiplied by itself, as in the margin, according to the contracted method mentioned at p. 149, gives the answer required.

Exercises.

1. Find the 5th power of 97, . . Ans. 8587340257.
2. " 10th " 18, . . . " 3570467226624.
3. " 7th " 29, . . . " 17249876309.
4. " 8th " 13, . . . " 815730721.
5. " 4th " $\frac{2}{3}$, . . . " $\frac{16}{81}$.
6. " 3d " $\frac{9}{34}$, . . . " $\frac{729}{39304}$.
7. " 6th " $\frac{3}{4}$, . . . " $\frac{729}{4096}$.
8. " 20th " 1·04 true to 6 places of decimals, Ans. 2·191123.

9. What is the value of $1{\cdot}05^{13}$ true to 6 places of decimals?
Ans. 1·885649.

10. 〃 〃 $1{\cdot}025^{30}$ 〃 7 places of decimals?
Ans. 2·0975676.

11. 〃 〃 $1{\cdot}06^{25}$ 〃 5 places of decimals?
Ans. 4·29187.

12. 〃 〃 $1{\cdot}035^{20}$ 〃 7 places of decimals?
Ans. 1·9897889.

13. If the side of a square table measures 62 inches, how many square inch. are in the surface of the table? Ans. 3844 sq. inches.

14. There is a square court, whose side measures 102 feet; how many square bricks will be required to pave it, supposing the side of a brick to measure 9 inches? . Ans. 18496 bricks.

15. The side of a cubic block of granite measures 6 feet; how many solid feet in the block?—and what is its weight, supposing a cubic foot of granite to weigh 2654 oz.?
Ans. Solidity, 216 feet; weight, 319 cwt. 3 qr. 17 lb.

16. There is a cistern in the form of a cube, and its side measures 3 feet 8 inches; how many imperial gallons will the cistern hold? . . . . Ans. $307\frac{30441}{138637}$.

EVOLUTION, OR THE EXTRACTION OF ROOTS.

EVOLUTION is the process of finding or extracting the roots of numbers.

The *Root* of any number is that number which, on being multiplied one or more times by itself, produces the given number.

The *Square* root is that number which, on being multiplied by itself, produces the given number; thus, 4 is the square root of 16, because 4 multiplied by 4 (4×4) produces 16.

The *Cube* root is that number which, on being multiplied by itself, and the product again multiplied by it, produces the given number; thus, 3 is the cube root of 27, because 3 multiplied by 3, and the product again by 3 ($3 \times 3 \times 3$) produces 27.

The sign $\sqrt{}$ (termed the radical sign) placed before a number, indicates that the *square* root of that number is to be extracted; thus $\sqrt{25} = 5$.

The sign $\sqrt[3]{}$ placed before a number indicates that the *cube* root of that number is to be extracted; thus $\sqrt[3]{729} = 9$.

The extraction of the square and cube roots serves various useful purposes in connection with the measurement of fields, walls, solid bodies, &c., as will appear from the exercises under the rules for extracting the square and cube roots of numbers.

Note.—The square and cube roots of some numbers can be found exactly—of others, only approximately.

EXTRACTION OF THE SQUARE ROOT.

RULE—1. Point off the given number, by means of commas, into periods of two figures each; beginning at the right in integers, and counting towards the *left;* and beginning at the left in decimals, and counting towards the *right:* thus, 6,98,01,64·37,65,4.

If, in pointing off the periods, one figure remain over at the left in integers, it is considered as a period; but if one remain at the right in decimals, a nothing is annexed to complete the period.

2. Find the greatest square *root* contained in the first period at the left, and place it as the first figure of the root; then subtract the *square* of this figure from the first period, and annex to the remainder, if any, the next period of two figures, to form a new sum or dividend.

Example 1.—What is the square root of 6980164?

```
      6,98,01,64 (2642 Ans.
      4
 46   298
  6   276
 524  2201
   4  2096
 5282 10564
      10564
```

Here, the greatest square root contained in the first period, 6, is 2, which is placed as the first figure of the root. The square of 2—namely, 4—is then subtracted from the first period, and to the remainder, 2, is annexed 98, the next period, forming the dividend, 298.

3. Write down as a *partial* divisor the *double* of the root already got; and find how often it is contained in the dividend, exclusive of its last figure; place the quotient* as the next figure of the root; and also annex it to the *partial* divisor, to form a *complete* divisor.

The *double* of the root, 2—namely, 4—is then written as a partial divisor, and 4 being contained 7 times in 29—the dividend 298 with its last figure left out, the quotient 7 is apparently the next figure of the root; but as, on proceeding further with the process, it is found to be too many, a less quotient, 6, is placed as the next figure of the root, and is also annexed to the partial divisor, 4, to form the complete divisor, 46.

4. Multiply the completed divisor by this same quotient figure, and subtract the product from the dividend; then annex to any remainder, the next period of two figures, to form another dividend.

Here, the divisor, 46, is multiplied by the quotient, 6, and the product, 276, is subtracted from the dividend, 298: to the remainder, 22, is then annexed the next period, 01, to form another dividend.

* Sometimes a less figure requires to be taken, as in the above example. See Note 2, page 213. See also Note 3, for another variety of the rule.

5. Form a new partial divisor by adding to the previous divisor its last figure; then find the next figure of the root, and complete the divisor, as in paragraph 3; and form another dividend, as in paragraph 4.

Here, to the previous divisor, 46, is added 6, its last figure, making 52 for a new partial divisor; and 52 being contained 4 times in 220—the dividend, 2201, with its last figure left out—the 4 is placed as the next figure of the root, and is also annexed to the partial divisor, 52, to form the complete divisor, 524.

6. Proceed in the same way till all the periods have been brought down and operated upon; when, if there is no remainder, the extraction of the root is completed.

The divisor, 524, is then multiplied by the quotient, 4, and the product, 2096, is subtracted from the dividend, 2201; to the remainder, 105, is annexed the next period, 64, making 10564; and the extracting of the next figure of the root, 2, is then proceeded with as before. There being now no more periods to bring down, and no remainder, the process is completed.

NOTE 1.—When there is a remainder after all the periods have been brought down, annex a period of two nothings to form a new dividend; and then proceed with the further extraction of the root: the figure of the root thus obtained is a decimal. The process may be carried to any degree of minuteness by annexing more nothings.

There are always as many figures in the root as there are periods in the given number; and those figures are decimals in the root, which are extracted from the decimals in the given number.

NOTE 2.—It sometimes happens that the completed divisor obtained in the manner described in the previous page, paragraph 3, proves to be too large, as on multiplying it, according to paragraph 4, it is found to be greater than the dividend: when this is the case, the quotient placed in the root and annexed to the divisor must be reduced as much as is found necessary.

NOTE 3.—If at any time, on bringing down a new period to form a dividend, the *partial* divisor is found to be greater than the dividend, a nothing [0] must be placed in the root, and also annexed to the divisor, to express this: the next period is then brought down, and annexed to the dividend, and the extraction of a new figure of the root proceeded with as before.

NOTE 4.—The square root of a fraction is found by extracting the roots of the numerator and denominator.

Example 2.—What is the square root of 67081?

```
       6,70,81(259
       4
 45    270
  5    225
509     4581
        4581
```

Pupil. I first divide the number 67081 into periods containing two figures, by placing a comma between 0 and 8, and another between 6 and 7. Now, the greatest square root contained in 6 is 2, which I place as the first figure of the root. Subtracting 4, the square of 2, from 6, the remainder is 2; and bringing down the next period, 70, I obtain 270 as a partial dividend. Doubling 2, I get 4 for a partial divisor: fours in 27, 6 times; but as 6 times is too great, I try 5 as the second figure of the root; and annexing 5 to 4, I get 45 for the complete divisor. I now multiply and subtract, and to the remainder annex 81, the next period, and get 4581 for a new partial dividend. Adding 5 to 45, I get 50 for a new partial divisor: fives in 45, 9 times. I place 9 as the third figure of the root, and annexing it also to 50, I get 509 as the complete divisor; and multiplying by 9, I find that there is no remainder. Hence the exact root is 259.

Example 3.—What is the square root of 372·69357 to six places of decimals?

```
          3,72·69,35,70(19·305273              3,72·69,35,70(19·305273
          1                                    1
29        272                       29         272
 9        261                        9         261
383        1169                     383         1169
  3        1149                       3         1149
38605        203570                 38605         203570
    5        193025                     5         193025
386102        1054500               38,6,1,0       10545
     2         772204                               7722
3861047        28229600                             2823
      7        27027329                             2703
38610543        120227100                            120
                115831629                            116
                  4395471                              4
```

In this example, the number is divided into periods by proceeding towards the left in the integral part 372, but towards the right in the decimal part ·69357, adding a cipher to complete the last period, which does not alter the value of the decimal (p. 136). The root is then extracted in the usual way, as on the left of the page. When the number is a mixed decimal, as in the present case, the work may be considerably shortened by finding one more than half the number of figures required in the usual way, and then proceeding to obtain the other figures of the root by the contracted method of division of decimals (p. 152), as on the right of the page. If the decimal had been a recurring one, instead of bringing down ciphers, as above, we would have brought down the figures of the recurring decimal.

REASON OF THE RULE.—The principles on which the rule depends are:

1. 'That since the local values of the figures in the root increase in a tenfold ratio by each removal to the left, their squares will increase 100 times.' Thus, the square of 4 is 16; the square of 40 is 1600, 100 times more than the square of 4; and the square of 400 is 160000, 100 times more than the square of 40; and so on. Hence the reason for dividing the number into periods of two figures each.

2. 'That the first figure in the root is first found, and then the square of its value is taken from the given number; that from the remainder the second figure is found, and the square of the sum of their values is taken from the given number; and so on.' But in order to shew how this *abrupt* method becomes the *continuous* one prescribed in the rule, the following proposition must be premised:

The square of the sum of two numbers is equal to the squares of the two numbers, together with twice their product.

Take any two numbers, as 20 and 5; their sum is 25, then

$$25^2 = 20^2 + 5^2 + 2 \times 20 \times 5.$$

$$\begin{aligned} \text{For}\quad 25^2 = 25 \times 25 &= 25 \times (20 + 5) \\ &= 25 \times 20 + 25 \times 5 \\ &= (20 + 5) \times 20 + (20 + 5) \times 5 \\ &= 20 \times 20 + 5 \times 20 + 20 \times 5 + 5 \times 5 \\ &= 20^2 + 2 \times 20 \times 5 + 5^2; \end{aligned}$$

$$\text{since } 20^2 + 2 \times 20 \times 5 + 5^2 = 20^2 + (2 \times 20 + 5) \times 5;$$

$$\therefore\ 25^2 = 20^2 + (2 \times 20 + 5) \times 5.$$

If the number consist of three digits, we have similarly,

$$250^2 = 200^2 + (2 \times 200 + 50) \times 50;$$

and subtracting 200^2 from both sides of this equality, we get

$$250^2 - 200^2 = (2 \times 200 + 50) \times 50.$$

Hence we perceive that, after subtracting the square of the value of the first figure, the remainder must contain *twice* the value of the first figure plus the value of the second, the number of times that there are units in the value of the second figure.

Example 2, fully wrought out, will afford an illustration.

$$
\begin{array}{rr}
2 \times 200 = 400 & 67081\,(200 \\
\underline{50} & 200^2 = \underline{40000}\quad 50 \\
2 \times 200 + 50 = 450 & 27081 \quad \underline{9} \\
\underline{50} & (2 \times 200 + 50) \times 50 = \underline{22500}\quad 259 = 67081^{\frac{1}{2}} \\
2 \times 250 = 2 \times 200 + 2 \times 50 = 500 & 67081 - 250^2 = 4581 \\
\underline{9} & (2 \times 250 + 9) \times 9 = \underline{4581} \\
2 \times 250 + 9 = 509 & 67081 - 259^2 = 0
\end{array}
$$

And thus, if the given number had contained more periods, we would have gone on to obtain the other successive figures of the root. By omitting the ciphers in the above operation, and those parts that are explanatory, the work will stand as in Example 2.

Exercises.

1.	Required the square root of	9216, . .	Ans. 96.
2.	〃 〃 〃	27225, .	〃 165.
3.	〃 〃 〃	119025, .	〃 345.
4.	〃 〃 〃	717409, .	〃 847.
5.	〃 〃 〃	62504836, .	〃 7906.
6.	〃 〃 〃	97535376,	〃 9876.
7.	〃 〃 〃	15241578750190521,	Ans. 123456789.
8.	〃 〃 〃	7619, .	〃 87·286883.
9.	〃 〃 〃	761·9, .	〃 27·602536.
10.	〃 〃 〃	5, . .	〃 2·236068.
11.	〃 〃 〃	17, . .	〃 4·1231056.
12.	〃 〃 〃	1·7, . .	〃 1·30384048.
13.	〃 〃 〃	237·615, .	〃 15·4147656.
14.	〃 〃 〃	$7894{\cdot}61\dot{9}\dot{3}$, .	〃 88·8516707.
15.	〃 〃 〃	$12{\cdot}3\dot{1}\dot{4}$, .	〃 3·509151.
16.	〃 〃 〃	$42{\cdot}\dot{3}4\dot{5}$, .	〃 6·5073301.
17.	〃 〃 〃	$\frac{121}{144}$, . .	〃 $\frac{11}{12}$.
18.	〃 〃 〃	$\frac{1521}{632025}$, .	〃 $\frac{39}{795}$.
19.	〃 〃 〃	$\frac{3}{8}$, . .	〃 ·612372436.
20.	〃 〃 〃	·0003841, .	〃 ·019598.

IF THE MEANS OF A PROPORTION ARE EQUAL, either is called the *mean proportional* between the extremes. Thus, since $4 : 6 :: 6 : 9$ is a proportion, the 6 is a mean proportional between the 4 and 9.

Since $\frac{6 \times 6}{4} = 9$, $\therefore\ 6 \times 6 = 9 \times 4$; that is, $6^2 = 9 \times 4$; or the square of the mean is equal to the product of the extremes. Hence—

TO FIND THE MEAN PROPORTIONAL BETWEEN TWO GIVEN NUMBERS.

Multiply the one number by the other, and extract the square root of their product.

Example.—Find the mean proportional between 4 and 9.

$4 \times 9 = 36$, and $\sqrt{36} = 6$, the mean proportional required.

Exercises.

21. Find the mean proportional between 16 and 144. Ans. 48.

22. " " " " 19 " 41. Ans. 27·91057.

23. A cheese, when put into one scale of a false balance, was found to weigh 43 lb., but when put into the other, it weighed 89 lb.; what was the true weight of the cheese? Ans. 61·86275 lb.

TO FIND THE SIDE OF A SQUARE WHEN ITS AREA IS GIVEN.

RULE.—Extract the square root of the given area.

THE REASON of the rule is plain, as the area of a square is the product of the side multiplied by itself.

Exercises.

24. The area of a circle is 7085 square inches; what is the side of a square whose area is equal to that of the circle? Ans. 84·1724 inches.

25. A gentleman has three fields: the first measures 2 ac. 1 ro. 30 po.; the second, 3 ac. 2 ro. 15 po.; and the third, 1 ac. 3 ro. 27 po. He wants another of a square form equal in area to all the three; how many poles must its side measure? Ans. 35·665109.

26. A person has a field measuring 3 ac. 1 ro. 35 po., which he wishes to exchange for a square one of inferior quality, but $3\frac{1}{2}$ times as large; how many poles in the length of its side?

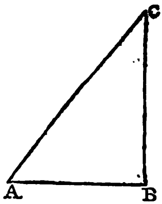
Ans. 44·0738.

In the solution of exercises 27 to 32, the pupil must understand that—

I. 'THE AREAS OF CIRCLES are to one another as the squares of their respective diameters.' Thus, if the diameter of a circle measure 2 feet, and that of another 3 feet, then—

the first circle : the second :: $2^2 : 3^2$, or as 4 : 9.

II. 'THE SQUARE OF THE HYPOTENUSE of a right-angled triangle is equal to the sum of the squares of the base and perpendicular.'



In the annexed diagram, AC is the hypotenuse, AB the base, and BC the perpendicular; and $AC^2 = AB^2 + BC^2$, $\therefore AC = (AB^2 + BC^2)^{\frac{1}{2}}$.

27. The paving of a circular space 50 feet in diameter costs £74, 16*s*. 8*d*.; what will the paving of another 120 feet in diameter cost at the same rate? Ans. £431, 0*s*. $9\frac{1}{2}$*d*. $\frac{2}{5}$.

28. If a pipe whose diameter is 1·5 inches fill a cistern in 5 hours, in what time will another, whose diameter is 3·5 inches, fill the same? . . . . . . Ans. 55 min. $6\frac{6}{49}$ sec.

29. If a sluice 3 feet in diameter draw off the water in a pond in 20 hours, what must be the diameter of another that would draw it off in one-fifth of the time? . . Ans. 6·7082 feet.

30. The base of a right-angled triangle is 342 feet, and the perpendicular 475 feet; required the length of the hypotenuse,

Ans. 585·311028.

31. The height of a wall was 31 feet, and the breadth of a ditch surrounding it was 24 feet; what must be the length of a ladder that will reach from the edge of the ditch to the top of the wall?

Ans. 39·20459.

32. Close by the side of a river rises a precipice to the height of 261 feet, and a line, reaching from its top to the opposite bank of the river, measures 582 feet; what is the breadth of the river?

Ans. 520·19515.

33. London stands on an area of 122 square miles, find the side of a square equal in area to the surface of London,

Ans. 11·04536 miles nearly.

34. Great Britain and Ireland embraces an area of 122550 square miles, required the side of a square equal in area to the surface of the United Kingdom of Great Britain and Ireland,

Ans. 350·07142 miles.

35. The true weight of a body is a mean proportional between the apparent weights, when weighed successively in the two scales of a false balance; what is the true weight of a body which being successively weighed in the scales of a false balance appears to be 36 and 38½ lb., respectively?

Ans. 37·229 lb. very nearly.

36. What is the true weight of a body which being weighed successively in the two scales of a false balance, shews 25 lb. and 36 lb.? . . . . Ans. 30 lb.

37. A heavy body falling freely at the surface of the earth, falls 16·1 feet in the first second of time, and the space fallen through in any time is equal 16·1 feet multiplied into the square of the time; find the time in seconds in which a heavy body will fall through a space of 2318·4 feet, . . . Ans. 12 seconds.

38. In what time will a heavy body fall freely through a space of 300 feet? . . . . . Ans. 4·31665 + seconds.

39. Find a mean proportional between 12 and 75, and another between 75 and 363, ... Ans. 30 and 165.

40. The areas of two circles are in the ratio of 7 to 13; the diameter of the first is 21, what is the diameter of the second?

Ans. 28·618 +.

41. The hypotenuse of a right-angled triangle is 725, and one of the sides is 465; find the other side, . . Ans. 556·237 +.

42. A room is 24 feet long, 18 feet wide, and 12 feet high; it is required to find the length of a cord that would reach from one corner on the floor to the opposite corner at the ceiling,

Ans. 32·311 feet.

43. From a point in a street a ladder 55 feet long reached to a window 44 feet high on one side, and being turned over it reached to another 40 feet high on the other side of the street; what was the width of the street? . . . Ans. 70·7492 feet nearly.

44. The diameter of a pipe which empties a cistern in 40 minutes is one inch; required the diameter of another pipe that would empty the same cistern in 15 minutes, Ans. 1·632993 inches.

EXTRACTION OF THE CUBE ROOT.

RULE—1. Point off the given number, by means of commas, into periods of three figures each; beginning at the right in integers, and counting towards the *left*; and beginning at the left in decimals, and counting towards the *right*: thus, 1,784,453·547,46.

If, in pointing off the periods, *one* or two figures remain over, at the left in integers, this remainder is considered as a period, although not consisting of three figures; but if at the right in decimals, a nothing or nothings must be annexed to complete the period.

2. Find the greatest cube *root* contained in the first period at the left, and place it at the right of the given number, as the first figure of the root: then subtract the *cube* of this figure from the first period, and to the remainder, if any, annex the next period of three figures, to form a new sum or dividend.

Example.—What is the cube root of 178453547?

			178,453,547 (563 *Ans.*
			125
$5^2 \times 300$	$=$	7500	53453
$(5 \times 30 + 6) \times 6$	$=$	936	
		8436	50616
		36	
		940800	2837547
$(56 \times 30 + 3) \times 3$	$=$	5049	
		945849	2837547

Here, the greatest cube root contained in the first period, 178, is 5, which is placed as the first figure of the required root: the cube of 5*—namely, 125—is then subtracted from the first period, and to the remainder, 53, is annexed the next period, 453, forming the dividend, 53453.

* $5 \times 5 \times 5 = 125$.

3. Form a DIVISOR for this dividend as follows:

Multiply the *square* of the root found by 300, for a *partial* divisor.

Find how often this partial divisor is contained in the dividend,* and place the quotient† as the next figure of the required root:

Here, 25, the square of 5, is multiplied by 300, making the partial divisor 7500.

The partial divisor is contained 6 times in the dividend;* the quotient, 6, is therefore

* See Note 2, page 222.

† Sometimes a less figure requires to be taken, as in the present example.

* It is here contained 7 times in the dividend, but as it is found, on carrying the process further, that 7 is too many, 6, the next lower figure, is taken.

then multiply the *previous* figure of the root by 30, and add to the product the figure of the root just found; multiply the result by the same figure, and place the product below the partial divisor, then add the two sums together, and their amount is the *complete* divisor.

placed as the next figure of the required root: the root already found, 5, is then multiplied by 30, and 6 is added to the product 150, making 156; this multiplied by 6, makes 936, which is placed below the partial divisor, 7500. The two sums, 7500 and 936, are then added together, forming the complete divisor, 8436.

4. Multiply the complete divisor by the last figure of the root; subtract the product from the dividend; and to any remainder, annex the next period of three figures, to form a new dividend.

Here, the divisor, 8436, is multiplied by 6, the last figure of the root, and the product is subtracted from the dividend, 53453: to the remainder, 2837, is brought down the next period of three figures, forming the new dividend, 2837547.

5. Form a new DIVISOR as follows:

Place the *square* of the last figure of the root under the previous divisor; add it to the two lines of figures above it; and to the amount annex two nothings, to form a partial divisor.

Find how often the partial divisor is contained in the dividend; and, as in paragraph 3, place the quotient as the next figure of the required root: then multiply the *previous* figures of the root by 30, and add to the product the figure of the root just found; multiply the result by the same figure, and place the product below the partial divisor: then add the two sums together, and their amount is the *complete* divisor.

Here, 36, the square of 6, the last figure of the root, is placed below the complete divisor, 8436; it is then added to the two lines above it—namely, 8436 and 936 — and their amount, with two nothings annexed, forms the partial divisor, $940800 = 56^2 \times 300$.

The partial divisor is contained 3 times in the dividend; the quotient, 3, is therefore placed as the next figure of the root: the root already found, 56, is then multiplied by 30, and 3 is added to the product; this multiplied by 3 makes 5049, which is placed under the partial divisor. The two sums, 940800 and 5049, are then added together, forming the complete divisor, 945849.

6. Multiply the complete divisor, and so on, as already described in paragraph 4.

7. Proceed to form new divisors according to paragraph 5; and go on with the rest of the process according to paragraph 4, till all the periods have been brought down and operated upon; when, if there is no remainder, the extraction of the root is completed.

Here, the divisor, 945849, is multiplied by 3, the last figure of the root, and the product is subtracted from the dividend, 2837547: there being no remainder, and no more periods to bring down, the extraction of the root is completed.

NOTE 1.—When there is a remainder after all the periods have been brought down, annex a period of three nothings, to form a new dividend; and then proceed with the further extraction of the root: the figure of the root thus obtained is a decimal. The process may be carried to any degree of minuteness by annexing more nothings.

There are always as many figures in the root as there are periods in the given number; and those figures are decimals in the root, which are extracted from the decimals in the given number.

NOTE 2.—If at any time, on bringing down a new period to form a dividend, the *partial* divisor is found to be greater than the dividend, a nothing must be placed in the root, and two nothings annexed to the partial divisor: the next period is then brought down, and annexed to the dividend, and the extraction of a new figure of the root proceeded with as before.

NOTE 3.—The cube root of a fraction is found by extracting the roots of the numerator and denominator.

THE REASON of the Rule is shewn in 'Algebra,' p. 105.

Exercises.

1.	Required the cube root of	110592, . .	Ans.	48.
2.	" "	373248, . .	"	72.
3.	" "	843908625, .	"	945.
4.	" "	219365327791, .	"	6031.
5.	" "	3·539, . .	"	1·52391305.
6.	" "	11, . . .	"	2·22398.
7.	" "	24, . . .	"	2·884499.
8.	" "	7·52, . .	"	1·959172.
9.	" "	$9613\dot{\cdot}\dot{9}2$, .	"	21·2634.

10. A box, whose length, breadth, and depth are equal, contains 216 cubic feet; what are its dimensions? Ans. 6 feet each way.

11. A person has a box 5 feet long, 4 feet broad, and $6\frac{1}{4}$ feet deep, and wishes another box to contain the same number of cubic feet, whose length, breadth, and depth shall be equal; what are the required dimensions? . . . . Ans. 5 feet each.

GLOBES are in proportion to each other as the cubes of their diameters: hence to find the diameter of a globe that shall contain 8 times more than another whose diameter is 2 feet, multiply the cube of 2 feet, the given diameter, by 8, and extract the cube root of the product.

CUBES are in proportion to each other as the cubes of a side of each.

12. If a globe, whose diameter is 4 inches, weigh 5 lb., what is the diameter of another globe which weighs 40 lb.? Ans. 8 inches.

13. If the side of a box, whose length, breadth, and depth are equal, is 4 feet long, what is the length of a side of another cubical box, that contains 27 times as many cubic feet? . Ans. 12 feet.

14. The cube root of the solid content of a body is the side of a cube of equal content or volume; find the side of a cube that will contain a bushel = 2218·192 cubic inches, Ans. 13·041665 inches.

15. A French kilolitre is 61028·79179 cubic inches; find the side of a cube that will contain a kilolitre,

Ans. 39·37008 inches = 1 mètre.

16. The diameter of a sphere is the cube root of the quotient of its solidity divided by ·5236: find the diameter of a sphere whose solidity is 4188800 cubic inches, . . Ans. 200 inches.

17. Find the diameter of a sphere whose solid content is 5254725974·3748 cubic miles, . . . Ans. 2157 miles.

18. Kepler's third law is, that the *squares* of the periodic times of any two planets are to each other in the same proportion as the *cubes* of their mean distances from the sun. The earth's periodic time is 365·256384 days, and that of Mars 686·076119 days; if the earth's distance from the sun be 1, what is that of Mars? . . . . Ans. 1·522357.

19. From the facts stated in the last question, find the mean distance of Jupiter from the sun, his periodic time being 4332·5693 days nearly, ... Ans. 5·20114.

SERIES.

A SERIES is a succession of numbers, that are derived from one another according to a certain law.

The first and last terms of any series are called the *extremes*, and the others the *means*.

I. EQUIDIFFERENT SERIES, OR ARITHMETICAL PROGRESSION.

A SERIES, in which the difference between any two consecutive terms is the same, is called an *equidifferent* series. If the terms increase, it is called an *increasing* series; when they decrease, a *decreasing* series.

THE CONSTANT DIFFERENCE between any two successive terms is called the *common difference.* Thus, 1, 5, 9, 13, 17, &c., is an *increasing* equidifferent series, the common difference of which is 4; and 30, 27, 24, 21, &c., is a *decreasing* equidifferent series, the common difference being 3.

THE LAST TERM OF AN EQUIDIFFERENT SERIES is equal to the first term, increased, when the series is increasing, by the product of the common difference multiplied into a number which is one less than the number of terms; but diminished by it, when the series is decreasing.

Take any equidifferent series, as 3, 7, 11, 15, 19, 23, whose common difference is 4. It is obvious that this series may be arranged thus: $3, 3 + 1 \times 4, 3 + 2 \times 4, 3 + 3 \times 4, 3 + 4 \times 4, 3 + 5 \times 4$, where it is plain that any term as the 6th is equal to the first 3, increased by the common difference, 4, multiplied by 5, a number 1 less than 6, the number of terms. Similarly when the series is decreasing.

Exercises.

1. The first term of an increasing equidifferent series is 5, the common difference 11, and the number of terms 100. What is the last term? . . . . Ans. 1094.

2. The first term of a decreasing equidifferent series is 59, the common difference 2, and the number of terms 24. Required the last term, ... Ans. 13.

THE SUM OF THE TERMS OF AN EQUIDIFFERENT SERIES is equal to the sum of the first and last terms multiplied by half the number of terms.

Take any equidifferent series, as 3, 7, 11, 15, 19, 23, consisting of any number of terms, as 6; then if s denote the sum of the series, $s = \frac{(3 + 23) \times 6}{2}$

For, $s = 3 + 7 + 11 + 15 + 19 + 23$, then arranging the series backwards

$s = 23 + 19 + 15 + 11 + 7 + 3$ adding vertically, we have

$2s = 26 + 26 + 26 + 26 + 26 + 26 = 26 \times 6.$

$$\therefore s = \frac{26 \times 6}{2} \text{ or } \frac{(3 + 23) \times 6}{2}$$

Exercises.

3. The first term of an equidifferent series is 17, the last 85, and the number of terms 17. What is the sum of the series?
Ans. 867.

4. The first term of an equidifferent series is 100, the last 21, and the number of terms 85. What is the sum of the series?
Ans. $5142\frac{1}{2}$.

5. How many times does a common clock strike in a day?
Ans. 156.

6. A debt can be discharged in 52 weeks by paying 1*s*. the first week, 3*s*. the second, 5*s*. the third, and so on. Required the amount of the debt, and the last payment,
Ans. Amount, £133, 18*s*., and last payment £5, 3*s*.

7. From two towns, A and B set out to meet each other; A went 3 miles the first day, 5 the second, 7 the third, and so on; B went 4 miles the first day, 6 the second, 8 the third, and so on; they met in 8 days. What was the distance between the towns?
Ans. 168 miles.

II. Equirational series, or Geometrical progression.

A series, in which the ratio of any two consecutive terms is the same, is called an *equirational* series. It is called an *increasing* or *decreasing* series, according as the terms increase or decrease.

The ratio of any term to the preceding one is called the *common ratio*.

Thus, 7, 14, 28, 56, is an increasing equirational series, whose common ratio is 2; and 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, is a decreasing one, whose common ratio is $\frac{1}{2}$.

The last term of an equirational series is equal to the first term multiplied by that power of the common ratio whose index is one less than the number of terms.

Take any equirational series, as 3, 6, 12, 24, 48, 96, whose common ratio is 2; it is obvious that this series may be written thus—3, 3×2, 3×2^2, 3×2^3, 3×2^4, 3×2^5; where it is plain that any term, as the 6th, is equal to the first 3, multiplied by 2, the common ratio, raised to that power whose index is 5—that is, one less than 6, the number of terms.

Exercises.

8. The first term of an equirational series is 4, the common ratio 3, and the number of terms 13. What is the last term?

Ans. 2125764.

9. The first term of an equirational series is 100, the common ratio $\frac{1}{2}$, and the number of terms 17. What is the last term?

Ans. $\frac{25}{16384}$.

The sum of the terms of an equirational series is found by multiplying the first term by the difference between 1 and that power of the common ratio whose index is equal to the number of terms, and then dividing the product by the difference between 1 and the common ratio.

Take any equirational series, as 2, 12, 72, 432, 2592, whose common ratio is 6; let s represent the sum, then $s = \frac{(6^5-1)\times 2}{6-1}$.

For, as 1 time $s = 2 + 2 \times 6 + 2\times 6^2 + 2\times 6^3 + 2\times 6^4$; multiply by com. ratio, 6 times $s = 2\times 6 + 2\times 6^2 + 2\times 6^3 + 2\times 6^4 + 2\times 6^5$;

$$\therefore 5 \quad '' \quad s = 2\times 6^5 - 2 = 2\times(6^5 - 1).$$

and
$$s = \frac{2\times(6^5-1)}{5} = 3110.$$

Exercises.

10. The first term of an equirational series is 5, the common ratio 3, and the number of terms 14. What is the sum of the series? . . . . Ans. 11957420.

11. The first term of an equirational series is 8, the common ratio 4, and the number of terms 12. Required the sum of the series, ... Ans. 44739240.

12. The first term of an equirational series is 1, the common ratio $\frac{1}{5}$, and the number of terms 10. What is the sum of the series? . . . . Ans. $1\frac{488281}{1953125}$.

Note.—In this exercise, $s = \dfrac{(1 - \frac{1}{5^{10}})}{1 - \frac{1}{5}}$. It may be observed that when the number of terms is indefinitely great, the fraction in the numerator of the sum becomes indefinitely small, and its value may be regarded as 0; hence, when the series is decreasing, and the number of terms infinite (as it is called), the sum is found by dividing the first term by the difference between 1 and the common ratio.

LOGARITHMS.

THE RULES by which the following exercises are to be worked, with an explanation of Logarithms, will be found at pages v. to xv. of the Introduction to 'Mathematical Tables' of this Course.

Exercises.

1.	Find the logarithm of 3·729416, . .	Ans. 0·5716409.
2.	〃 〃 184·372, . .	〃 2·2656950.
3.	〃 〃 73·6148, . .	〃 1·8669651.
4.	〃 〃 ·0473827, . .	〃 $\bar{2}$·6756198.
5.	Find the number whose log. is 2·7169325, .	〃 521·1137.
6.	〃 〃 〃 4·1763927,	〃 15010·41.
7.	〃 〃 〃 $\bar{1}$·9537346,	〃 ·8989481.
8.	〃 〃 〃 0·5379182,	〃 3·450787.
9.	Multiply 73·627, ·01852, and 9·31654 together,	〃 12·70377.
10.	Find the continued product of ·68715, ·00354, 839·612, and ·5183, ...	〃 1·058558.
11.	Divide 68·3167 by 983·7, . . .	〃 ·0694487.
12.	〃 ·05685 by ·73254, ...	〃 ·07760669.
13.	Find the 3d power of 1·03721, . .	〃 1·115835.
14.	〃 20th 〃 1·05, ...	〃 2·653297.
15.	〃 31st 〃 1·025, . . .	〃 2·150011.

16. Extract the square root of 11, . . . Ans. 3·316624.
17. " " cube " 92, . . . " 4·514357.
18. " " fifth " 5·16, . . . " 1·388449.
19. " " tenth " ·16354, . . . " ·8343771.

20. Find the value of $\frac{1{\cdot}716 \times {\cdot}073459 \times 78{\cdot}615 \times 54{\cdot}728}{69{\cdot}73 \times {\cdot}969312 \times 81{\cdot}7346}$ by logarithms, ... Ans. ·098172.

21. Find the value of $\frac{7{\cdot}854^3 \times 3{\cdot}14169^{\frac{1}{2}}}{5{\cdot}12796^4}$, . . . " 1·02616.

22. " " $\frac{1{\cdot}83^{10} \times {\cdot}6075^{\frac{1}{2}}}{7{\cdot}19^2}$, ... " 7·19349.

COMPOUND INTEREST.

Compound Interest is the Rule employed, as mentioned at p. 175, when the Interest of money is *added* to the principal as it becomes due, and this amount reckoned as the principal on which interest is to be reckoned for the next period or term.

Example.—What is the amount of £320, 10*s.* for 4 years, at 5 per cent. compound interest?

	£	s.	d.	
	320	10	0	= principal for the 1st year.
5 = $\frac{1}{20}$ of 100	16	0	6	= interest " "
	336	10	6	= principal for the 2d year.
5 = $\frac{1}{20}$ of 100	16	16	6½	= interest " "
	353	7	0½	= principal for the 3d year.
5 = $\frac{1}{20}$ of 100	17	13	4	= interest " "
	371	0	4½	= principal for the 4th year.
5 = $\frac{1}{20}$ of 100	18	11	0	= interest " "
	389	11	4½	= answer required.

Here, since 5 is the $\frac{1}{20}$ of 100, the interest of £320, 10*s.* for 1 year is computed accordingly. When the time is long, this method of calculation becomes very laborious; it is therefore preferable to proceed according to the following rules.

Compound Interest is also reckoned by means of Tables such as are given at p. 269.

I. To find the amount of £1, for any number of terms at Compound Interest.

Rule.—Find the amount of £1 for *one* term, and raise it to that power denoted by the number of terms: the result will be the amount required.

It is by this Rule that Table I, p. 269, is constructed.

Example.—What is the amount of £1 for 5 years at 4 per cent. compound interest, supposing the interest payable yearly?—half-yearly?—quarterly?

Ans. Yearly, £1·2166529; half-yearly, £1·218994; quarterly, £1·22019.

In the first question the term is 1 *year;* and 4 being the rate per cent., the amount of £100 for 1 year is £104, hence it is plain that the amount of £1 for 1 year is equal to 1·04; then $1·04^5$ = amount required.

In the second question the term is 1 *half-year;* ∴ the amount of £100 for this term is £102, and the amount of £1 is equal to 1·02; then $1·02^{10}$ = amount required, 10 being the number of *half-years* in five years.

Similarly, in the third question, $1·01^{20}$ = amount required, 20 being the number of terms.

For the method of finding the powers, and an example wrought out at length, the pupil is referred to Example 3, p. 210.

Exercises.

Find the amount of the following:

1. £1 for	7 years at	3	per cent.	Ans.	£1·2298739.
2. £1 "	10 "	2½	"	"	£1·2800845.
3. £1 "	35 "	6	"	"	£7·6860868.
4. £1 "	50 "	5	"	"	£11·4673998.
5. £1 "	100 "	3½	"	"	£31·191408.

II. To find the amount of any sum at Compound Interest, for any number of terms.

Rule.—Find the amount of £1 for the given time, and rate; then multiply by the given sum. The result will be the amount required.

The interest of any sum at compound interest for a given time is found by subtracting the principal from the amount of the sum for that time.

Example.—What is the amount of £527, 13*s.* 4*d.* for 25 years at $4\frac{1}{2}$ per cent. compound interest?

			£	s.	d.	
	$1{\cdot}045^{25} =$ £ 3·0054345	= am. of £1 for 25 years.				
	527					
	210380415					
	60108690					
	150271725					
	1583·8639815	= am. of	527	0	0	for 25 years.
10*s.* = $\frac{1}{2}$ of £1,	1·5027172	= 〃	0	10	0	〃
3*s.* 4*d.* = $\frac{1}{3}$ 〃 10*s.*	·5009057	= 〃	0	3	4	〃
	£1585·8676044	= 〃	527	13	4	
	£1585, 17*s.* $4\frac{1}{4}$*d.* nearly.					

Here the amount of £1 for 25 years is equal to £3·0054345, and this, multiplied by 527, gives the amount of £527. Taking aliquot parts for 13*s.* 4*d.*, we find the whole amount of £527, 13*s.* 4*d.* to be £1585·8676044, or £1585, 17*s.* $4\frac{1}{4}$*d.* nearly.

By decimals.

```
  3·0054345
  6666·725
  1502717
    60109
    21038
     1803
      180
       18
        2
£1585·867
£1585, 17s. 4¼d.
```

By Logarithms.

Amount $= 1{\cdot}045^{25} \times 527{\cdot}6666.$

$$25 \log. 1{\cdot}045 = 0{\cdot}4779073$$
$$\log. 527{\cdot}6666 = 2{\cdot}7223596$$
$$\log. 1585{\cdot}867 = 3{\cdot}2002669$$

or £1585, 17*s.* $4\frac{1}{4}$*d.*

The pupil is recommended to work out the following exercises by each of these methods: the logarithms of the amount of £1 for the more common rates per cent. will be found at page 269. of this volume. The answers to the exercises are given to the nearest farthing. When the exercises are solved by the aid of logarithms, answers may come out sometimes more and sometimes less, especially when the periods are long.

Reason of the Rules.—The reason of the Rules may be shewn thus: Take any rate per cent., as 5; the amount of £100 for 1 year will be £105, and it is obvious that the amount of £1 will be the $\frac{1}{100}$ part of this; that is, £1·05; and this, by the nature of compound interest, becomes the principal for the second year. Now, *as* the principal 1 *is to* its amount 1·05, *so is* the principal 1·05 *to* $1{\cdot}05^2$ = amount of £1 for 2 years; and this, by the nature of compound interest, becomes the principal for the third year; and so on. It appears, then, that the index of the power is always equal to the number of years. Similar reasoning would apply were the terms half-years, &c.; hence Rule I. The amount of £1 being found, Rule II. follows at once, since the amounts are proportional to the principals.

	£1·05 =	am. of £1 for 1 yr.
$1 : 1{\cdot}05 :: 1{\cdot}05 : 1{\cdot}05^2$	=	" 2 "
$1 : 1{\cdot}05 :: 1{\cdot}05^2 : 1{\cdot}05^3$	=	" 3 "
$1 : 1{\cdot}05 :: 1{\cdot}05^3 : 1{\cdot}05^4$	=	" 4 "

Exercises.

6. Find the amount of £325, 12*s.* 6*d.* for 17 years at 4 per cent. compound interest, . . . Ans. £634, 5*s.* 8¼*d.*

7. What sum will £724, 13*s.* 4*d.* amount to in 50 years at 5 per cent.? . . . . Ans. £8310, 0*s.* 10*d.*

8. What is the amount of £987, 15*s.* 9*d.* for 19 years at 4½ per cent.? . . . . Ans. £2279, 13*s.* 6*d.*

9. To what sum will £1728, 16*s.* 8*d.* amount to in 64 years at 4¾ per cent.? . . . . Ans. £33698, 11*s.*

10. To what sum will £618, 11*s.* 6*d.* amount to in 61 years at 2½ per cent.? . . . . . . Ans. £2789, 12*s.* 9½*d.*

11. Required the amount of £837, 7*s.* 6*d.* for 22 years at 4 per cent., supposing the interest payable half-yearly,
Ans. £2001, 7*s.* 5*d.*

12. Required the amount of £235, 15*s.* for 15 years at 4 per cent., supposing the interest payable quarterly,
Ans. £428, 5*s.* 8½*d.*

13. A legacy of £1200 was left to a boy 3 years of age; what sum will he have to receive at the age of 21, supposing the legacy improved by compound interest at 6 per cent.?
Ans. £3425, 4*s.* 1¾*d.*

III. To find the *present worth* of a sum for a given time, and at a given rate, compound interest.

Rule.—Divide the given sum by the amount of £1 for the given time, and at the given rate; the quotient will be the present worth required.

This rule is obviously the converse of the last. Hence the present worths of £1 for any time and rate are the reciprocals of the amounts of £1 for the same time and rate: it is thus that TABLE II., p. 270, is constructed. The pupil should take the present worth of £1 from the table when he can.

Example.—What is the present worth of £527, 10*s.* 6*d.* due 9 years hence, at 4 per cent. compound interest?

Here $1{\cdot}04^9 = £1{\cdot}4233118$; therefore $\dfrac{527{\cdot}525}{1{\cdot}4233118} = \overset{£}{370{\cdot}632}$

$= £370,\ 12s.\ 7\tfrac{3}{4}d.$ *Ans.*

By Table II.	By Logarithms.
$\dfrac{1}{1{\cdot}04^9} = {\cdot}7025867$	pr. worth $= \dfrac{527{\cdot}525}{1{\cdot}04^9}$
and ${\cdot}7025867 \times 527{\cdot}525 = \overset{£}{370{\cdot}632}$	log. $527{\cdot}525 = 2{\cdot}7222430$
$= £370,\ 12s.\ 7\tfrac{3}{4}d.$	9 log. $1{\cdot}04 = 0{\cdot}1532997$
	log. $370{\cdot}632 \quad 2{\cdot}5689433$
	or £370, 12*s.* $7\tfrac{3}{4}d.$

Exercises.

14. What is the present worth of £231, 10*s.* due 11 years hence, reckoning compound interest at 5 per cent.?
Ans. £135, 7*s.* $0\tfrac{3}{4}d.$

15. What sum of money will now discharge a debt of £1000 due 5 years hence, at $3\tfrac{1}{2}$ per cent. compound interest?
Ans. £841, 19*s.* $5\tfrac{1}{4}d.$

16. Required the present worth of £719, 17*s.* 6*d.* due 22 years hence, at $2\tfrac{1}{2}$ per cent. compound interest, Ans. £418, 2*s.* $11\tfrac{1}{4}d.$

17. Find the present worth of £837, 15*s.* 6*d.* due 27 years hence, at $4\tfrac{1}{2}$ per cent. compound interest, . Ans. £255, 5*s.* 3*d.*

18. Whether is it better to sell an estate for £4000 payable at present, £4000 payable at the end of 10 years, and £4000 payable at the end of 20 years; or to sell it for £12000 payable at the end of 10 years, reckoning compound interest at 3 per cent.?
Ans. Better at 3 payments by £261, 19*s.* $0\tfrac{1}{2}d.$

Rule II. may be applied to the calculation of the number of inhabitants in a town or country, when the excess of the births above the deaths for any year bears a constant ratio to the number of the inhabitants for that year. In such calculations, however, many other circumstances require to be taken into account, which it would be altogether out of place to mention here.

19. The population of England for the year 1851 was estimated at 17927609, what may the population for 1861 be estimated at, supposing the annual increase to be at the rate of 1·8 per cent.?
Ans. 21428920.

ANNUITIES.

AN ANNUITY is any periodical income, payable at equal intervals, such as yearly, half-yearly, quarterly, &c.

Annuities are called CERTAIN when they are to continue for a fixed number of years; when they are to be paid only so long as one or more individuals shall live, they are called CONTINGENT or LIFE annuities. When an annuity is to continue for ever, it is called a PERPETUITY.

When an annuity is payable at present, it is said to be in *possession;* but when the payment is not to commence until some time has elapsed, it is called a *deferred* or a *reversionary* annuity.

When the payment of an annuity has been forborne for a certain number of terms, the sum of all the payments, each improved at compound interest from the period of its becoming due until the payment of the whole, is called the *amount.*

The *present value* of an annuity is that sum which, improved at compound interest, would exactly pay the annuity as it becomes due.

ANNUITIES CERTAIN.

I. TO FIND THE AMOUNT OF AN ANNUITY, CERTAIN.

RULE.—From the amount of £1 for the given time and rate, subtract 1, divide the difference by the interest of £1 for a year; the quotient, multiplied by the given annuity, will give the amount required.

Note.—One of the equal intervals at which the annuity is payable is always supposed to elapse between entering on possession and the first payment of the annuity.

Example.—Find the amount of an annuity of £50, 12*s.* 6*d.* for 15 years at 4½ per cent. compound interest.

$$\text{Amount} = \frac{1{\cdot}045^{15} - 1}{{\cdot}045} \times 50{\cdot}625.$$

(Table I., p. 269) $\quad 1{\cdot}045^{15} = 1{\cdot}9352824$

$$\begin{array}{r}
1{\cdot}9352824 \\
1{\cdot} \\
\hline
{\cdot}045){\cdot}9352824 \\
\hline
20{\cdot}78405 \\
526{\cdot}05 \\
\hline
10392025 \\
124704 \\
4157 \\
1039 \\
\hline
1052{\cdot}1925
\end{array} = £1052,\ 3s.\ 10d.$$

REASON OF THE RULE.—Suppose that we were required to find the amount of an annuity of £1 for any number of years, as 7, and that the rate is 5 per cent. It is obvious that the 7th payment would be simply £1, as it is paid when it becomes due; that the 6th payment would be 1·05, as the annuity has been at interest for 1 year; that the 5th would be $1{\cdot}05^2$, since it has been at interest for 2 years; and so on, backwards, until the first, which would be $1{\cdot}05^6$. Now, if a denote the amount, it will be equal to the sum of all these—

$$\therefore a = 1 + 1{\cdot}05 + 1{\cdot}05^2 + 1{\cdot}05^3 + 1{\cdot}05^4 + 1{\cdot}05^5 + 1{\cdot}05^6.$$

As this is plainly an equirational series, whose common ratio is 1·05,

$$a = \frac{1{\cdot}05^7 - 1}{{\cdot}05}.$$

The rule follows at once from this, since the amounts are proportional to the annuities.

Exercises.

1. What will be the amount of an annuity of £160 for 25 years, at 4 per cent.? . . . . Ans. £6663, 6*s.* 10¾*d.*

2. What will a pension of £78, 10*s.* per annum, payable yearly, amount to in 7 years, at 3½ per cent.? . Ans. £610, 13*s.* 8*d.*

3. If an annual salary of £346, payable half-yearly, be forborne 19 years, what will be the amount at 5 per cent.?

Ans. £10765, 6*s.* 5¼*d.*

4. If an annuity of £725, 13*s.* 4*d.*, payable yearly, be omitted to be paid for 16 years, find the amount at 3 per cent.

Ans. £14627, 3*s.* 6¼*d.*

II. TO FIND THE *present value* OF AN ANNUITY, CERTAIN.

RULE.—Subtract the present value of £1 for the given time and rate from 1, divide the difference by the interest of £1 for a year; the quotient, multiplied by the given annuity, will be the present value required.

THE REASON of the rule may be shewn in a somewhat similar manner to the last.

When the annuity is perpetual, its present value is found by dividing the annuity by the interest of £1 for a year. It will be convenient to denote the reciprocal of $1{\cdot}05^7$, for instance, by $1{\cdot}05^{\bar{7}}$.

Example.—What is the present value of an annuity of £50, 12s. 6d., payable yearly for 15 years, at 4½ per cent.?

$$\text{Present value} = \frac{(1 - 1{\cdot}045^{-15})}{{\cdot}045} \times 50{\cdot}625.$$

$$
\begin{array}{lr}
 & 1{\cdot} \\
\text{(Table II., p. 270)}\ \dfrac{1}{1{\cdot}045^{15}} \text{ or } 1{\cdot}045^{-15} = & {\cdot}5167204 \\
 & {\cdot}045\overline{){\cdot}4832796} \\
 & 10{\cdot}73954 \\
 & 526{\cdot}05 \\
 & 5369770 \\
 & 64437 \\
 & 2148 \\
 & 537 \\
 & 543{\cdot}6892 = £543,\ 13s.\ 9\tfrac{1}{4}d.
\end{array}
$$

Exercises.

5. What is the present value of an annuity of £126, payable yearly for 17 years, at 3 per cent.? . . Ans. £1658, 18s. 7¼d.

6. What is the present value of an annual pension of £225, payable yearly for 20 years, at 5 per cent.? Ans. £2803, 19s. 11¼d.

7. What is the present value of a yearly rent of £62 for 25 years, at 3½ per cent.? . . . . Ans. £1021, 17s. 0¾d.

8. A tenant took a farm for 19 years at a rent of £550, and after 10 years of the lease were expired, his landlord wished him to pay up the rent for the rest of the time, promising him compound interest at 4 per cent., and a present discount of £300. What sum ought the farmer to pay down? Ans. £3789, 8s. 7¾d.

9. What is the present value of a perpetuity of £160, interest being at 2½ per cent.? . . . . Ans. £6400.

III. To find the present value of a *deferred* annuity.

Rule.—From the present value of £1 for the time until the annuity is entered upon, subtract the present value of £1 from the present time until the annuity terminates, and divide the remainder by the interest of £1 for a year; the quotient, multiplied by the given annuity, will be the present value required.

Example.—Required the present value, at 4 per cent., of a yearly annuity of £75, 16s. 8d. in reversion, commencing at the end of 20 years, and continuing for 15 years.

$$\text{Present value} = \frac{(1{\cdot}04^{-20} - 1{\cdot}04^{-35})}{{\cdot}04} \times 75{\cdot}833.$$

$$\begin{array}{lrcr}
 & & & £ \\
\text{(Table II., p. 270)} & \dfrac{1}{1{\cdot}04^{20}},\ \text{or } 1{\cdot}04^{-20} & = & {\cdot}4563870 \\
 & \dfrac{1}{1{\cdot}04^{35}},\ \text{or } 1{\cdot}04^{-35} & = & {\cdot}2534155 \\
 & & {\cdot}04) & {\cdot}2029715 \\
 & & & 5{\cdot}0742875 \\
 & & & 33338{\cdot}57 \\
 & & & 3552001 \\
 & & & 253714 \\
 & & & 40594 \\
 & & & 1522 \\
 & & & 152 \\
 & & & 15 \\
 & & & 2 \\
 & & & 384{\cdot}8000 = £384,\ 16s.
\end{array}$$

Reason of the Rule.—From a little consideration, it will appear that the present value of the deferred annuity is equal to an equivalent annuity from the present time till the end of its continuance, diminished by the present value of the same annuity for the time that must elapse before it commences. In the above Example, suppose the annuity £1, then,

$$\text{Present value for 35 years} = \frac{1 - 1{\cdot}04^{-35}}{{\cdot}04}$$

$$\text{〃} \quad \text{〃} \quad 20 \quad \text{〃} = \frac{1 - 1{\cdot}04^{-20}}{{\cdot}04};\ \text{subtracting, we obtain}$$

$$\text{〃} \quad \text{〃} \ \text{deferred ann.} = \frac{1{\cdot}04^{-20} - 1{\cdot}04^{-35}}{{\cdot}04}$$

The rest is obvious.

Exercises.

10. What is the present value of an annuity of £627 a year, to commence 17 years hence, and to continue 25 years, at $3\frac{1}{2}$ per cent.? . . . . Ans. £5758, 1*s.* $10\frac{1}{2}$*d.*

11. What is the present worth of an annuity of £450, which commences 10 years hence, and continues for 90 years, interest at 4 per cent.? . . . . Ans. £7377, 6*s.* $11\frac{1}{4}$*d.*

12. What is the present worth of the reversion of a lease of £547, 18*s.* per annum, to continue for 18 years, but not to commence till the end of 6 years, allowing interest at 5 per cent. to the purchaser? . . . . . . Ans. £4779, 6s. 1d.

13. The reversion of a freehold estate of £1265, 7*s.* 6*d.* per annum, to continue 30 years, and to commence 12 years hence, is to be sold; what is it worth just now, allowing the purchaser 4½ per cent. for his money? . . . Ans. £12153, 17*s.* 9*d.*

14. What is the present worth of the reversion of a freehold estate, to continue for ever, but not to be entered upon until the expiry of 35 years, the annual rent being £925, and the rate of interest 2½ per cent.? . . . . Ans. £15590, 14*s.* 7*d.*

Miscellaneous Exercises.

In the solution of the following exercises the pupil will find little or no difficulty. It may be remarked that an annuity is said to be worth as many *years' purchase* as there are pounds in the present value of an annuity of £1. Freehold estates are generally bought and sold at so many years' purchase, or, what amounts to the same thing, at so many years' rent.

1. What is the present worth of a perpetual annuity of £724, 16*s.*, payable half-yearly, interest being 3¾ per cent.? Ans. £19328.

2. A gentleman purchased a freehold estate for £19400; what ought the annual rent to be when it is bought at 16 years' purchase? . . . . Ans. £1212, 10*s.*

3. I paid down £1000 for a perpetual annuity; what was the yearly annuity, supposing interest to be at 4½ per cent.? Ans. £45.

4. A merchant commenced business with a capital of £1800; how much will he be worth at the end of 25 years, supposing that, after paying all expenses, he increases his capital each year by a tenth part of itself? . . . Ans. £19502, 9*s.* 4¾*d.*

5. A person purchased a perpetual annuity of £320 a year for £10000; what rate of interest had he for his money? Ans. £3, 4*s.*

6. The annual rent of a freehold, which was purchased for £6265, 12*s.* 6*d.*, is £250, 12*s.* 6*d.*; at how many years' purchase was it bought? . . . . Ans. 25 years.

7. At how many years' purchase must a freehold estate be bought that the purchaser may have 5 per cent. for his money? Ans. 20 years.

8. What is the difference between the present value of a leasehold estate of £325 a year for 70 years to come, and that of a freehold or perpetuity of the same sum, reckoning compound interest at 5 per cent.? . . . . . Ans. £213, 12*s.* 7½*d.*

9. A father dying, bequeathed to the younger of his two sons the first 15 years of his freehold estate, whose annual rent was £1728, and the reversion after the 15 years were expired to his elder son; what is the present value of their respective legacies, interest at 3 per cent.?

Ans. Younger son, £20628, 15*s.*; elder, £36971, 5*s.*

10. Suppose I would add 5 years to a running lease of 15 years to come, the improved rent being £186, 7*s.* 6*d.* per annum, what ought I to pay down for this favour, reckoning compound interest at 4 per cent.? . . . . Ans. £460, 14*s.* 1$\frac{3}{4}$*d.*

11. A landlord grants a lease of 21 years, to commence 8 years hence; he receives £800 from the tenant at present, and agrees to indemnify him in the course of the lease by a regular deduction from each year's rent. What must the deduction be, allowing the tenant 5 per cent. for his money at compound interest?

Ans. £72, 4*s.* 7$\frac{3}{4}$*d.*

LIFE OR CONTINGENT ANNUITIES.

In the calculation of life-annuities, there are two things that require particular attention—1st, The probability of the continuance of the annuity; 2d, The rate of interest.

The probability of the duration of life is calculated from Tables; the table used here is that generally known as the 'Carlisle rate of mortality.' The general principle on which all the tables have been constructed is, that from a certain number of human beings whose births have been recorded, an estimate has been made of the number alive at different ages. Thus, in Table V. (p. 273), of 10000 that are born alive, 8461 survive the first year, &c.

In general, the probability of any event happening is *measured* by the ratio of the number of cases that are favourable to all the possible number of cases.

Thus, by Table V., the number living at the age of 21 is 6047; at the age of 31, 5585: the probability of a person aged 21 reaching 31 is therefore measured by the ratio 5585 : 6047, or by the fraction $\frac{5585}{6047}$; and the probability of his dying during the interval is measured by $\frac{462}{6047}$, where the numerator 462 is the number that die between the ages of 21 and 31; and so on in other cases.

Method of calculating Table VI., p. 274.

That a person may have an *absolute* right to £1 at the end of a year, he must pay down the present value of £1 at the beginning of the year; but if the receipt of the £1 at the end of the year be made dependent upon the person's living to the end of that year, it is obvious that the sum to be paid down must be that fraction of the present value which expresses the probability of his receiving it. Now, if the present values of all the successive annual payments be multiplied by the respective probabilities that the annuitant will be alive to receive them, the sum of *all* the payments which may *possibly* be received will be the value of an annuity of £1 dependent on the annuitant's life.

Thus, suppose that the rate of interest is 3 per cent.; then, according to the Carlisle table, out of 3 persons alive at 103, only 1 is alive at the end of the year; the probability of living to the end of 1 year is therefore $\frac{1}{3}$; hence

$1{\cdot}03^{-1} \times \frac{1}{3}$, or £·9708738 × $\frac{1}{3}$ = ·3236, &c., or ·324 nearly is put down as the present value of an annuity of £1 granted to a person aged 103.

To continue the calculation at 102: the probability of the person's living to 103 is $\frac{3}{5}$.

∴ £·9708738 × $\frac{3}{5}$ = ·582526, &c. = for the probability of this year.

£·9425959 × $\frac{1}{5}$ = ·188519, &c. = 〃 〃 next 〃

∴ £·771045, or ·771 is the present value of an annuity of £1 granted to a person aged 102. In this manner Table VI. is constructed. Table VII. is constructed in a somewhat similar way; the only difference is, that two probabilities instead of one are used.

Note.—We will find it convenient to denote the value of an annuity of £1, during a life aged 15, for instance, by a_{15}; during two joint lives, 10 and 15, by $a_{10, 15}$.

I. To find the present value of an Annuity on a *single* life.

Example 1.—What is the value of an annuity of £63 during the life of a person aged 32, interest at 4 per cent.?

Ans. £1042, 15*s*. 6$\frac{1}{4}$*d*.

$$\text{By Table VI., } a_{32} = 16{\cdot}552;$$
$$\therefore 16{\cdot}552 \times 63 = \text{answer.}$$

Example 2.—What is the amount of an annuity that a person aged 75 may purchase for £1000, interest at 5 per cent.?

Ans. £200, 8*s*. 9$\frac{3}{4}$*d*.

$$\text{By Table VI., } a_{75} = 4{\cdot}989;$$
$$\therefore \frac{1000}{4{\cdot}989} = \text{answer.}$$

Exercises.

1. What is the present value of an annuity of £225 during the continuance of a life aged 47, interest at 3 per cent.?

Ans. £3441, 8*s*.

2. What annuity may a person aged 60 purchase for £820, interest at 4 per cent.? . . . . Ans. £84, 17*s*. 2$\frac{1}{4}$*d*.

3. What is the value of an annuity of £75 during the life of a person whose age is 57, interest at 3 per cent.? . Ans. £871, 1s.

4. A young merchant marries a widow lady of 40 years of age, with a jointure of £256, and wants to dispose of the jointure for ready money; what sum ought he to receive, reckoning interest at 4 per cent.? . . . . . . Ans. £3858, 18s. $10\frac{1}{2}d.$

5. What annuity may a person aged 51 purchase for £7200, interest at 3 per cent.? . . . . . Ans. £516, 15s. $10\frac{3}{4}d.$

II. To find the present value of an Annuity on *two joint* lives.

Example 1.—What is the value of an annuity of £80 a year, to continue during the joint lives of two persons whose ages are 15 and 30 respectively, interest at 5 per cent.? . Ans. £1055, 12s.

$$\text{By Table VII., } a_{15, 30} = 13{\cdot}195;$$
$$\therefore £13{\cdot}195 \times 80 = \text{answer}.$$

Example 2.—An annuity of £72 is to continue during the joint continuance of the lives of two persons whose ages are 37 and 60; what is its present value, interest at 3 per cent.?

Ans. £672, 3s. 10d.

These ages not being found in the table, we can only approximate to the value of the annuity by proportion; thus,

$$a_{35, 60} = 9{\cdot}410$$
$$a_{40, 60} = 9{\cdot}224$$
$$\therefore \text{difference} = {\cdot}186$$

Now, as the difference between 35 and 40 is 5, and that between 35 and 37 is 2,

$$\therefore a_{37, 60} = 9{\cdot}410 - {\cdot}186 \times \tfrac{2}{5} = 9{\cdot}336 \text{ nearly}.$$
$$\therefore 9{\cdot}336 \times 72 = \text{answer}.$$

Exercises.

6. What is the present value of an annuity of £450 during the joint lives of two persons whose ages are 45 and 70, interest at 4 per cent.? . . . . Ans. £2750, 17s.

7. What is the present value of an annuity of £57, 10s., payable during the joint lives of a man and his wife whose respective ages are 39 and 35, interest at 3 per cent.? Ans. £815, 9s. $3\frac{1}{4}d.$

8. What is the annuity that may be purchased for £900 on the joint lives of two gentlemen aged 45 and 50, interest at 5 per cent.? . . . . Ans. £92, 8s. $7\frac{1}{4}d.$

III. TO FIND THE PRESENT VALUE OF AN ANNUITY ON THE *longer* OF TWO LIVES.

RULE.—Grant an annuity to both, on condition that a like annuity be restored so long as both persons survive; or what is the same thing, from the sum of the values on the single lives, subtract the value of the annuity during their joint lives.

Exercises.

9. What is the value of an annuity of £166, 6*s.* 8*d.* during the longer of two lives, whose ages are 27 and 35, interest at 3 per cent.? . . . . Ans. £3853, 2*s.* 2¾*d.*

10. What is the value of an annuity of £78, 15*s.* 7*d.* on the longer of two lives, whose ages are 30 and 50, interest at 5 per cent.? . . . . Ans. £1258, 16*s.* 3*d.*

11. What single payment ought a gentleman to pay down just now, to secure an annuity of £362, 14*s.* during the joint existence of himself and wife, and also during the life of the survivor, their respective ages being 50 and 42, and interest at 4 per cent.?
Ans. £6090, 1*s.* 11*d.*

IV. TO FIND THE PRESENT VALUE OF AN ANNUITY ON A LIFE A, AFTER THE FAILURE OF ANOTHER LIFE B.

RULE.—Grant an annuity to A for his whole life, on condition that he restore it so long as B is alive; or, what is the same thing, from the value of an annuity on the life A, subtract the value of an annuity during the joint lives of A and B.

If this reversion be secured by an annual premium, it will be found on dividing the single payment, by 1 increased by the present value of an annuity of £1 during the joint lives.

Example.—What is the value of an annuity of £60 a year during the continuance of the life of a person aged 25, after the death of another aged 40, interest at 4 per cent.?—and what annual premium would be required?

Ans. Pre. value, £266, 11*s.* 7¼*d.*; an. prem. £18, 15*s.* 4¾*d.*

$$\begin{aligned} a_{25} &= 17{\cdot}645 \\ a_{25,\,40} &= 13{\cdot}202 \\ & \overline{4{\cdot}443} \times 60 = 266{\cdot}580. \end{aligned}$$

$$\text{Annual premium} = \frac{266{\cdot}580}{1+13{\cdot}202} = 18{\cdot}770.$$

Exercises.

12. A person whose age is 45 is to enjoy an annuity of £438, 18*s*. 6*d*. after the death of the present annuitant, whose age is 75; what is the present value of the younger person's expectation, interest 4 per cent.? . . . . . Ans. £4062, 5*s*. 0¼*d*.

13. A father, whose age is 65, is desirous that his only daughter, whose age is 20, should enjoy an annuity of £125, 12*s*. after his decease; what sum should he at present pay down, interest at 5 per cent.?—or what annual payment during their joint lives ought to be paid? . . . . . . Ans. Single payment, £1062, 1*s*. 5¾*d*.; annual premium, £127, 0*s*. 6¼*d*.

14. A enjoys a living of £256 a year, and B wishes to purchase it for his life after A's death; what ought B to pay for it, interest at 4 per cent., A's age being 75, and B's 35?

Ans. £2843, 12*s*. 11¼*d*.

V. To find the present value of a deferred Life Annuity.

Rule.—Add the number of years the annuity is deferred to the given age. Find the present value of an annuity of £1 on a life whose age is the above sum; multiply it by the present value of £1 for the given deferred time, and by the probability of the continuance of the given life to the end of that time. The result, multiplied by the given annuity, will be the present value required.

The value of an annuity upon a given life for a number of years is found by subtracting the value of an annuity deferred the given number of years from the present value of an annuity on the whole life.

The Reason of the Rule is obvious: the rule also applies to joint lives.

Exercises.

15. What is the present value of an annuity of £73, 8*s*. 6*d*., to be entered upon after 9 years have expired, and then to continue during the life of a person whose present age is 32, interest at 3 per cent.? . . . . Ans. £861, 4*s*. 8½*d*.

$$\text{Present value} = a_{41} \times 1{\cdot}03^{-9} \times \frac{5009}{5528} \times 73{\cdot}425.$$

16. What is the present value of an annuity of £32, 15*s*., to be entered upon after 20 years, and then to continue during the joint lives of two persons whose ages are 30 and 45, interest at 5 per cent.? . . . . Ans. £41, 15*s*. 1½*d*.

$$\text{Present value} = a_{50, 65} \times 1{\cdot}05^{-20} \times \frac{4397}{5642} \times \frac{3018}{4727} \times 32{\cdot}75.$$

17. Find the value of a life-annuity of £560, secured upon a freehold property, receivable after the termination of a lease of which 7 years have to run; the age of the person being 25, and the interest of money reckoned at 4 per cent. Ans. £6628, 4s. 6d.

18. What is the value of an annuity of £85, 17s. 6d., receivable during the joint continuance of two lives of 21 and 25, but not to commence until 10 years have expired, interest at 5 per cent.?

Ans. £533, 14s. 4¼d.

19. What is the present value of an annuity of £30 for the next 15 years, dependent upon the existence of a life whose age is 19, interest at 4 per cent.? . . Ans. £316, 1¼d. nearly.

$$\text{Present value} = \left(a_{19} - a_{34} \times 1{\cdot}04^{-15} \times \frac{5417}{6133}\right) \times 30.$$

VI. To find the present value of a Perpetuity to be entered upon after the failure of a given Life or Lives.

Rule.—From the present value of the perpetuity subtract the present value of an annuity on the given life or lives; the remainder will be the value of the perpetuity in reversion.

The Reason of the rule is obvious.

Exercises.

20. What is the value of a freehold estate of £1721, 10s. yearly rent, to be entered upon after the decease of the present possessor, whose age is 64, interest at 4 per cent.? . Ans. £28244, 13s.

21. What is the value of a freehold estate of £1721, 10s. yearly rent, to be entered upon after the failure of one of two lives, whose ages are 35 and 40, interest at 3 per cent.?

Ans. £33199, 14s. 0¼d.

VII. To find the present value of a given sum payable on the failure of a given Life or Lives.

Rule.—Multiply the value of an annuity of £1 on the given life or lives by the interest of £1 for a year, and subtract the product from 1; multiply the remainder by the present value of £1 for a year, and then by the given sum; the result will be the value in a single payment. The annual payment is found by dividing the single payment by the value of an annuity on the given life or lives increased by 1.

Or, Subtract the present value of £1 for 1 year from 1, and multiply the remainder by the value of an annuity of £1 on the given life or lives; subtract this product from the present value of £1 for 1 year, and the remainder, multiplied by the given sum, will be the value in a single payment.

This is called an assurance on the given life or lives. The annual premiums are in all cases supposed to be payable in advance.

Example.—What ought a gentleman, whose age is 36, to pay just now to secure to his heirs £700 payable at his death?—and what ought the annual payments to be, during his life, to assure the same, interest at 5 per cent.?

First Method.

a_{36} =	13·987
	·05
Subtract	·69935
from	1·
	·30065
$1·05^{-1}$ inverted,	83259·
	270585
	15033
	601
	90
	24
	·286333
Given sum inverted,	007
	200·4331
Single payment,	£200, 8*s.* 8*d.*

Second Method.

	1·
$1·05^{-1}$ =	·952381
	·0476190
a_{36} inverted,	789·31
	476190
	142857
	42857
	3810
	333
	·666047
$1·05^{-1}$ =	·952381
	·286334
	007
	200·4338
	£200, 8*s.* 8*d.*

$$\text{Annual payment} = \frac{200·433}{14·987} = 13·373 = £13,\ 7s.\ 5\tfrac{1}{4}d.$$

Reason of the Rule.—This Rule may be made to depend upon Rule VI., by converting the sum assured into a perpetual annuity, the amount of which will manifestly be the interest of the sum assured for 1 year. In the case of a perpetual annuity, however, the first payment is supposed to be made at that term next succeeding the death of the present occupant; but in the case of a sum assured, payable at the same term, one year would require to elapse before the first year's interest could be drawn: hence the present value of a reversionary perpetuity, equal to the interest of the sum assured, is more valuable than the present value of the reversionary sum assured by one year's purchase. It follows, then, that the result obtained by last rule, multiplied by the present value of £1 for 1 year, will be the present value of the sum assured in a single payment.

Thus, in the Example, suppose the sum assured is £1: since the rate per cent. is 5, ∴ the present value of a perpetuity of £1 is

$\frac{1}{\cdot 05}$; hence, by Rule VI.,

the present value of a reversionary perpetuity of £1 $= \frac{1}{\cdot 05} - a_{36}$.

Now, if $\frac{1}{\cdot 05} - a_{36}$ be multiplied by ·05, the interest of £1 for a year, we get $1 - \cdot 05 \times a_{36}$. For the reason above stated, this requires to be multiplied by $1\cdot 05^{-1}$, to obtain the present value of £1 assured on a life aged 36; that is,

$$\text{present value} = (1 - \cdot 05 \times a_{36}) \times 1\cdot 05^{-1}.$$

Hence the first Rule: the second is only a modification of this.

Exercises.

22. What single premium, or what annual premium, would be required to secure £739, 7*s.* 6*d.*, payable at the end of the year in which the existence of a person now aged 29 shall fail, interest at 3 per cent.? Ans. Single premium, £292, 5*s.* 7¾*d.*; annual premium, £14, 1*s.* 6¾*d.*

23. What is the value of £2300 payable on the failure of two joint lives, aged 55 and 60, interest at 4 per cent.? Ans. £1541, 10*s.* 7¼*d.*

24. What annual premium must be paid during the joint existence of two lives aged 25 and 30, to secure the payment of £869, 13*s.* 6*d.* after the extinction of the longer of the two lives, interest at 4 per cent.? . . . Ans. £10, 11*s.* 2¼*d.*

25. What ought a person aged 51 to pay annually to assure £1000 on his life, interest at 5 per cent.? . Ans. £32, 19*s.* 2¾*d.*

26. What ought a person aged 30 to pay annually for an assurance of £350 on his life, interest at 3 per cent.? Ans. £6, 16*s.* 7¾*d.*

Miscellaneous Exercises.

1. A father is desirous of providing a dowry of £1560 for his daughter on her arriving at the age of 21; what sum should he pay to secure it, interest at 4 per cent., his daughter's age at present being 12? . . . . . . Ans. £1035, 11*s.* 7¼*d.*

2. What is the annual premium during the joint existence of two lives aged 45 and 50, for an assurance of £7000, payable on the death of the last survivor, interest at 3 per cent.? Ans. £239, 0*s.* 2¼*d.*

3. Required the single payment for an annuity of £120 to a lady aged 37, after the decease of her husband, aged 40, in the event of her surviving him, interest at 5 per cent. Ans. £816, 8*s.* 9¼*d.*

4. Required the annual premium payable during marriage, for an annuity of £400 on the life of a lady aged 30, in the event of her surviving her husband, whose age is 35, interest at 4 per cent. Ans. £92, 15*s*. 5¾*d*.

5. What is the price of an annuity of £250 during the joint lives of a gentleman aged 70 and a lady aged 55, interest at 3 per cent.? . . . . Ans. £1504, 15*s*.

6. What is the present value of the next presentation to a living, the age of the present incumbent being 86, the rate of interest 4 per cent.; assuming that the age of the presentee is 30, and that the annual value of the living is £420? Ans. £5873, 14*s*.

7. A gentleman assures his life for £2000; what annual premium ought he to pay, supposing that he pays £500 just now, that his age is 47, and that the rate of interest is 4 per cent.? Ans. £25, 7*s*. 7¾*d*.

8. What is the value of a perpetual annuity of £827, to be entered upon after the decease of a lady aged 61, interest at 5 per cent.? . . . . Ans. £9335, 3*s*. 6¼*d*.

9. A party proposed to lay out £400 in the purchase of an annuity, to be entered on at the end of 9 years, and to continue so long as a life, now aged 32, shall survive that time; what sum per annum will he be entitled to, interest at 3 per cent.? Ans. £34, 2*s*. 0½*d*.

10. A society of married men pay each £10 of entry, besides an annual contribution during marriage, depending upon their ages at entering; what ought a man aged 48 to contribute annually to secure an annuity of £75 for his wife, aged 30, in the event of her surviving him, interest at 4 per cent.? . Ans. £23, 4*s*. 11½*d*.

11. A person aged 40 is desirous of purchasing an annuity of £200 upon his life, to commence on his arriving at the age of 65, and to be paid for by an annual premium until that time; what is the annual premium, the rate of interest being 4 per cent.? Ans. £26, 1*s*. 2¼*d*.

12. A man and his wife, aged 35 and 27, wish to sink £2000 upon two annuities; one during their joint lives, and another, of half the value, during the remainder of the surviving life: what annuities ought to be granted, interest at 5 per cent.? Ans. During their joint lives, one for £137, 0*s*. 5¼*d*.; during the surviving life, one for £68, 10*s*. 2¾*d*.

EXCHANGE.

Exchange is the rule by which sums in the money of one country are converted into sums of equivalent value, in the money of another.*

Par of exchange means that sum in the currency of one country which, in *intrinsic* or *real* value, is equal to a given sum estimated in the currency of another. Thus, according to the mint regulations of Great Britain and France, £1 sterling is equal to 25 fr. 22 cents: this is called the par of exchange.

The course of exchange, at any date, means that sum in the currency of one country which *at that date* is equal to a given sum estimated in the currency of another. The course of exchange is seldom at par, but is sometimes above, and sometimes below par, according to circumstances.

In the calculation of exchanges, a *variable* sum in one country is allowed for a *certain* sum in another. Thus, London gives France £1 sterling for a *variable* number of francs, and Portugal a variable number of pence for 1 milrea.

The calculations in Exchange are performed by Simple Proportion and Practice. The pupil will find no difficulty in the solution of any of the exercises.

MEASURES OF FOREIGN MONEY.

Austria.—4 *pfennings* = 1 *creutzer*, 60 *creutzers* = 1 *florin*. The money of account and exchange is the florin, which is equal to 2*s*. 0¼*d*. sterling nearly; the par of exchange with London is 9 fl. 50 cr. per £1 sterling.

Austrian Italy.—100 *centisimi* = 1 *lira Austriacha*. The money of account and exchange is the lira Austriacha, which is equal to 8⅛*d*. sterling nearly; the par of exchange with London is 6 lire Austriache per 48¾*d*. sterling.

Canada.—100 *cents* = 1 *dollar*. The dollar is equal to about 4*s*. 1¼*d*. sterling. In exchange, the dollar is estimated at 4*s*. 6*d*. sterling, and bills on London sell at a premium of 9¼ per cent. Accounts were formerly kept in pounds, shillings, and pence *currency*: £100 currency were equal to £90 sterling, giving a premium for bills on London.

France.—100 *centimes* = 1 *franc*. The money of account and exchange is the franc, which is equal to 9½*d*. sterling nearly; the par of exchange with London is 25 fr. 22 ct. per £1 sterling.

* For full information, see Chambers's *Commercial Tables*, pp. 242 to 253.

FRANKFORT-ON-THE-MAINE.—4 *creutzers* = 1 *batze*, 15 *batzen* = 1 *florin*, 1½ *florins* = 1 *rixdollar* current. The batze is equal to 1⅝*d.* sterling nearly; the par of exchange with London is 148 batzen per £1 sterling.

GENOA.—100 *centisimi* = 1 *lira nuova*. The lira nuova is equal to the French franc, or 9½*d.* sterling nearly.

HAMBURG.—12 *pfennings* = 1 *schilling*, 16 *schillings* = 1 *mark*, 3 *marks* = 1 *rixdollar* or *reichsthaler*. These denominations have different values, according as they represent *current* or *banco* money: the difference in value between sums reckoned in banco and current money is called *agio*, which varies from 20 to 25 per cent. The mark banco is equal to 1*s.* 5½*d.* sterling nearly; and the par of exchange with London is 13 mk. ban. 10½ sch. per £1 sterling.

HOLLAND.—100 *cents* or 20 *stivers* = 1 *florin* or *guilder*. The florin is equal to 1*s.* 8*d.* sterling nearly; the par of exchange with London is 12 florins per £1 sterling.

INDIA.—12 *pice* = 1 *anna*, 16 *annas* = 1 *rupee*. The *Company's rupee* is equal to 1*s.* 10¼*d.* sterling; accounts are also stated in the *sicca rupee*, which is worth 1*s.* 11¾*d.* or nearly 2*s.* sterling. Exchanges are effected in both rupees.

NAPLES.—10 *grani* = 1 *carlino*, 10 *carlini* = 1 *ducato di regno*. The ducat is equal to 3*s.* 3¾*d.* sterling nearly; the par of exchange with London is 1 ducat per 39¾*d.* sterling.

PORTUGAL.—400 *reas* = 1 *crusado of exchange*, 480 *reas* = 1 *new crusado*, 1000 *reas* = 1 *milrea*. The milrea is equal to 4*s.* 9½*d.* sterling nearly; the par of exchange with London is 1 milrea per 57½*d.* sterling.

PRUSSIA.—12 *pfennings* = 1 *silver groschen*, 30 *silver groschen* = 1 *dollar*, or *thaler*. The Prussian dollar is equal to 2*s.* 10¾*d.* sterling nearly.

RUSSIA.—100 *copecs* = 1 *silver ruble*. The silver ruble is equal to 3*s.* 1½*d.* sterling; the par of exchange with London is 6 ru. 40 cop. per £1 sterling.

UNITED STATES.—100 *cents* = 1 *dollar*, which is denoted thus, $1. The dollar is equal to 4*s.* 1¾*d.* sterling nearly. In exchange, the dollar is estimated at 4*s.* 6*d.* sterling, and bills on London sell at a premium of about 9 per cent.

Exercises.

1. How many francs must be paid at Paris to receive £160 in London, exchange 25 fr. 70 ct. per £1 sterling? . Ans. 4112 fr.

2. A merchant in London remits £750, 15*s.* to his correspondent in Paris; what is the value in French money, exchange 25 fr. 40 ct. per £1 sterling? . . . Ans. 19069 fr. 5 ct.

3. A gentleman in Portugal being desirous to remit to his correspondent in London 6470 milreas, for what sum in sterling money will he be credited in London, exchange 4*s.* 4*d.* sterling per milrea? . . . . Ans. £1401, 16*s.* 8*d.*

4. Remitted from London to Rotterdam a bill for £7600, 18*s.* sterling; what is the equivalent sum in Dutch money, exchange 12 fl. 5½ stiv. per £1 sterling? . . . Ans. 93301 fl. 4¾ ct.

5. A merchant in Amsterdam remits 6854 florins 64 cents to be paid in London, how much sterling money must he draw for, exchange 12 fl. 2¾ stiv. per £1 sterling? Ans. £564, 14*s.* 11½*d.* $\frac{5}{971}$.

6. If I pay here a bill of £2500, for what may I draw my bill at Hamburg, exchange 13 mk. ban. 12¾ sch. per £1 sterling?

Ans. 34492 mk. 8 sch.

7. A merchant in Quebec wishes to remit to London 5110 dollars; for what sum in sterling money must the bill be drawn when bills on London sell at a premium of 9⅓ per cent.?

Ans. £1050.

8. A merchant in New York wishes to pay a debt of £760 to his correspondent in England; how many dollars must he deposit with his banker to discharge this debt, reckoning the dollar at 4*s.* 6*d.* sterling, and bills on London selling at a premium of 10 per cent.? . . . . Ans. \$3715, 55$\frac{5}{9}$.

9. If I pay a bill in London of £1600, for what must I draw on my correspondent at Oporto, exchange 4*s.* 10*d.* per milrea?

Ans. 6620 mil. 689$\frac{19}{29}$ reas.

10. A factor has sold goods at Naples for 6230 ducati di regno; what is the value in sterling money, exchange 3*s.* 11*d.* per ducato?

Ans. £1220, 0*s.* 10*d.*

11. What is the value, in Austrian money, of goods sold in London for £375, 18*s.* 6*d.*, exchange 10 fl. 3 cr. per £1 sterling?

Ans. 3778 fl. 2$\frac{31}{40}$ cr.

12. How many pounds sterling are equal to 7832 fl. 45 cr., exchange 9 fl. 50 cr. per £1 sterling? . Ans. £796, 11*s.* 0$\frac{0}{4}$*d.* $\frac{48}{59}$.

13. Change 8643 lire Austr. 75 centisimi to sterling money, exchange 50½*d.* per 6 lire Austr. . . . Ans. £303, 2*s.* 7½*d.* ¼.

14. A merchant in London remits to his correspondent in Venice £623, 11*s.* 6*d.*; how much will his correspondent draw from his banker there, when the exchange is 47*d.* per 6 lire Austr.? . . . . Ans. 19105 lire Austr. 27$\frac{31}{47}$ cent.

15. Change 7467 fr. 80 centimes to sterling money, exchange 25 fr. 46 cent. per £1 sterling, . . Ans. £293, 6*s.* 3½*d.* $\frac{514}{1273}$.

16. How much French money is equivalent to £840, 11*s.* 6*d.* sterling, exchange 25 fr. 25 cent. per £1 sterling?

Ans. 21224 fr. 51⅞ cent.

17. A merchant at Frankfort-on-Maine has to remit to London 1253 fl. 10 batzen; for what sum must he be credited in London, exchange 150¾ batzen per £1 sterling? Ans. £124, 14*s.* 10¼*d.* $\frac{47}{201}$.

18. How many lire nuove are equivalent to £572, 10*s.* 6*d.* sterling, exchange 25 lire nuove 80 cent. per £1 sterling?

Ans. 14771 lire nuove 14½ cent.

19. How much British money is equal to 6052 lire nuove 75 cent., exchange 25 lire nuove 70 cent. per £1 sterling?
Ans. £235, 10*s*. $3\frac{1}{2}$*d*. $\frac{242}{257}$.

20. How many rupees are equivalent to £638, 17*s*. 5*d*., exchange 2*s*. $1\frac{1}{2}$*d*. per rupee? . . . . . Ans. 6012 rup. $14\frac{22}{51}$ an.

21. Change 7965 rup. 12 an. to British money, exchange 1*s*. $11\frac{3}{4}$*d*. per rupee, ... Ans. £788, 5*s*. $6\frac{1}{2}$*d*. $\frac{1}{4}$.

22. Change £712, 16*s*. 8*d*. to Russian money, exchange 6 ru. 60 cop. per £1 sterling, ... Ans. 4704 ru. 70 cop.

23. How much sterling money is equal to 3296 ru. 85 cop., exchange 6 ru. 40 cop. per £1 sterling? . Ans. £515, 2*s*. $7\frac{3}{4}$*d*. $\frac{1}{2}$.

24. Required the value, in Hamburg bank money, of a bill for £3276, 15*s*. 6*d*., exchange 13 mk. $12\frac{1}{2}$ sch. per £1 sterling,
Ans. 45158 mk. $0\frac{71}{80}$ sch.

25. Find the value, in British money, of goods sold for 9127 mk. 13 sch., when the exchange is 13 mk. $9\frac{3}{4}$ sch. per £1 sterling,
Ans. £670, 14*s*. $0\frac{3}{4}$*d*. $\frac{288}{871}$.

26. How many American dollars are equivalent to £5827, 14*s*. 8*d*. sterling, reckoning the dollar at 4*s*. 6*d*. sterling, and bills on London selling at a premium of 8 per cent.? . Ans. $25573, 12.

27. How much sterling money is equivalent to $3239, 24, reckoning the dollar at 4*s*. 6*d*. sterling, and bills on London selling at a premium of 4 per cent.? Ans. £1782, 10*s*. $6\frac{1}{2}$*d*. $\frac{10}{13}$.

POSITION.

Position is the rule by which, on making use of *assumed* numbers, and working with them as if they were the true ones, we discover the number that answers certain conditions proposed in a question. It is divided into **Single Position** and **Double Position**.

I. Single Position.

Single Position is employed when the required number is increased or diminished by some part of itself, or when it is multiplied or divided by a given number; the answer will in this case be obtained by making *one supposition;* hence the name *Single Position.*

Rule—1. Assume any number to be the number sought, and perform on it the operations indicated in the question.

2. Then say, *as* the result obtained *is to* the true result, *so is* the assumed number *to* the number required.

The Rule is merely a particular application of Simple Proportion.

Example.—What number is that which, increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, makes a sum equal to 625?

		Proof.
Suppose 60		300
$\frac{1}{2}$ of 60 = 30		$\frac{1}{2}$ of 300 = 150
$\frac{1}{3}$ " 60 = 20		$\frac{1}{3}$ " 300 = 100
$\frac{1}{4}$ " 60 = 15		$\frac{1}{4}$ " 300 = 75
$\therefore$ 125 : 625 :: 60 : 300		625

Exercises.

1. What number is that which, multiplied by 2, and the product divided by 7, gives 16 for a quotient? . . Ans. 56.

2. A's age is double of B's, and B's is triple of C's, and the sum of all their ages is 150; what is each person's age?
Ans. A's age is 90; B's, 45; C's, 15.

3. A gentleman had a certain sum of money in his purse; he gave one half of what he had to his tailor, one half of what remained to his shoemaker, and found that he had £8, 15*s.* left. How much money had he at first? . . . . Ans. £35.

4. Required a number to which, if $\frac{1}{2}$ and $\frac{1}{3}$ of itself be added, and then $\frac{1}{4}$ of the sum subtracted, the remainder shall be 106,
Ans. 160.

5. A sum of money, amounting to £1000, is to be divided among A, B, and C, in such a manner that the $\frac{1}{2}$ of A's share, the $\frac{1}{3}$ of B's, and the $\frac{1}{3}$ of C's, are to be equal. Find their respective shares,
Ans. A's share, £266, 13*s.* 4*d.*; B's, £333, 6*s.* 8*d.*; C's, £400.

6. A market-woman bought a certain number of eggs at 2 a penny, and as many more at 3 a penny, and afterwards sold them all at the rate of 5 for twopence. She then found that she had lost 8*d.* by the transaction. How many eggs did she purchase?
Ans. 480.

II. Double Position.

Double Position is employed when the required number is increased or diminished by some part or multiple of itself, and also by some number which is no known part or multiple of itself. *Two suppositions* are used; hence the name *Double Position.*

Rule—1. Assume any two numbers, perform on them separately the operations indicated in the question, and find the *errors*; marking an error of *excess* by +, an error of *defect* by −.

2. Then say, *as* the *difference* of the errors, when they are both +, or both −; or, *as* the *sum* of the errors, when one is +, and the other −, *is to* the *difference* of the assumed numbers,

so is either error to the correction of the supposition which produces that error. This correction added to the supposition, when it is too small, but subtracted from it when it is too great, will give the number required.

The Rule is founded on the supposition that the 'differences between the true and assumed numbers are proportional to the differences between the true and erroneous results.' This principle is *accurately* true for some questions, but only *approximately* so for others.

Example.—Find a number such that, if 27 be subtracted from its half, $\frac{2}{3}$ of the remainder may be equal to 43.

$$\begin{array}{lr}
\text{Suppose} & 60 \\
\tfrac{1}{2} \text{ of } 60 = & 30 \\
\text{Subtract} & \underline{27} \\
 & 3 \\
 & 2 \\
 & 3\overline{)6} \\
 & 2 \\
 & \underline{43} \\
 & -41
\end{array}
\qquad\qquad
\begin{array}{lr}
\text{Suppose} & 72 \\
\tfrac{1}{2} \text{ of } 72 = & 36 \\
\text{Subtract} & \underline{27} \\
 & 9 \\
 & 2 \\
 & 3\overline{)18} \\
 & 6 \\
 & \underline{43} \\
 & -37
\end{array}$$

$\therefore$ 41 − 37 : 72 − 60 : 41 : 123 correction of the first supposition.
60 first supposition.

$\therefore$ 183 is the number required.

Exercises.

7. It is required to find such a number that, if 5 be subtracted from it, $\frac{7}{8}$ of the remainder shall be 35, . . . Ans. 45.

8. A post is $\frac{1}{4}$ in mud, $\frac{1}{3}$ in water, and 10 feet above the water. What is the length of the post? . . . . . Ans. 24 feet.

9. What number is that, which multiplied by 21, the product diminished by 39, and the remainder divided by 17, gives the number itself for the quotient? . . . . . . Ans. $9\frac{3}{4}$.

10. A merchant has spirits at 9*s.* and 13*s.* per gallon; how many gallons of each must he take to make a mixture of 100 gallons, so that the mixture may be worth 12*s.* a gallon?
Ans. 25 at 9*s.*; 75 at 13*s.*

11. A sum of money, amounting to £1000, is to be divided among A, B, and C, in such a manner that B is to have twice as much as A, and £6 more, C as much as A and B together, and £20 more. Find their respective shares,
Ans. A gets £161, 6*s.* 8*d.*; B, £328, 13*s.* 4*d.*; C, £510.

12. A labourer engaged to serve for 30 days, on condition that for every day he worked he should receive 4*s.*, but for every day he was idle he should forfeit 5*s.* At the end of the time he had to receive £1, 19*s.* How many days did he work? Ans. 21 days.

ALLIGATION.

Alligation teaches how to compound or mix together several Simples of different qualities, so that the Composition may be of some intermediate quality. It is commonly distinguished into **Alligation Medial**, and **Alligation Alternate**.

ALLIGATION MEDIAL.

Alligation Medial is the method of finding the rate or quality of the Composition, from having the quantities and rates or qualities of the several simples given. It is performed as follows:

Rule.—Multiply the quantity of each ingredient by its rate or quality; add all the products together into one sum, and all the quantities into another sum; then divide the sum of the products by the sum of the quantities—and the quotient will be the required rate or quality of the composition.

Example.—If four different qualities of gunpowder be mixed together—namely, 60 lb. at 12*d.* per pound, 50 lb. at 10*d.*, 36 lb. at 9*d.*, and 30 lb. at 8*d.* per pound; what is the value of a pound of the composition?

Here 60, 50, 36, and 30 are the quantities, and 12, 10, 9, and 8 are the rates or qualities; hence

$$\begin{array}{lcr}
60 \times 12 & \ldots\ldots = & 720 \\
50 \times 10 & \ldots\ldots = & 500 \\
36 \times 9 & \ldots\ldots = & 324 \\
30 \times 8 & \ldots\ldots = & \underline{240} \\
176) & \ldots\ldots & 1784(10\tfrac{3}{22}d. \\
 & & \underline{1760} \\
 & & 24
\end{array}$$

$$\frac{24}{176} = \frac{3}{22}.$$

Therefore, the rate or price of the composition is $10\frac{3}{22}d.$ per pound.

The Reason of the rule is obvious from the example, for each of the products is the price in pence of the given quantity from which it arises; hence the sum of the products is the price of the composition in pence, which, being divided by the number of pounds in it, gives the price of one pound of the mixture in pence.

Exercises.

1. A grocer mixes 24 lb. of tea at 6*s.* a pound, with 20 lb. at 5*s.*, and 30 lb. at 4*s.* 6*d.* a pound; what was the value of a pound of the mixture? . . . . Ans. 5*s.* $1\frac{1}{4}$*d.* $\frac{31}{37}$.

2. A composition being made of 10 gallons at 7*s.*, 18 gal. at 8*s.* 6*d.*, and 15 gal. at 5*s.* 10*d.*; what is a gallon of the mixture worth? . . . . Ans. 7*s.* $2\frac{1}{2}$*d.* $\frac{26}{43}$.

3. Mixed 4 lb. of tea at 4*s.* 10*d.* per pound, with 10 lb. at 5*s.* 3*d.* per pound, and 16 lb. at 5*s.* 8*d.* per pound; what is the value of a pound of this mixture? . . . . . . Ans. 5*s.* 5*d.*

4. Having mixed 18 gallons of spirits at 12*s.* 6*d.* per gallon, with 24 gal. at 10*s.* 6*d.*, 12 gal. at 9*s.*, and 30 gal. at 8*s.* 6*d.*; I wish to know what a gallon of the mixture is worth, . Ans. 10*s.*

5. Having melted together 7 lb. of gold of 22 carats fine, with 10 lb. at 21 carats fine, and 19 lb. at 19 carats fine; what is the fineness of the composition? . . . Ans. $20\frac{5}{36}$ carats fine.

6. Find the average price of 25 qr. wheat at 40*s.*, 36 at 44*s.*, 16 at 48*s.*, 15 at 54*s.*, and 18 at 60*s.* . . Ans. £2, 7*s.* $7\frac{3}{4}$*d.* $\frac{23}{55}$.

ALLIGATION ALTERNATE.

Alligation Alternate is the method of finding what quantity of any number of Simples, whose rates or prices are given, will compose a Mixture of a given rate or price, intermediate between the rates of the simple quantities.

Rule.—Set the rates or prices of the Simples in a column under each other. Link, or connect with a line, the rate of each Simple which is *less* than that of the mixture, with one or more of those that are greater than the mixture; and connect each *greater* rate with one or more of the less.

Write the *difference* between the rate of the mixture and the rate of each simple, opposite the rate or rates with which each is linked. Then if only *one* difference stand opposite a rate, it will be the quantity belonging to that rate; but if there be *several* differences, their sum will be the quantity belonging to the rate.

Questions in this rule often admit of a variety of correct answers, which are obtained by linking the rates in a variety of different ways. This will be evident from the following example:

Example.—A merchant would mix wines at 14*s*., 19*s*., 15*s*., and 22*s*. per gallon, so that the mixture may be worth 18*s*. What quantity of each may he take?

Ans. 1.

$$18s.\left\{\begin{array}{l|}14 & 1\\ 15 & 4\\ 19 & 4\\ 22 & 3\end{array}\right.$$

Ans. 2.

$$18s.\left\{\begin{array}{l|}14 & 4\\ 15 & 1\\ 19 & 3\\ 22 & 4\end{array}\right.$$

Ans. 3.

$$18s.\left\{\begin{array}{ll}14 & 1+4=5\\ 15 & 1=1\\ 19 & 3+4=7\\ 22 & 4=4\end{array}\right.$$

Ans. 4.

$$18s.\left\{\begin{array}{ll}14 & 4=4\\ 15 & 1+4=5\\ 19 & 3=3\\ 22 & 3+4=7\end{array}\right.$$

Ans. 5.

$$18s.\left\{\begin{array}{ll}14 & 1+4=5\\ 15 & 4=4\\ 19 & 4=4\\ 22 & 4+3=7\end{array}\right.$$

Ans. 6.

$$18s.\left\{\begin{array}{ll}14 & 1=1\\ 15 & 1+4=5\\ 19 & 3+4=7\\ 22 & 3=3\end{array}\right.$$

Ans. 7.

$$18s.\left\{\begin{array}{ll}14 & 1+4=5\\ 15 & 1+4=5\\ 19 & 3+4=7\\ 22 & 3+4=7\end{array}\right.$$

The Reason of the rule may be shewn thus: Take any two quantities which result from one linking, as, for instance, 4 and 3, in the first solution, which result from the linking of 15*s*. with 22*s*.; since 15*s*. is less than 18*s*. by 3*s*., the gain on 4 gallons is 3*s*. × 4 = 12*s*., and since 22*s*. is greater than 18*s*. by 4*s*., the loss on 3 gallons is 4*s*. × 3 = 12*s*.; therefore the gain and loss are equal. In the same manner, it may be shewn that the gain and loss on the quantities resulting from any other linking are equal, and hence the gain and loss on the aggregate of the linkings are equal.

Note 1.—When the whole composition is limited to a certain quantity, find an answer by linking; then say, *as* the sum of the quantities thus determined *is to* the given quantity, *so is* the quantity of each ingredient found by linking, *to* the required quantity of each.

Note 2.—When one of the ingredients is limited to a certain quantity, link as before; then say, *as* the quantity found of that which is limited, *is to* the limited quantity, *so is* any other quantity found *to* the quantity of it to be taken to form the mixture.

Exercises.

7. A merchant would mix wines at 16*s*., at 18*s*., and at 23*s*. per gallon, so that the mixture may be worth 20*s*. per gallon; what quantity of each must he take?

Ans. 3 gal. at 16*s*., 3 gal. at 18*s*., and 6 gal. at 23*s*., or 1, 1, and 2, or any other quantities in the same proportion.

8. How much corn at 2*s.* 8*d.*, 3*s.* 4*d.*, 3*s.* 8*d.*, and 4*s.* per bushel, must be taken to form a compound worth 3*s.* 6*d.* per bushel?

Ans. 6, 2, 2, and 10 or 2, 8, 12, 2, or 2, 6, 10, 2.

9. How many pounds of sugar at 4*d.*, at 6*d.*, and at 7½*d.* per lb. must be mixed together to form a mixture at 5¾*d.* per lb.?

Ans. 8 lb., 7 lb., and 7 lb.

10. A goldsmith has four sorts of gold—namely, of 24 carats fine, of 22, 20, and 15 carats fine, and wishes to mix of each sort together so as to have 63 oz. of 17 carats fine. How much must he take of each sort? Ans. 6 oz. of 24, 6 oz. of 22, 6 oz. of 20, and 45 oz. of 15 carats fine.

11. A distiller would mix 48 gallons of French brandy at 18*s.* per gallon, with British at 10*s.* 6*d.*, and spirits at 6*s.* per gallon. What quantity of each sort must he take that the mixture may be worth 12*s.* per gallon? Ans. 48 gallons French brandy, 48 British, and 36 of spirits; or 48 gallons French brandy, 64 British, and 32 of spirits, &c.

12. A grocer has teas at 5*s.*, 4*s.* 6*d.*, and 3*s.* per pound, from which he makes up two parcels; the first contained 28 lb. at 4*s.* per lb., and the second 56 lb. at 4*s.* 3*d.* How many pounds of each sort was taken to form the parcels? Ans. 8 lb. at 5*s.*, 8 lb. at 4*s.* 6*d.*, and 12 lb. at 3*s.*, to form the first; 20 lb. at 5*s.*, 20 lb. at 4*s.* 6*d.*, and 16 lb. at 3*s.* per lb., to form the second.

13. A farmer would mix 30 bushels of wheat at 6*s.* per bushel with other sorts worth 5*s.*, 4*s.* 6d., and 4*s.* per bushel, so that the mixture may be worth 4*s.* 9*d.* per bushel. How many bushels of each sort may he take? Ans. 30 bushels at 6*s.*, 50 bushels at 4*s.*, and any equal quantities at 5*s.* and 4*s.* 6*d.*

DUODECIMAL MULTIPLICATION, OR MENSURATION.

Duodecimal Multiplication is that which is employed in the measurement of walls, flooring, &c.; and solid bodies, such as logs of wood, in which feet, inches, and their subdivisions are multiplied together, to ascertain the required dimensions.

It is so named from *duodecim*, a Latin word, signifying twelve, because, in multiplying the feet, inches, &c., the number *carried* from one denomination to another is 12, instead of 10, as in decimal multiplication.

In measurement, some line of a determinate length is employed, as *an inch*, *a foot*, *a yard*, &c. The assumed line is called the *unit of measure*.

The number of times that the unit of measure is contained in any line is called its *length* or its *measure*.

Surface has two dimensions—length and breadth; the unit of measure for surfaces is the *Square* described on the unit of length. Thus, the unit may be *a square inch, a square foot, a square yard*, &c.

The number of times that any surface contains the unit of measure is called its *Area*, or its *Content*. Thus, if a surface contain a square foot 30 times, its area is 30 square feet.

A solid has three dimensions—length, breadth, and thickness; the unit of measure for solids is the *Cube* described on the unit of length. Thus, the unit may be *a cubic or solid inch, a cubic or solid foot, a cubic or solid yard*, &c.

The number of times that any solid contains the unit of measure is called its *Volume*, or *Solidity*. Thus, if a solid contain a cubic or solid inch 30 times, its solidity is 30 solid inches.

Note.—For further definitions, rules, and their demonstrations, the pupil is referred to the *Treatise on Practical Mathematics* of this Course.

RULE FOR MULTIPLICATION.

1. Place the *feet* of the multiplier below the lowest denomination of the multiplicand; the inches, one place further to the right; the seconds, beyond the inches; and so on.
2. Multiply each denomination of the given quantity, by the *feet* of the multiplier, as in Compound Multiplication; the *twelves* being taken out of each product, and carried to the denomination above it, and the remainder written below the denomination multiplied.
3. Multiply each denomination in the same way, by the *inches* of the multiplier, carrying the twelves as before, but writing the remainders of each product one place further to the *right*, than those in the previous line; the first remainder of the product being thus written immediately below the *inches* of the multiplier.
4. Multiply in the same way by the seconds, and so on; always writing the remainders in each new line of products, one place further to the right than those in the previous line—the first remainder in each line, being thus written immediately below the figure used as the multiplier.
5. Then add all the products together for the answer, which is in *square* or *cubic* measure, as the case may be.

THE PRODUCT of feet, inches, &c., multiplied by feet, inches, &c., is expressed in *feet, firsts, seconds, thirds;* and so on. A *first* is 1-12th of a square foot; a *second*, 1-12th of a first, &c., each denomination being 1-12th of that preceding it. Each lower denomination is written a place further to the right than the one above it.

In multiplying by inches, seconds, &c., each *product* is of a lower denomination than the number multiplied, as shewn below, and must be converted to the highest denomination that it admits of, before being written down. Thus, in multiplying 346 feet by 4 seconds, the product is 1384 seconds (feet multiplied by seconds producing seconds); these are therefore divided by 12, to convert them to *firsts;* the answer is 115 firsts, and 4 seconds over; the 115 firsts are then divided by 12, to convert them to feet, making 9 feet and 7 firsts over.

The following are the duodecimal divisions of feet, used to express the product of a multiplication:

1 Square Foot	= 12 firsts.	1 Third, marked 1‴	= 12 fourths.
1 First, marked 1′	= 12 seconds.	1 Fourth 〃 1⁗	= 12 fifths.
1 Second 〃 1″	= 12 thirds.	1 Fifth 〃 1′⁗	= 12 sixths.

The result of multiplying by feet, inches, and seconds, is as follows:

Feet multiplied	by	feet	give	feet.
〃	〃	〃	inches	〃	firsts.
〃	〃	〃	seconds	〃	seconds.
Inches	〃	〃	feet	〃	firsts.
〃	〃	〃	inches	〃	seconds.
〃	〃	〃	seconds	〃	thirds.
Seconds	〃	〃	seconds	〃	fourths.

Examples.—Multiply 1 ft. 8 in. 4 sec. by 1 ft. 2 in. 6 sec.; and 346 ft. 6 in. 5 sec. by 2 ft. 3 in. 4 sec.

(1.)

	ft.	*in.*	*sec.*		
	1	8	4		
			1	2	6
	1	8	4		
		3	4	8	
			10	2	
sq. ft.	2	0′	6″	10‴	

(2.)

	ft.	*in.*	*sec.*		
	346	6	5		
			2	3	4
	693	0	10		
	86	7	7	3	
	9	7	6	1	8
sq. ft.	789	3′	11″	4‴	8⁗

Here it must be remembered, in multiplying by inches and seconds, that the products are of lower denominations than the numbers multiplied, and must be converted to their highest denominations before the figures are written in the product: thus, in example 2, the 346 feet multiplied by 3 inches produce firsts, which must be converted into feet; and the 346 feet multiplied by 4 seconds produce seconds, which must be converted into firsts, and then into feet, as explained above, before the figures are written in the product.

In square measure, the answer is in *square* feet, firsts, seconds, &c. In expressing the product of a multiplication, the denomination next to feet is often erroneously termed *inches;* but although the number stands below the inches of the multiplicand, it does not express inches, but *firsts;* thus, in example 2, the figure 3 of the product under inches, means 3 firsts, or 3-12th of a square foot, and is equal to 36 square inches. If the 3 had been square inches, it would have been in value only 1-48th of a square foot.

In Cubic Measure, the answer is in *cubic* feet, and proportional subdivisions.

When the number of feet is large, it is often the most convenient method to multiply by the feet of the multiplier, and then to take aliquot parts for the inches, &c.—thus, for 6 inches, take the $\frac{1}{2}$ of 1 foot; and so on.

I. To find the area of a rectangle.

Rule.—Multiply the length by the breadth, and the product will be the area.

Example 1.—What is the area of a rectangle, whose length is 4 feet and breadth 3 feet?

$$4 \times 3 = 12 \text{ square feet.}$$

Example 2.—What is the area of a rectangular pane of glass, whose length is 1 foot 8 in. 4 sec., and breadth 1 foot 2 in. 6 sec.?

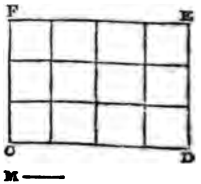
ft.	*in.*	*sec.*	
1	8′	4″	
	1	2	6
1	8	4	
	3	4	8
	10	2	0
2	0	6	10

In the product, the 2 denotes square feet; the next place denotes 12th parts of a square foot; the figure in the third place, 6, denotes 6-12ths of a 12th; that is, 6-144th parts of a square foot, &c.

1·84
1·26
1·84
·348
·0a20
2·06a0

Taking a to denote 10, and b to denote 11, the work, by the duodecimal scale, would stand as in the margin.

Reason of the Rule.—By referring to Example 1, if CD, the length of the rectangle, be divided into 4 equal parts, each of the parts will measure 1 foot; and if CF, the breadth, be divided into 3 equal parts, each of the parts will also be a foot. Now, if lines be drawn through these points of division, as in the annexed figure, the horizontal lines will divide the rectangle into three rows, and each row obviously contains as many *square feet* as there are units in the length; that is, in the present example, each row contains 4 square feet; and since there are 3 rows, ∴ 4 square feet, multiplied by 3, gives 12 square feet for the area.



Hence we may conclude that whatever be the number of units in the length and breadth, the area will contain as many square units as there are units in the product of these numbers.

Exercises.

Find the area of a rectangle whose dimensions are—

	Length.		Breadth.		
1.	4 ft. 2 in.	by	3 ft. 7 in.	Ans.	14 ft. 11′ 2″.
2.	7 ft. 8 in.	"	5 ft. 4 in.	"	40 ft. 10′ 8″.
3.	9 ft. 5 in.	"	3 ft. 10 in.	"	36 ft. 1′ 2″.
4.	11 ft. 7 in.	"	9 ft. 8 in.	"	111 ft. 11′ 8″.
5.	13 ft. 5 in. 4″	"	7 ft. 4 in.	"	98 ft. 7′ 1″ 4‴.
6.	7 ft. 6 in. 9″	"	3 ft. 7 in.	"	27 ft. 1′ 2″ 3‴.
7.	10 ft. 11 in.	"	7 ft. 4 in. 9″.	"	80 ft. 8′ 10″ 3‴.

Find the area of a rectangle whose dimensions are—

	Length.		Breadth.	
8.	96 ft. 5 in. 7″	by	23 ft. 6 in. 3″,	Ans. 2268 ft. 11′ 3″ 10‴ 9⁗.
9.	179 ft. 10 in. 11″	"	78 ft. 9 in. 8″,	Ans. 14177 ft. 10′ 7″ 6‴ 4⁗.
10.	764 ft. 3 in. 10″	"	43 ft. 7 in. 11″,	Ans. 33369 ft. 11′ 8″ 4‴ 2⁗.
11.	77 ft. 11 in. 8″	"	37 ft. 11 in. 8″,	Ans. 2960 ft. 9′ 4″ 1‴ 4⁗.
12.	88 ft. 10 in. 4″	"	47 ft. 3 in. 7″,	Ans. 4203 ft. 0′ 1″ 0‴ 4⁗.
13.	73 ft. 5 in. 8″	"	33 ft. 6 in. 4″,	Ans. 2463 ft. 4′ 3″ 10‴ 8⁗.
14.	96 ft. 8 in. 9″	"	55 ft. 11 in. 9″,	Ans. 5418 ft. 11′ 9″ 9‴ 9⁗.

15. Find the content of a board 32 ft. 5 in. long, and 2 ft. 5 in. broad, ... Ans. 78 ft. 4′ 1″.

16. Required the area of a pavement whose length is 23 ft. 11 in., and breadth 3 ft. 4 in. 6″, . . . Ans. 80 ft. 8′ 7″ 6‴.

17. How many square feet of flooring in a room 24 ft. 7 in. long, and 16 ft. 4 in. broad? . . . Ans. 401 ft. 6′ 4″.

18. Required the area of a plank whose length is 20 ft. 3 in., and breadth 20 in. . . . . Ans. 33 ft. 9′.

19. What is the surface of a board 41 ft. 8 in. long, 2 ft. 8 in. broad at one end, and 1 ft. 10 in. at the other? . Ans. 93 ft. 9′.

Note.—When the breadth is not the same throughout, the mean or average breadth is to be taken. Thus, in the above exercise, 2 ft. 8 in. and 1 ft. 10 in. are added, and the sum, 4 ft. 6 in., is divided by 2, to give the mean breadth.

II. To find the solidity of a rectangular solid.

Rule.—Find the continued product of the length, breadth, and thickness; the result will be the solidity.

Note.—For the demonstration of this rule, the pupil is referred to *Practical Mathematics*, page 158.

Exercises.

20. What is the solidity of a rectangular block of marble, whose length is 10 ft. 4 in. 8″, breadth 7 ft. 2 in. 6″, and thickness 4 ft. 6″? . . . . . Ans. 301 ft. 7′ 10″ 3‴ 9^{iv} 9^{v}.

21. What is the solidity of a log of wood 20 ft. long, 18 in. broad, and 14 in. thick? . . . . Ans. 35 ft.

22. How many solid feet of air in a room 68 ft. 10 in. long, 35 ft. 7 in. broad, and 20 ft. 3 in. high? . Ans. 49598 ft. 8′ 7″ 6‴.

23. Required the solid content of a stone 13 ft. 8 in. long, 7 ft. 9 in. broad, and 3 ft. 11 in. thick, . . Ans. 414 ft. 10′ 1″.

24. What is the solid content of a log of beech 19 ft. $4\frac{1}{2}$ in. long, 2 ft. $8\frac{1}{8}$ in. broad, and 1 ft. $9\frac{2}{3}$ in. thick?

Ans. 79 ft. 2′ 4″ 4‴ 1^{iv}.

The solid content of round timber is commonly ascertained by girding it in two or more places, and dividing the sum of the girts by their number for the mean girt, then taking the *one-fourth* of the mean girt for the mean breadth and thickness.

25. What is the solid content of a round tree 25 ft. 8 in. long, its girts at four different places being 7 ft., 5 ft., 4 ft., and 2 ft.?

Ans. 32 ft. 5′ 9″ 9‴.

26. How many solid feet of timber in a tree 28 ft. 6 in. long, its girts being 60 in., 52 in., 48 in., and 40 in.?

Ans. 30 ft. 11′ 1″ 1‴ 6^{iv}.

27. Required the solid content of a tree 31·52 feet long, its girts at five different places being 9·72 ft., 7·56 ft., 6·32 ft., 5·96 ft., and 4·48 ft. . . . . Ans. 91·30726208 ft.

Miscellaneous Exercises.

1. How many square feet of flooring in a room 42 ft. 8 in. long, and 31 ft. 10 in. broad? and what will it cost at 2*s.* $4\frac{1}{2}$*d.* per square yard? . . Ans. 1358 ft. 2′ 8″; cost, £17, 18*s.* $5\frac{1}{27}$*d.*

2. A window is 7 ft. 8 in. high, and 3 ft. $6\frac{1}{2}$ in. wide; what will be the expense of glazing it at 1*s.* 8*d.* per square foot?

Ans. £2, 5*s.* $3\frac{1}{18}$*d.*

3. How many imperial gallons of water will a cistern hold whose dimensions are 6 ft. 8 in., 3 ft. 10 in., and 5 ft. 4 in.?

Ans. $849\frac{57187}{138637}$.

4. A man was hired to dig a cellar 20 ft. long, 14 ft. broad, and 10 ft. 6 in. deep; how many solid feet must he excavate to complete it? and how much will he have for his labour at 2*s.* 3*d.* per solid yard? . . . . . . Ans. 2940 ft.; £12, 5*s.*

5. How many feet of boards will it take to make a box 6 ft. long, 4 ft. wide, and 3 ft. deep, without a top? and how many bushels will the box hold? . . Ans. 84 ft.; $56\frac{12328}{138637}$ bush.

6. How many square feet of painting on the walls of a room 45 ft. 6 in. long, 24 ft. 10 in. broad, and 13 ft. 4 in. high? and what will be the expense of painting the walls at $10\frac{1}{2}d.$ per square yard, and whitewashing the ceiling at $1\frac{1}{4}d.$ per square yard?

Ans. 1875 ft. 6′ 8″; expense, £9, 18*s.* $0\frac{101}{218}d.$

7. How many square feet of paving in a court 68 ft. 4 in. by 60 ft. 6 in.? and what will it cost at $3\frac{1}{4}d.$ per square yard?

Ans. 4134 ft. 2′; cost, £6, 4*s.* $4\frac{3}{4}d.$ $\frac{31}{54}$.

8. A house has 3 tier of windows, 5 in each tier: the height of the under tier is 7 ft. 9 in.; of the second, 6 ft. 10 in.; of the third, 5 ft. 6 in.; and their common breadth is 3 ft. 8 in.; what is the expense of glazing them at 1*s.* 6*d.* per square foot?

Ans. £27, 12*s.* $3\frac{1}{2}d.$

9. How many square feet of flooring are in a house of 4 floors, 60 ft. by 30 within the walls, deducting from each floor the vacancy for the stair, 12 ft. 4 in. by 8 ft. 6 in.? and what is the value at 3*s.* 6*d.* per square yard?

Ans. 6780 ft. 8′; value, £131, 16*s.* $11\frac{1}{3}d.$

10. How many square feet of plastering in the roof and walls of a room 32 ft. 6 in. long, 16 ft. 6 in. broad, and 9 ft. 3 in. high, deducting a door 6 ft. 6 in. by 3 ft.? and what will it cost at $4\frac{1}{2}d.$ per square yard? . . Ans. 1423 ft. 3′; cost, £2, 19*s.* $3\frac{1}{2}d.$ $\frac{1}{2}$.

11. How many deals 12 ft. 6 in. long, and $8\frac{1}{2}$ in. broad, will be required to floor a room 50 ft. by 32 ft.? . . Ans. $180\frac{12}{17}$.

12. What will be the expense of lining a water-cistern with lead, allowing 10 lb. to the square foot, the length of the cistern being 2 ft. 10 in., its depth 2 ft. 6 in., and breadth 2 ft., and the cost of the lead £1, 18*s.* 9*d.* per cwt.? . Ans. £5, 3*s.* $2\frac{1}{2}d.$ $\frac{13}{28}$.

13. How many solid yards of earth must be dug to form the foundation of a house $46\frac{1}{2}$ feet long and $37\frac{1}{2}$ feet wide, the depth being $7\frac{3}{4}$ feet; and what will it cost at 2*s.* per solid yard?

Ans. $500\frac{25}{48}$ yards; cost, £50, 1*s.* $0\frac{1}{2}d.$

14. How many yards of carpet, yard wide, will be required to carpet a room $25\frac{1}{2}$ feet long, and $19\frac{1}{2}$ feet wide; and what will it cost at 3*s.* 4*d.* per yard? . . Ans. $55\frac{1}{4}$ yards; cost, £9, 4*s.* 2*d.*

MISCELLANEOUS QUESTIONS.

1. For the year ending Dec. 31, 1857, the revenue of Great Britain derived from customs was £22,464,358; from excise, £17,472,000; from stamps, £7,269,224; from land and assessed taxes, £3,104,020; from property-tax, £15,137,996; from post-office, £2,992,000; from crown-lands, £273,654; from various other sources, £1,677,096; the expenditure for the same year was £70,354,246. What was the whole revenue?—and what was the surplus income? . . . . Ans. £70,390,343; £36,097.

2. Two travellers depart at the same time from the same place; the one goes 20 miles a day, the other $23\frac{1}{2}$: how far will they be from one another at the end of 14 days, if they both travel in the same direction?—and how far, if in contrary directions?

Ans. 49 miles; 609 miles.

3. What must I give for 8 pieces of cloth, each 24 yards, at 15*s*. 6*d*. per yard? . . . . Ans. £148, 16*s*.

4. A person commenced business with a capital of £8000; he gains at the rate of £6000 in 4 years by trading in cotton, and at the rate of £6000 in 6 years by trading in shawls, and he spends annually £3000; how long will it be before he become bankrupt? . . . . Ans. 16 years.

5. In a certain division the remainder is 108, and the divisor $\frac{25}{6}$ of the remainder, while the integral quotient is 5 times the divisor; find the divisor, dividend, and quotient,

Ans. Divisor, 450; dividend, 1012608; quotient, $2250\frac{108}{450}$.

6. How long will it require to count eight hundred millions, at the rate of 250 per minute, reckoning 10 hours to a day?

Ans. 14 yr. 223 d. 3 hr. 20 min.

7. If 3 gallons of water be mixed with 18 gallons of rum at 15*s*. 9*d*., what will be the value of a gallon of the mixture?

Ans. 13*s*. 6*d*.

8. Bought $14\frac{1}{2}$ dozen of port wine for £27, 7*s*. $4\frac{1}{2}$*d*.; how much is that per dozen, and per bottle?

Ans. £1, 17*s*. 9*d*. per dozen; 3*s*. $1\frac{3}{4}$*d*. per bottle.

9. If a person lay up $2\frac{1}{2}$*d*. per day, how long will he be in laying up £150? . . . . . . Ans. 39 years 165 days.

10. What does the rent of a house amount to from May 15 to January 20, at the rate of £15, 10*s*. a year?

Ans. £10, 12*s*. $3\frac{3}{4}$*d*. $\frac{57}{73}$.

11. A merchant failing in trade owes A £500, 10*s*.; B, £350, 15*s*. 6*d*.; C, £245, 0*s*. $6\frac{1}{2}$*d*.; D, £56, 18*s*. $1\frac{1}{2}$*d*.; and E, £337, 1*s*. 8*d*.; he has in cash £192, 7*s*. 1*d*., and his effects amount to £739, 1*s*. $6\frac{3}{4}$*d*.: what will each creditor receive per pound, supposing his cash and effects delivered to them?

Ans. 12*s*. 6*d*. per pound.

12. Shipped on an adventure to Lisbon 300 barrels of salmon at £3, 18*s*. 6*d*., 450 yards of linen at 2*s*. 7½*d*., 1200 yards of broad cloth at 16*s*. 4*d*.; paid for insurance and other charges, £53, 18*s*. 6*d*.; the net proceeds, as per account sales, was £2463, 17*s*. 9*d*. What was the whole gain or loss?—gain or loss per cent.? Ans. £193, 8*s*. gain on the whole; £8, 10*s*. 4¼*d*. $\frac{42827}{161639}$ gain per cent.

13. Shipped for Bordeaux 1200 qr. of wheat, at 46*s*. 10*d*. per qr., 600 qr. barley at 25*s*. 6*d*., 400 yards of cloth at 15*s*. 8¾*d*.; insurance on £2200 at 2 guineas per cent., policy-duty 5*s*. per cent., and other charges, £118, 10*s*.: the net proceeds were £4000. Required whole gain or loss, . . . Ans. Loss, £51, 9*s*.

14. A spirit-dealer purchased a puncheon of whisky, containing 90 gallons, at 8*s*. 6*d*. per gallon; he mixes it with 30 gallons of water, and sells the mixture at 3½*d*. per gill: what does he gain by the sale of a puncheon? . . . . Ans. £17, 15*s*.

15. Bought paper at 17*s*. 6*d*. per ream; at what must I sell it per ream to gain £26 on 310 reams, and give six months' credit?

Ans. 19*s*. 7¾*d*. $\frac{82}{155}$.

16. If 17 guineas be lost by the sale of 460 lb. of raw silk at £1, 6*s*. 4*d*. per lb., what was the prime cost per lb.?—loss per cent.?

Ans. £1, 7*s*. 1¼*d*. $\frac{22}{115}$ per lb.; £2, 17*s*. 3¾*d*. $\frac{19473}{37411}$ loss per cent.

17. A merchant consigns to his factor in Jamaica 32 bales of cloth, with orders to dispose of it at £172, 5*s*. 10*d*. per bale, and after retaining commission at 6 per cent., to make remittance for value, one-half in sugar at £2, 10*s*. per cwt., and the other half in rum at 13*s*. 6*d*. per gallon; what quantity of rum and sugar must be returned by the factor?

Ans. 3838$\frac{74}{81}$ gallons of rum; 1036$\frac{38}{75}$ cwt. of sugar.

18. A has a quantity of pepper, weighing 7800 lb., at 1*s*. 3½*d*. per lb., which he barters with B for equal quantities of ginger at 7½*d*. per lb., and cinnamon at 4*s*. 9*d*. per lb.; how many pounds of each did A receive? . . . Ans. 1874$\frac{18}{43}$ lb. of each.

19. A wood-merchant exchanged 345 loads of Quebec oak, valued at £7, 9*s*. 6*d*. per load, for ⅓ of the value in cash, and quantities of pine and teak in the ratio of 11 to 3; how many loads of pine and teak did he receive, supposing the pine valued at £3, 15*s*., and the teak at £12, 5*s*. per load?

Ans. 242$\frac{11}{24}$ loads of pine; 66⅛ loads of teak.

20. A tradesman has velvets at 10*s*. 6*d*. per yard ready money, but in bartering them for my flannels, he raises the price to 12*s*. per yard; to what must I raise my flannels, which I sell at 1*s*. 9*d*. per yard ready money, so that he may obtain no advantage?

Ans. 2*s*. per yard.

21. A barters with B 272 lb. of tea at 5*s*. 6*d*., 148 lb. at 4*s*., and 256 lb. at 3*s*. 8*d*., for 11 cwt. 3 qr. 24 lb. of sugar at 8½*d*. per lb., 10 cwt. 1 qr. 8 lb. at 6¼*d*. per lb., and the balance in money; how much money did A receive? . . . Ans. £78, 15*s*. 5*d*.

22. Four men received a prize in time of war valued at £8190; but being of different ranks, it was divided as follows: the first received $\frac{1}{3}$ of the whole, and $\frac{1}{10}$ more; the second, $\frac{1}{4}$ of the whole, and $\frac{1}{10}$ more; the third, $\frac{1}{6}$ of the whole. What part of the whole did the fourth receive?—and what was the amount of each man's share?

Ans. The fourth received $\frac{1}{20}$; and their respective shares were £3549, £2866, 10*s*., £1365, £409, 10*s*.

23. A and B engaged in trade with a capital of £4000, of which A's share was to B's as 7 : 5; at the end of 15 months their whole stock was increased to £5680; what was each partner's share of the gain? . . . . Ans. A, £980; B, £700.

24. Three persons, A, B, and C, purchased a ship, of which A paid for $\frac{2}{9}$, B for $\frac{3}{5}$, and C paid £400; what part of the ship had C, and what did A and B pay?

Ans. C had $\frac{8}{45}$; A paid £500, and B £1350.

25. A merchant had $5\frac{3}{8}$ cwt. of sugar at $6\frac{3}{4}$*d*. per lb., which he bartered for tea at $8\frac{5}{8}$*s*. per lb.; how much tea did he receive for the sugar? . . . . Ans. $48\frac{1}{69}$ lb.

26. Bought $48\frac{5}{8}$ yards of canvas for £3·646875; what did it cost per yard? . . . . Ans. 1*s*. 6*d*.

27. Two men travelled from the same town—the one north, 28 miles per day; the other west, 36 miles per day: how far were they distant from each other after travelling 6 days?

Ans. 273·642102 miles.

28. There are three boxes: the content of one is 10000 solid inches; of another, 16656; and of the third, 20000: required the side of a cubical box that will contain as much as all the three,

Ans. 36 inches.

29. A linen manufacturer purchases 12 mats of rough flax at 25 guilders per mat, at $23\frac{3}{4}$*d*. per guilder; he dresses the flax, the expense of which is £6, 10*s*.; and when dressed, it yields $32\frac{1}{2}$ stones of tow, and 680 lb. of flax. He sells the tow at $5\frac{1}{2}$*d*. per lb., and delivers the flax to be spun into yarn; every 4 lb. produces 5 spindles, and every spindle makes $3\frac{1}{5}$ yards of linen; expense of spinning, 1*s*. $4\frac{1}{4}$*d*. per spindle; weaving, $5\frac{3}{4}$*d*.; and bleaching, $2\frac{1}{2}$*d*. per yard. He then sells $\frac{1}{4}$ of the linen at 1*s*. 9*d*., $\frac{1}{4}$ at 1*s*. 8*d*., and the remainder at 1*s*. 6*d*. per yard. Required his gain or loss, ... Ans. Gain, £41, 7*s*. 1*d*.

30. What will the digging of the foundation of a house 68 feet long, 33 broad, and 5 deep, come to at 1*s*. 3*d*. per solid yard?

Ans. $415\frac{5}{9}$ yards; £25, 19*s*. $5\frac{1}{4}$*d*. $\frac{1}{3}$.

31. A contractor engages to supply a ship with provisions, and receives 15*s*. per month of 28 days for each private man, thrice as much for each officer, and six times as much for the captain. The crew consists of the captain, 8 officers, and 160 private men. He provides 35 barrels of beef at £1, 15*s*. 6*d*.; 16 do. of pork at £2, 12*s*.; 8 tons $15\frac{1}{2}$ cwt. of bread at £1, 5*s*. per cwt.; 82 bushels of

pease at 5*s.*; 26 cwt. 2 qr. 21 lb. cheese at £1, 10*s.* per cwt.; 18 cwt. 3 qr. 16 lb. of butter at £3, 14*s.* 8*d.* per cwt.; 26½ qr. of oatmeal at 19*s.*; 8 cwt. of fish at £1, 6*s.*; other small articles to the amount of £1, 7*s.* 3½*d.* At the return of the ship, he finds his profit is £185, 15*s.* 5*d.* How long was the voyage?

Ans. 4 months 21 days.

32. How many deals 14 ft. 6 in. long, and 9½ in. broad, will floor a room 70 feet by 30? . . . . . Ans. 182$\frac{518}{551}$.

33. A Memel log 14 ft. 9 in. long was sawed into 11 deals, each 2 ft. 3 in. broad; how many square feet did they contain?—and what is the sawyers' wages at 1½*d.* per square foot?

Ans. 365 ft. 9″; £2, 5*s.* 7½*d.* ⅜.

34. A house is painted at 9½*d.* per square yard. The first room is 28 yards in circumference by 3½ high; the second, 18 yards by 3½; the third, 12 yards by 3¼; the fourth, 14 yards by 3; and the fifth, 10 yards by 3. The painter uses 16 stones of white-lead at 5*s.* 8*d.* per stone, 30 pints of linseed oil at 1*s.* 3*d.* per pint, and other materials to the value of 16*s.* How much has he for his work? . . . . Ans. £3, 11*s.* 2*d.*

35. How many square yards of paving in a street ¾ of a mile long, and 66½ feet broad?—and what will it cost at 6½*d.* the square yard? . . . . Ans. 29260 yards; £792, 9*s.* 2*d.*

36. How many guineas should I receive in change for 350 five-pound notes and 20 crowns? . . Ans. 1671 guin. 9*s.*

37. If needles be bought for 2*s.* 6*d.* a gross, how many may be sold for 1*d.* to gain 20 per cent.? . . . . . Ans. 4.

38. I bought 360 yards of cloth at 5*s.* 4*d.* per yard, of which I sold 210 yards at 7*s.* 2*d.*; but the article advancing in price, I sold the remainder at 9*s.* per yard: what did I gain on the whole?—and how much per cent.?

Ans. Gained, £46, 15*s.*; per cent., £48, 13*s.* 11¼*d.*

39. A gentleman left among 5 sons £1500 in cash, and 5 bills of £48, 10*s.* 6*d.* each; he ordered £29 to be expended upon his funeral, and his debts, amounting to £464, 10*s.*, to be paid. The remainder was to be divided among his sons; the eldest was to receive ⅓ of the whole, and the balance was equally divided among the other 4. Required the share of each, Ans. Eldest son's share, £416, 7*s.* 6*d.*; each of the others, £208, 3*s.* 9*d.*

40. A, B, C, and D got the present of a guinea, of which A claimed ½, B, ⅓, C, ¼, and D, ⅕; but found it too little: it is required to determine their shares of it in proportion to the above fractions, Ans. A's share, 8*s.* 2$\frac{0}{4}$*d.* $\frac{8}{11}$; B's, 5*s.* 5¼*d.* $\frac{9}{11}$; C's, 4*s.* 1$\frac{0}{4}$*d.* $\frac{4}{11}$; D's, 3*s.* 3¼*d.* $\frac{1}{11}$.

41. A farmer sold 115 qr. of wheat at £2, 10*s.*; 120 qr. of barley at £1, 15*s.*; 275 qr. of oats at 20*s.* 8*d.*; 3000 stones of hay at 8*d.* per stone; and other articles to the amount of £96, 15*s.* The yearly rent of the farm is £317, 10*s.* 6*d.*; he has 4 men-servants at £20 a year, 3 maid-servants at £14; 6 horses at £2…

a year, each. Required his annual profit, allowing £60 for reapers' wages, and £30, 15*s.* for tear and wear of implements,
Ans. £298, 2*s.* 10*d.*

42. Sent to Bombay 800 pieces of linen, each 25¼ yards, which my correspondent sold at 1 sic. rup. 12 an. per yard; what do I gain or lose, the prime cost of the linen being 2*s.* 7¼*d.* per yard, freight, 3¼*d.* per piece; my correspondent's commission, 5 per cent.; and the exchange, 2*s.* 1¼*d.* per sicca rupee? . .
Ans. Gain, £342, 17*s.* 3¾*d.* ⅘.

43. If 100 stones are placed on the ground in a straight line at the distance of a yard from each other, how far will a person travel who will bring them all one by one to a basket placed 1 yard from the first stone? . . Ans. 5 miles 1300 yards.

44. A tank can be filled with water by two separate pipes: by the first it would be filled in 10 hours; by the second, in 8 hours. It is begun to be filled by the first alone; but when half full, the supply from the second pipe is turned on. What was the whole time of filling the tank? . . . Ans. 7 hours 18⅓ min.

45. A man aged 45 has a pension of £300 a year during his own life; but he wishes to exchange it for another, to continue not only during his own life, but also during that of his wife, aged 40: what will this pension be, reckoning compound interest at 5 per cent.? . . . . Ans. £245, 15*s.* 0¼*d.*

46. If a person lend £7000 at 6 per cent. compound interest, and allow the interest to accumulate in the hands of the creditor, except £240 per annum, which he draws for family expenditure; how much will the creditor owe him at the end of 16 years?
Ans. £11621, 1*s.* 1¼*d.*

47. A church, which contains 1500 persons, is built for £1200; one-third of the seats are let at 3*s.* 6*d.*, another at 2*s.* 6*d.*, and the remaining third at 1*s.* The minister's stipend, £100, is paid from these rents, and the remainder applied to reimburse the expense of building: how much per cent. is paid annually? Ans. £6, 5*s.*

48. 2000 copies of an octavo work, of 600 pages, or 37½ sheets, are printed. The paper cost 12*s.* per ream of 500 sheets; type-work, 1*s.* per page; press-work, 2*s.* 6*d.* per ream; other charges at printing, 5*s.* per sheet; binding, 8½*d.* per copy; advertising and other expenses, £3, 19*s.* 2*d.* At what price must a copy be sold that the editor's profit may be 100 per cent.? Ans. 4*s.* 5½*d.*

49. An architect employs 24 masons at 2*s.* 6*d.*, 10 labourers at 1*s.* 8*d.*, and 6 miners at 2*s.* for every working-day from Feb. 4 to Nov. 11; but reduces their wages during winter as follows: masons, 2*s.*, labourers, 1*s.* 4*d.*, and miners, 1*s.* 8*d.* per day. Required the yearly income of each, Ans. A mason's income, £37, 6*s.*; a labourer's, £24, 17*s.* 4*d.*; a miner's, £30, 1*s.* 8*d.*

50. A person possessed ⅗ of a copper-mine, and sold ¾ of his share for £1710; what was the value of the whole mine at the same rate? . . . . Ans. £3800.

51. The stock employed by a paper-manufacturer is £2500. There is consumed annually 48 tons of rags, the $\frac{1}{4}$ of which cost £36 per ton, and the remainder £20 per ton. He employs 24 hands, whose wages on an average amount to 8*s*. each a week; repairs, and other expenses, £205 per annum. He manufactures 500 reams of paper at a guinea per ream, 800 at 18*s*., 1200 at 15*s*., 1600 at 10*s*. 6*d*., and 650 at 7*s*. 6*d*.; the duty on paper is 20 per cent. Required the gain or loss, allowing 5 per cent. interest on the stock, ... Ans. Gain, £537, 16*s*.

52. An estate, consisting of 130 acres at 15*s*. an acre, 200 at 12*s*., and 250 at 9*s*., is to be divided between two sisters; the elder gets the best ground, but allows in return a greater quantity to the younger, so as to render the value equal. How much land has each? Ans. Elder, 242 ac. 2 ro.; younger, 337 ac. 2 ro.

53. A cargo of corn cost £1200; 200 quarters being damaged, were sold at 22*s*., and the loss sustained on them was £80; but the remainder was sold at such a price that there was a gain of £100 on the whole. Required the quantity, the cost per quarter, and the selling price of a quarter of the undamaged part,

Ans. 800 qr.; cost, 30*s*. per qr.; selling price, 36*s*. per qr.

54. One Sessa, in India, having invented the game of chess, shewed it to his prince, who was so delighted with it, that he promised him any reward he should ask; on which Sessa requested that he might be allowed 1 grain of wheat for the first square on the chess-board, 2 for the second, 4 for the third, and so on, doubling continually to 64, the whole number of squares. Now, supposing a pint to contain 7680 of these grains, and 1 quarter to be worth £1, 7*s*. 6*d*., it is required to compute the value of all the corn, . Ans. £6450468216285, 17*s*. $3\frac{3}{4}$*d*. $\frac{32767}{32768}$.

55. A person observed the flash of a cannon 7 seconds before he heard the report; how far was the cannon distant, sound moving at the rate of 1142 feet per second? . Ans. 1 m. 904 yd. 2 ft.

56. A draper commenced business with a capital of £2000, and at the end of the year proceeds to 'take his stock;' that is, to estimate the value of all the goods in his shop. In the various articles of linen he finds 765 yards at 2*s*. 2*d*., 372 yards at 2*s*. 6*d*., 465 yards at 2*s*. 8*d*., and 82 whole pieces of Holland, each $48\frac{1}{2}$ yards at $10\frac{1}{2}$*d*. In the various sorts of woollen cloths he has $369\frac{1}{2}$ yards broad at 19*s*. 6*d*., 374 yards narrow at 7*s*. 6*d*., and of lower-priced cloths 37 yards of each sort, at the different prices of 7*s*., 6*s*. 3*d*., 4*s*. 7*d*., 8*s*. 9*d*., and 2*s*. 6*d*. The value of all the other articles which he sells is £1379, 17*s*. 4*d*. His book-debts amount to £1000, and he has cash on hand £1780. He owes various sums amounting to £2300. What was his net profit, allowing 5 per cent. interest on his capital? . . Ans. £652, 7*s*. $3\frac{1}{2}$*d*.

57. A tobacconist has on hand 3 hogsheads of leaf-tobacco, weighing net, 42 cwt. 3 qr. 12 lb.; prime cost, $5\frac{3}{4}$*d*. per lb.; duty, 1*s*. 9*d*. per lb. He makes one-third of it into twist-tobacco, outcome,

2 oz. per lb.; expense of twisting, $1\frac{1}{2}d.$ per lb.; the remainder he manufactures into snuff, which loses 2 oz. per lb.; expense, $1d.$ per lb. on the snuff. He sells one-half of the twist-tobacco at $3s.$ $2d.$, and the other half at $2s.$ $10d.$ per lb.; one-half of the snuff at $3s.$, and the other at $2s.$ $6d.$ per lb. What was the gain or loss?

Ans. Gain, £97, $1s.$ $8d.$

58. A wine-merchant has 54 dozen of Madeira wine, which he can dispose of just now at $36s.$ per dozen, but by keeping it two years, he expects to sell it at $41s.$ per dozen; but besides losing the interest of the value, the leakage will be 2 dozen, cellar-rent and other charges, $15s.$ Whether will he gain or lose by keeping it? . . . . Ans. Loss, £1, $1s.$ $4\frac{3}{4}d.$ $\frac{1}{5}$.

59. The first term of an increasing equidifferent series is 12, the common difference 15, and the number of terms 38. Find the last term, and the sum of the series,

Ans. Last term, 567; sum of the series, 11001.

60. A, B, and C, dividing a quantity of goods which cost £120, mutually agreed that B should have a third part more than A, and C a fourth part more than B. What must each pay?

Ans. A, £30; B, £40; C, £50.

61. The first term of an equirational series is 19, the common ratio, 13, and the number of terms, 8. Find the last term, and the sum of the series,

Ans. Last term, 1192221823; sum of the series, 1291573640.

62. A man has two silver cups of unequal weight, and a cover, which fits both, weighing 5 ounces. Now, when the cover is put on the less cup, the weight is double the weight of the greater; but when put on the greater, the weight is triple that of the less cup. What is the weight of each? . . Ans. 3 oz., and 4 oz.

63. A man bought 17 yards of cloth, and gave for the first yard $2s.$, and for the last $10s.$, the prices of the 17 yards making an equidifferent series. What was the price of the cloth? Ans. £5, $2s.$

64. A man bought a horse, and by agreement was to give a farthing for the first nail that the horse had in his shoe, 2 farthings for the second, and so on, doubling the price for every nail in the four shoes. What was the value of the horse, supposing that he had 24 nails in all his shoes? Ans. £17476, $5s.$ $3\frac{3}{4}d.$

65. A person mixed 60 gallons of wine, part worth $8s.$ per gallon, and the remainder worth $10s.$ per gallon, so that the value of the mixture was $8s.$ $6d.$ per gallon. What quantities of each were taken? . . . Ans. 45 gal. at $8s.$; 15 at $10s.$

66. Supposing that the expenses of constructing a railway are two million pounds sterling, of which $\frac{1}{4}$ part was money borrowed on debenture at 5 per cent., and the remaining $\frac{3}{4}$ were held in shares: it is required to determine what must be the average weekly receipts, so as to pay the shareholders 6 per cent., the expenses of working the railway being 45 per cent. of the gross *receipts*, ... Ans. £4020, $19s.$ $6\frac{3}{4}d.$ [illegible]

TABLES OF COMPOUND INTEREST AND ANNUITIES.

Table I.—Shewing the sum to which £1, accumulating at Compound Interest, will amount in any number of years from 1 to 50.

Example.—Required the sum to which £1000, accumulating at 4½ per cent. compound interest, will amount in 15 years. Amount of £1 in 15 years at 4½ per cent., £1·9352824 × 1000, Ans. £1935, 5s. 7¾d. = £1935·2824000.

Year.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1·0250000	1·0300000	1·0350000	1·0400000	1·0450000	1·0500000
2	1·0506250	1·0609000	1·0712250	1·0816000	1·0920250	1·1025000
3	1·0768906	1·0927270	1·1087179	1·1248640	1·1411661	1·1576250
4	1·1038129	1·1255088	1·1475230	1·1698586	1·1925186	1·2155063
5	1·1314082	1·1592741	1·1876863	1·2166529	1·2461819	1·2762816
6	1·1596934	1·1940523	1·2292553	1·2653190	1·3022601	1·3400956
7	1·1886858	1·2298739	1·2722793	1·3159318	1·3608618	1·4071004
8	1·2184029	1·2667701	1·3168090	1·3685691	1·4221006	1·4774554
9	1·2488630	1·3047732	1·3628974	1·4233118	1·4860951	1·5513282
10	1·2800845	1·3439164	1·4105988	1·4802443	1·5529694	1·6288946
11	1·3120867	1·3842339	1·4599697	1·5394541	1·6228530	1·7103394
12	1·3448888	1·4257609	1·5110687	1·6010322	1·6958814	1·7958563
13	1·3785110	1·4685337	1·5639561	1·6650735	1·7721961	1·8856491
14	1·4129738	1·5125897	1·6186945	1·7316764	1·8519449	1·9799316
15	1·4482982	1·5579674	1·6753488	1·8009435	1·9352824	2·0789282
16	1·4845056	1·6047064	1·7339860	1·8729813	2·0223701	2·1828746
17	1·5216183	1·6528476	1·7946756	1·9479005	2·1133768	2·2920183
18	1·5596587	1·7024331	1·8574892	2·0258165	2·2084788	2·4066192
19	1·5986502	1·7535061	1·9225013	2·1068492	2·3078603	2·5269502
20	1·6386164	1·8061112	1·9897889	2·1911231	2·4117140	2·6532977
21	1·6795819	1·8602946	2·0594315	2·2787681	2·5202412	2·7859626
22	1·7215714	1·9161034	2·1315116	2·3699188	2·6336520	2·9252607
23	1·7646107	1·9735865	2·2061145	2·4647155	2·7521663	3·0715238
24	1·8087260	2·0327941	2·2833285	2·5633042	2·8760138	3·2250999
25	1·8539441	2·0937779	2·3632450	2·6658363	3·0054345	3·3863549
26	1·9002927	2·1565913	2·4459586	2·7724698	3·1406790	3·5556727
27	1·9478000	2·2212890	2·5315671	2·8833686	3·2820096	3·7334563
28	1·9964950	2·2879277	2·6201720	2·9987033	3·4297000	3·9201291
29	2·0464074	2·3565655	2·7118780	3·1186515	3·5840365	4·1161356
30	2·0975676	2·4272625	2·8067937	3·2433975	3·7453181	4·3219424
31	2·1500068	2·5000804	2·9050315	3·3731334	3·9138574	4·5380395
32	2·2037569	2·5750828	3·0067076	3·5080587	4·0899810	4·7649415
33	2·2588509	2·6523352	3·1119424	3·6483811	4·2740302	5·0031885
34	2·3153221	2·7319053	3·2208603	3·7943163	4·4663615	5·2533480
35	2·3732052	2·8138625	3·3335904	3·9460890	4·6673478	5·5160154
36	2·4325353	2·8982783	3·4502661	4·1039325	4·8773785	5·7918161
37	2·4933487	2·9852267	3·5710254	4·2680099	5·0968605	6·0814069
38	2·5556824	3·0747835	3·6960113	4·4388135	5·3262192	6·3854773
39	2·6195745	3·1670270	3·8253717	4·6163660	5·5658991	6·7047511
40	2·6850638	3·2620378	3·9592597	4·8010206	5·8163645	7·0399887
41	2·7521904	3·3598989	4·0978338	4·9930615	6·0781009	7·3919882
42	2·8209952	3·4606959	4·2412580	5·1927839	6·3516155	7·7615876
43	2·8915201	3·5645168	4·3897020	5·4004953	6·6374382	8·1496669
44	2·9638081	3·6714523	4·5433416	5·6165151	6·9361229	8·5571503
45	3·0379033	3·7815958	4·7023586	5·8411757	7·2482484	8·9850078
46	3·1138509	3·8950437	4·8669411	6·0748227	7·5744196	9·4342582
47	3·1916971	4·0118950	5·0372840	6·3178156	7·9152685	9·9059711
48	3·2714896	4·1322519	5·2135890	6·5705282	8·2714556	10·4012697
49	3·3532768	4·2562194	5·3960646	6·8333494	8·6436711	10·9213331
50	3·4371087	4·3839060	5·5849269	7·1066834	9·0326363	11·4673998

Table II.—Shewing the present value of £1, or the sum accumulating at Compound Interest that will amount to £1, payable at the end of any number of years from 1 to 50.

Example.—Required the present value of £500 (or the sum that will amount to £500), payable at the end of 12 years, reckoning compound interest at 5 per cent. Present value of £1, payable at the end of 12 years, £·5568374 × 500, Ans. £278, 8*s.* 4½*d.* = £278·4187000.

Year.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	·9756098	·9708738	·9661836	·9615385	·9569378	·9523810
2	·9518144	·9425959	·9335107	·9245562	·9157299	·9070295
3	·9285994	·9151417	·9019427	·8889964	·8762966	·8638376
4	·9059506	·8884871	·8714422	·8548042	·8385613	·8227025
5	·8838543	·8626088	·8419732	·8219271	·8024511	·7835262
6	·8622969	·8374843	·8135006	·7903145	·7678957	·7462154
7	·8412652	·8130915	·7859910	·7599178	·7348285	·7106813
8	·8207466	·7894092	·7594116	·7306902	·7031851	·6768394
9	·8007284	·7664167	·7337310	·7025867	·6729044	·6446089
10	·7811984	·7440939	·7089188	·6755642	·6439277	·6139133
11	·7621448	·7224213	·6849457	·6495809	·6161987	·5846793
12	·7435559	·7013799	·6617833	·6245971	·5896639	·5568374
13	·7254204	·6809513	·6394041	·6005741	·5642716	·5303214
14	·7077272	·6611178	·6177818	·5774751	·5399729	·5050679
15	·6904656	·6418619	·5968906	·5552645	·5167204	·4810171
16	·6736249	·6231669	·5767059	·5339082	·4944693	·4581115
17	·6571951	·6050164	·5572038	·5133733	·4731764	·4362967
18	·6411659	·5873946	·5383611	·4936281	·4528004	·4155207
19	·6255277	·5702860	·5201557	·4746424	·4333018	·3957340
20	·6102709	·5536758	·5025659	·4563870	·4146429	·3768895
21	·5953863	·5375493	·4855709	·4388336	·3967874	·3589424
22	·5808647	·5218925	·4691506	·4219554	·3797009	·3418499
23	·5666972	·5066917	·4532856	·4057263	·3633501	·3255713
24	·5528754	·4919337	·4379571	·3901215	·3477035	·3100679
25	·5393906	·4776056	·4231470	·3751168	·3327306	·2953028
26	·5262347	·4636947	·4088377	·3606892	·3184025	·2812407
27	·5133997	·4501891	·3950122	·3468166	·3046914	·2678483
28	·5008778	·4370768	·3816543	·3334775	·2915707	·2550936
29	·4886613	·4243464	·3687482	·3206514	·2790150	·2429463
30	·4767427	·4119868	·3562784	·3083187	·2670000	·2313775
31	·4651148	·3999871	·3442304	·2964603	·2555024	·2203595
32	·4537706	·3883370	·3325897	·2850579	·2444999	·2098662
33	·4427030	·3770263	·3213427	·2740942	·2339712	·1998725
34	·4319053	·3660449	·3104761	·2635521	·2238959	·1903548
35	·4213711	·3553834	·2999769	·2534155	·2142544	·1812903
36	·4110937	·3450324	·2898327	·2436687	·2050282	·1726574
37	·4010671	·3349829	·2800316	·2342969	·1961992	·1644356
38	·3912849	·3252262	·2705619	·2252854	·1877504	·1566054
39	·3817414	·3157536	·2614125	·2166206	·1796655	·1491480
40	·3724306	·3065568	·2525725	·2082890	·1719287	·1420457
41	·3633469	·2976280	·2440314	·2002779	·1645251	·1352816
42	·3544848	·2889592	·2357791	·1925749	·1574403	·1288396
43	·3458389	·2805429	·2278059	·1851682	·1506605	·1227044
44	·3374038	·2723718	·2201023	·1780464	·1441728	·1168613
45	·3291744	·2644386	·2126592	·1711984	·1379644	·1112965
46	·3211458	·2567365	·2054679	·1646139	·1320233	·1059967
47	·3133129	·2492588	·1985197	·1582826	·1263381	·1009492
48	·3056712	·2419988	·1918065	·1521948	·1208977	·0961421
49	·2982158	·2349503	·1853202	·1463411	·1156916	·0915639
50	·2909422	·2281071	·1790534	·1407126	·1107097	·0872037

Table III.—Shewing the sum to which £1 *per annum*, accumulating at Compound Interest, will amount in any number of years from 1 to 50.

Example.—Required the sum to which an annuity of £50, accumulating at 4 per cent. compound interest, will amount in 20 years. Thus—£1 per annum will amount in 20 years to £29·7780786; × 50, Ans. £1488, 18s. 0¾d.=£1488·9039300.

Year.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1·0000000	1·0000000	1·0090000	1·0000000	1·0000000	1·0000000
2	2·0250000	2·0300000	2·0350000	2·0400000	2·0450000	2·0500000
3	3·0756250	3·0909000	3·1062250	3·1216000	3·1370250	3·1525000
4	4·1525156	4·1836270	4·2149429	4·2464640	4·2781911	4·3101250
5	5·2563285	5·3091358	5·3624659	5·4163226	5·4707097	5·5256313
6	6·3877367	6·4684099	6·5501522	6·6329755	6·7168917	6·8019128
7	7·5474302	7·6624622	7·7794075	7·8982945	8·0191518	8·1420084
8	8·7361159	8·8923361	9·0516868	9·2142263	9·3800136	9·5491089
9	9·9545188	10·1591061	10·3684958	10·5827953	10·8021142	11·0265643
10	11·2033818	11·4638793	11·7313932	12·0061071	12·2882094	12·5778925
11	12·4834663	12·8077957	13·1419919	13·4863514	13·8411788	14·2067872
12	13·7955530	14·1920296	14·6019616	15·0258055	15·4640318	15·9171265
13	15·1404418	15·6177905	16·1130303	16·6268377	17·1599133	17·7129829
14	16·5189528	17·0863242	17·6769864	18·2919112	18·9321094	19·5986320
15	17·9319267	18·5989139	19·2956809	20·0235876	20·7840543	21·5785636
16	19·3802248	20·1568813	20·9710297	21·8245311	22·7193367	23·6574918
17	20·8647305	21·7615877	22·7050158	23·6975124	24·7417069	25·8403664
18	22·3863487	23·4144358	24·4996913	25·6454129	26·8550837	28·1323847
19	23·9460074	25·1168684	26·3571805	27·6712294	29·0635625	30·5390039
20	25·5446576	26·8703745	28·2796818	29·7780786	31·3714228	33·0659541
21	27·1832741	28·6764857	30·2694707	31·9692017	33·7831368	35·7192518
22	28·8628559	30·5367803	32·3289022	34·2479698	36·3033779	38·5052144
23	30·5844273	32·4528837	34·4604137	36·6178886	38·9370300	41·4304751
24	32·3490380	34·4264702	36·6665282	39·0826041	41·6891963	44·5019989
25	34·1577639	36·4592643	38·9498567	41·6459083	44·5652101	47·7270988
26	36·0117080	38·5530423	41·3131017	44·3117446	47·5706446	51·1134538
27	37·9120007	40·7096335	43·7590602	47·0842144	50·7113236	54·6691265
28	39·8598008	42·9309225	46·2906273	49·9675830	53·9933332	58·4025828
29	41·8562958	45·2188502	48·0107993	52·9662863	57·4230332	62·3227119
30	43·9027032	47·5754157	51·6226773	56·0849378	61·0070697	66·4388475
31	46·0002707	50·0026782	54·4294710	59·3283353	64·7523878	70·7607899
32	48·1502775	52·5027585	57·3345025	62·7014687	68·6662452	75·2988294
33	50·3540345	55·0778413	60·3412101	66·2095274	72·7562263	80·0637708
34	52·6128853	57·7301765	63·4531524	69·8579085	77·0302565	85·0669594
35	54·9282074	60·4620818	66·6740127	73·6522249	81·4966180	90·3203073
36	57·3014126	63·2759443	70·0076032	77·5983139	86·1639658	95·8363227
37	59·7339479	66·1742226	73·4578693	81·7022464	91·0413443	101·6281388
38	62·2272966	69·1594493	77·0288947	85·9703363	96·1382048	107·7095458
39	64·7829791	72·2342328	80·7249060	90·4091497	101·4644240	114·0950231
40	67·4025535	75·4012597	84·5502778	95·0255157	107·0303231	120·7997742
41	70·0876174	78·6632975	88·5095375	99·8265364	112·8466876	127·8397629
42	72·8398078	82·0231965	92·6073713	104·8195978	118·9247885	135·2317511
43	75·6608030	85·4838923	96·8486293	110·0123817	125·2764040	142·9933386
44	78·5523231	89·0484091	101·2383313	115·4128770	131·9138422	151·1430056
45	81·5161312	92·7199614	105·7816729	121·0293921	138·8499651	159·7001559
46	84·5540344	96·5014572	110·4840315	126·8705677	146·0982135	168·6851637
47	87·6678853	100·3965010	115·3509726	132·9453905	153·6726331	178·1194218
48	90·8595824	104·4083960	120·3882566	139·2632061	161·5879016	188·0253929
49	94·1310720	108·5406479	125·6018456	145·8337343	109·8593572	198·4269626
50	97·4843488	112·7968673	130·9979102	152·6670837	178·5030283	209·3479957

TABLE IV.—SHEWING THE PRESENT VALUE OF £1 *per annum*, OR THE SUM THAT WILL PURCHASE AN ANNUITY OF £1, FOR ANY NUMBER OF YEARS FROM 1 TO 50.

Example.—Required the sum that will purchase an annuity of £100 for 20 years, reckoning compound interest at 3 per cent. Present value of an annuity of £1 for 20 years, £14·8774748 × 100, Ans. £1487, 14*s*. 11½*d*. = £1487·7474800.

Year.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	0·9756098	0·9708738	0·9661836	0·9615385	0·9569378	0·9523809
2	1·9274242	1·9134697	1·8996943	1·8860947	1·8726678	1·8594104
3	2·8560236	2·8286114	2·8016370	2·7750910	2·7489644	2·7232480
4	3·7619742	3·7170984	3·6730792	3·6298952	3·5875257	3·5459505
5	4·6458285	4·5797072	4·5150524	4·4518223	4·3899767	4·3294767
6	5·5081254	5·4171914	5·3285530	5·2421369	5·1578725	5·0756921
7	6·3493906	6·2302830	6·1145440	6·0020547	5·8927009	5·7863734
8	7·1701372	7·0196922	6·8739555	6·7327449	6·5958861	6·4632128
9	7·9708655	7·7861089	7·6076865	7·4353316	7·2687905	7·1078217
10	8·7520639	8·5302028	8·3166053	8·1108958	7·9127182	7·7217349
11	9·5142087	9·2526241	9·0015510	8·7604767	8·5289169	8·3064142
12	10·2577646	9·9540040	9·6633343	9·3850738	9·1185808	8·8632516
13	10·9831850	10·6349553	10·3027385	9·9856479	9·6828524	9·3935730
14	11·6909122	11·2960731	10·9205203	10·5631229	10·2228253	9·8986409
15	12·3813777	11·9379351	11·5174109	11·1183874	10·7395457	10·3796580
16	13·0550027	12·5611020	12·0941168	11·6522956	11·2340151	10·8377696
17	13·7121977	13·1661185	12·6513206	12·1656689	11·7071914	11·2740662
18	14·3533636	13·7535131	13·1896817	12·6592970	12·1599918	11·6895869
19	14·9788913	14·3237991	13·7098374	13·1339394	12·5932936	12·0853209
20	15·5891623	14·8774748	14·2124033	13·5903264	13·0079365	12·4622103
21	16·1845486	15·4150241	14·6979742	14·0291600	13·4047239	12·8211527
22	16·7654132	15·9369166	15·1671248	14·4511153	13·7844248	13·1630026
23	17·3321105	16·4436084	15·6204105	14·8568417	14·1477749	13·4885739
24	17·8849858	16·9355421	16·0583676	15·2469631	14·4954784	13·7986418
25	18·4243764	17·4131477	16·4815146	15·6220800	14·8282090	14·0939446
26	18·9506111	17·8768424	16·8903523	15·9827692	15·1466115	14·3751853
27	19·4640109	18·3270315	17·2853645	16·3295858	15·4513028	14·6430336
28	19·9648887	18·7641082	17·6670188	16·6630632	15·7428735	14·8981273
29	20·4535499	19·1884546	18·0357670	16·9837146	16·0218885	15·1410736
30	20·9302926	19·6004413	18·3920454	17·2920333	16·2888885	15·3724510
31	21·3954074	20·0004285	18·7362758	17·5884936	16·5443910	15·5928105
32	21·8491780	20·3887655	19·0688655	17·8735515	16·7888909	15·8026767
33	22·2918809	20·7657918	19·3902082	18·1476457	17·0228621	16·0025492
34	22·7237863	21·1318367	19·7006842	18·4111978	17·2467580	16·1929040
35	23·1451573	21·4872200	20·0006611	18·6646132	17·4610124	16·3741943
36	23·5562511	21·8322525	20·2904938	18·9082820	17·6660406	16·5468517
37	23·9573181	22·1672354	20·5705254	19·1425788	17·8622398	16·7112873
38	24·3486030	22·4924616	20·8410874	19·3678642	18·0499902	16·8678927
39	24·7303444	22·8082151	21·1024999	19·5844848	18·2296557	17·0170407
40	25·1027751	23·1147720	21·3550723	19·7927739	18·4015844	17·1590864
41	25·4661220	23·4124000	21·5991037	19·9930518	18·5661095	17·2943680
42	25·8206068	23·7013592	21·8348828	20·1856267	18·7235498	17·4232076
43	26·1664457	23·9819021	22·0626887	20·3707949	18·8742103	17·5459120
44	26·5038495	24·2542739	22·2827910	20·5488413	19·0183831	17·6627733
45	26·8330239	24·5187125	22·4954503	20·7200397	19·1563474	17·7740698
46	27·1541696	24·7754490	22·7009181	20·8846536	19·2883707	17·8800665
47	27·4674826	25·0247078	22·8994378	21·0429361	19·4147088	17·9810157
48	27·7731537	25·2667066	23·0912443	21·1951309	19·5356066	18·0771578
49	28·0713695	25·5016569	23·2765645	21·3414720	19·6512981	18·1687217
50	28·3623117	25·7297640	23·4556179	21·4821846	19·7620078	18·2559255

TABLE V.—Carlisle Rate of Mortality.

Age.	Number who complete that year.	Number who die in their next year.	Age.	Number who complete that year.	Number who die in their next year.	Age.	Number who complete that year.	Number who die in their next year.
0	10000	1539	35	5362	55	70	2401	124
1	8461	682	36	5307	56	71	2277	134
2	7779	505	37	5251	57	72	2143	146
3	7274	276	38	5194	58	73	1997	156
4	6998	201	39	5136	61	74	1841	166
5	6797	121	40	5075	66	75	1675	160
6	6676	82	41	5009	69	76	1515	156
7	6594	58	42	4940	71	77	1359	146
8	6536	43	43	4869	71	78	1213	132
9	6493	33	44	4798	71	79	1081	128
10	6460	29	45	4727	70	80	953	116
11	6431	31	46	4657	69	81	837	112
12	6400	32	47	4588	67	82	725	102
13	6368	33	48	4521	63	83	623	94
14	6335	35	49	4458	61	84	529	84
15	6300	39	50	4397	59	85	445	78
16	6261	42	51	4338	62	86	367	71
17	6219	43	52	4276	65	87	296	64
18	6176	43	53	4211	68	88	232	51
19	6133	43	54	4143	70	89	181	39
20	6090	43	55	4073	73	90	142	37
21	6047	42	56	4000	76	91	105	30
22	6005	42	57	3924	82	92	75	21
23	5963	42	58	3842	93	93	54	14
24	5921	42	59	3749	106	94	40	10
25	5879	43	60	3643	122	95	30	7
26	5836	43	61	3521	126	96	23	5
27	5793	45	62	3395	127	97	18	4
28	5748	50	63	3268	125	98	14	3
29	5698	56	64	3143	125	99	11	2
30	5642	57	65	3018	124	100	9	2
31	5585	57	66	2894	123	101	7	2
32	5528	56	67	2771	123	102	5	2
33	5472	55	68	2648	123	103	3	2
34	5417	55	69	2525	124	104	1	1

LOGARITHMS.

Number.	Logarithm.	Number.	Logarithm.
1·0025	·00108,43813	1·0325	·01389,00603
1·0050	·00216,60618	1·0350	·01494,03498
1·0075	·00324,50548	1·0375	·01598,81054
1·0100	·00432,13738	1·0400	·01703,33393
1·0125	·00539,50319	1·0425	·01807,60636
1·0150	·00646,60422	1·0450	·01911,62904
1·0175	·00753,44179	1·0475	·02015,40316
1·0200	·00860,01718	1·0500	·02118,92991
1·0225	·00966,33167	1·0525	·02222,21045
1·0250	·01072,38654	1·0550	·02325,24596
1·0275	·01178,18305	1·0575	·02428,03760
1·0300	·01283,72247	1·0600	·02530,58653

TABLE VI.—Value of an Annuity of £1 on a Single Life.

Age.	3 per cent.	4 per cent.	5 per cent.	Age.	3 per cent.	4 per cent.	5 per cent.
0	17·380	14·283	12·083	52	13·558	12·258	11·154
1	20·085	16·556	13·995	53	13·180	11·945	10·892
2	21·501	17·728	14·983	54	12·798	11·627	10·624
3	22·683	18·717	15·824	55	12·408	11·300	10·347
4	23·285	19·233	16·271	56	12·014	10·966	10·063
5	23·693	19·594	16·590	57	11·614	10·625	9·771
6	23·846	19·747	16·735	58	11·218	10·286	9·478
7	23·867	19·792	16·790	59	10·841	9·963	9·199
8	23·801	19·766	16·786	60	10·491	9·663	8·940
9	23·677	19·693	16·742	61	10·180	9·398	8·712
10	23·512	19·585	16·669	62	9·875	9·137	8·487
11	23·327	19·460	16·581	63	9·567	8·872	8·258
12	23·143	19·336	16·494	64	9·246	8·593	8·016
13	22·957	19·210	16·406	65	8·917	8·307	7·765
14	22·769	19·082	16·316	66	8·578	8·010	7·503
15	22·582	18·956	16·227	67	8·228	7·700	7·227
16	22·404	18·837	16·144	68	7·869	7·380	6·941
17	22·232	18·723	16·066	69	7·499	7·049	6·643
18	22·058	18·608	15·987	70	7·123	6·709	6·336
19	21·879	18·488	15·904	71	6·737	6·358	6·015
20	21·694	18·363	15·817	72	6·373	6·026	5·711
21	21·504	18·233	15·726	73	6·044	5·725	5·435
22	21·304	18·095	15·628	74	5·752	5·458	5·190
23	21·098	17·950	15·525	75	5·512	5·239	4·989
24	20·885	17·801	15·417	76	5·277	5·024	4·792
25	20·665	17·645	15·303	77	5·059	4·825	4·609
26	20·442	17·486	15·187	78	4·838	4·622	4·422
27	20·212	17·320	15·065	79	4·592	4·394	4·210
28	19·981	17·154	14·942	80	4·365	4·183	4·015
29	19·761	16·997	14·827	81	4·119	3·953	3·799
30	19·556	16·852	14·723	82	3·898	3·746	3·606
31	19·348	16·705	14·617	83	3·679	3·534	3·406
32	19·134	16·552	14·506	84	3·454	3·329	3·211
33	18·910	16·390	14·387	85	3·229	3·115	3·009
34	18·675	16·219	14·260	86	3·033	2·928	2·830
35	18·433	16·041	14·127	87	2·873	2·776	2·685
36	18·183	15·856	13·987	88	2·776	2·683	2·597
37	17·928	15·666	13·843	89	2·665	2·577	2·495
38	17·669	15·471	13·695	90	2·499	2·416	2·339
39	17·405	15·272	13·542	91	2·481	2·398	2·321
40	17·143	15·074	13·390	92	2·577	2·492	2·412
41	16·890	14·883	13·245	93	2·687	2·600	2·518
42	16·640	14·695	13·101	94	2·736	2·650	2·569
43	16·389	14·505	12·957	95	2·757	2·674	2·596
44	16·130	14·309	12·806	96	2·704	2·628	2·555
45	15·863	14·105	12·648	97	2·559	2·492	2·428
46	15·585	13·889	12·480	98	2·388	2·332	2·278
47	15·294	13·662	12·301	99	2·131	2·087	2·045
48	14·986	13·419	12·107	100	1·683	1·653	1·624
49	14·654	13·153	11·892	101	1·228	1·210	1·192
50	14·303	12·869	11·660	102	0·771	0·762	0·753
51	13·932	12·566	11·410	103	0·324	0·321	0·317

Table VII.—Value of an Annuity of £1 on two joint Lives.

Ages.		3 per cent.	4 per cent.	5 per cent.
10	10	19·963	17·049	14·803
	15	19·410	16·643	14·500
	20	18·873	16·264	14·221
	25	18·189	15·768	13·850
	30	17·411	15·190	13·416
	35	16·596	14·590	12·963
	40	15·605	13·835	12·378
	45	14·601	13·066	11·785
	50	13·310	12·034	10·953
	55	11·667	10·664	9·799
	60	9·957	9·196	8·530
	65	8·537	7·969	7·463
	70	6·874	6·484	6·131
	75	5·353	5·093	4·855
	80	4·262	4·088	3·925
	85	3·167	3·056	2·953
	90	2·454	2·374	2·299
15	15	18·908	16·272	14·215
	20	18·423	15·922	13·959
	25	17·794	15·460	13·608
	30	17·063	14·918	13·195
	35	16·295	14·347	12·765
	40	15·348	13·623	12·201
	45	14·381	12·884	11·630
	50	13·131	11·882	10·822
	55	11·528	10·543	9·692
	60	9·852	9·103	8·446
	65	8·458	7·897	7·398
	70	6·818	6·433	6·084
	75	5·315	5·057	4·821
	80	4·235	4·062	3·901
	85	3·149	3·039	2·937
	90	2·441	2·361	2·287
20	20	17·993	15·610	13·724
	25	17·421	15·182	13·398
	30	16·749	14·677	13·008
	35	16·031	14·142	12·602
	40	15·131	13·449	12·062
	45	14·207	12·741	11·511
	50	12·995	11·769	10·727
	55	11·429	10·458	9·621
	60	9·782	9·043	8·394
	65	8·411	7·856	7·361
	70	6·790	6·407	6·061
	75	5·298	5·042	4·807
	80	4·225	4·053	3·893
	85	3·143	3·034	2·932
	90	2·437	2·358	2·283
25	25	16·916	14·800	13·101
	30	16·311	14·339	12·742
	35	15·660	13·848	12·365
	40	14·824	13·202	11·856
25	45	13·954	12·530	11·335
	50	12·793	11·599	10·581
	55	11·274	10·325	9·505
	60	9·669	8·943	8·306
	65	8·329	7·783	7·295
	70	6·736	6·358	6·017
	75	5·263	5·010	4·778
	80	4·203	4·033	3·874
	85	3·130	3·022	2·921
	90	2·428	2·349	2·276
30	30	15·784	13·930	12·419
	35	15·209	13·491	12·078
	40	14·449	12·897	11·607
	45	13·650	12·278	11·121
	50	12·551	11·393	10·404
	55	11·089	10·164	9·364
	60	9·529	8·820	8·196
	65	8·224	7·688	7·210
	70	6·662	6·291	5·954
	75	5·213	4·964	4·735
	80	4·168	3·999	3·843
	85	3·107	3·000	2·900
	90	2·411	2·333	2·260
35	35	14·720	13·111	11·780
	40	14·048	12·581	11·354
	45	13·331	12·019	10·912
	50	12·314	11·196	10·238
	55	10·919	10·020	9·240
	60	9·410	8·716	8·105
	65	8·140	7·614	7·143
	70	6·608	6·242	5·910
	75	5·179	4·933	4·706
	80	4·148	3·981	3·826
	85	3·096	2·989	2.890
	90	2·403	2·325	2·253
40	40	13·481	12·125	10·984
	45	12·868	11·641	10·598
	50	11·954	10·894	9·984
	55	10·658	9·796	9·046
	60	9·224	8·553	7·961
	65	8·006	7·493	7·034
	70	6·515	6·157	5·832
	75	5·115	4·872	4·650
	80	4·102	3·937	3·784
	85	3·065	2·961	2·863
	90	2·380	2·304	2·233
45	45	12·371	11·243	10·278
	50	11·560	10·591	9·737
	55	10·400	9·583	8·870
	60	9·063	8·417	7·846
	65	7·910	7·411	6·964

TABLE VII. (*continued*)—Value of an Annuity of £1 on two joint Lives.

Ages.		3 per cent.	4 per cent.	5 per cent.
45	70	6·465	6·113	5·793
	75	5·069	4·850	4·639
	80	4·067	3·994	3·773
	85	3·056	2·952	2·854
	90	2·375	2·299	2·227
50	50	10·942	10·059	9·291
	55	9·924	9 181	8·528
	60	8·729	8·132	7·601
	65	7·691	7·221	6·799
	70	6·338	6·001	5·695
	75	5·022	4·790	4·577
	80	4·054	3·894	3·746
	85	3·040	2·938	2·842
	90	2·365	2·290	2·220
55	55	9·103	8·465	7·900
	60	8·098	7·574	7·106
	65	7·219	6·798	6·418
	70	6·019	5·712	5·431
	75	4·813	4·598	4·400
	80	3·920	3·770	3·630
	85	2·961	2·863	2·772
	90	2·307	2·236	2·168
60	60	7·296	6·854	6·456
	65	6·589	6·225	5·895
	70	5·565	5·293	5·044
	75	4·498	4·304	4·125

Ages.		3 per cent.	4 per cent.	5 per cent.
60	80	3·695	3·558	3·430
	85	2·812	2·722	2·637
	90	2·199	2·132	2·070
65	65	6·047	5·738	5·456
	70	5·193	4·956	4·737
	75	4·257	4·082	3·981
	80	3·542	3·416	3·297
	85	2·719	2·635	2·565
	90	2·131	2·069	2·008
70	70	4·556	4·367	4·191
	75	3·804	3·661	3·528
	80	3·229	3·121	3·020
	85	2·523	2·449	2·380
	90	1·967	1·932	1·880
75	75	3·231	3·119	3·015
	80	2·790	2·704	2·623
	85	2·217	2·157	2·100
	90	1·758	1·712	1·669
80	80	2·459	2·390	2·324
	85	1·993	1·943	1·896
	90	1·589	1·551	1·515
85	85	1·657	1·619	1·583
	90	1·335	1·307	1·279
90	90	1·088	1·066	1·045

DECIMAL MONEY.

A NEW SYSTEM OF RECKONING MONEY DECIMALLY, by subdividing the pound into tenths, hundredths, and thousandths, has been for some time under discussion, with a view to its introduction, if found to be practicable.

The precise details of the system are not determined upon, nor the names to be given to the new denominations of money, but the leading principle in the proposed change is to divide the pound into 1000 parts, instead of, as at present, into 960 farthings, and to advance from one denomination to another, by *tens* (or combination of tens), as in simple numbers, instead of from 4 farthings to 1 penny, 12 pence to 1 shilling, 20 shillings to 1 pound. If this system be adopted, calculations in money will become as easy as those in simple numbers, and will be wrought by the rules of *Simple* Addition, Subtraction, &c.

It has been proposed by some, to divide the pound into 1000 parts, termed *mils*—10 mils to make one cent; 10 cents, 1 florin; and 10 florins, 1 pound. By others, it is proposed to term the 1000 parts into which the pound is to be divided, *cents*—100 cents to make 1 florin, and 10 florins, 1 pound.

As this latter plan is simpler than the other, we shall assume it to be the one adopted; and as the principle of dividing the pound into 1000 parts is the same in both, the mode of working questions will also be the same, whichever plan should ultimately be preferred.

THE FOLLOWING TABLE will shew the nature of the proposed decimal division of the pound. Both of the proposed plans are given.

	Florins.		Cents.
	1	=	100
£1 =	10	=	1000

or,

	Florins.		Cents.		Mils.
			1	=	10
	1	=	10	=	100
£1 =	10	=	100	=	1000

Accounts would thus be kept in pounds, florins, cents; and shillings, pence, and farthings would be disused.

As a cent is the 1000th part of a pound, whilst a farthing is the 960th part, it follows that a farthing is equal to $1\frac{1}{24}$ cent; and a cent to $\frac{24}{25}$ of a farthing—that is, $\frac{1}{25}$ less than a farthing; 25 cents, therefore, will be the same as 24 farthings. A florin, or 100 cents, is equal in value to 2*s*.

Besides florins and cents used in keeping accounts, various other coins, each representing so many cents, will be required. The coins under the new system may probably be nearly as follows:—

			s.	*d.*		Cents.
Gold	Sovereign or pound, ...				=	1000
"	$\frac{1}{2}$ do.				=	500
Silver	Florin, . . .	=	2	0	. =	100
"	$\frac{1}{2}$ do. or shilling, .	=	1	0	. =	50
"	$\frac{1}{4}$ do. or present sixpence,	=	0	6	. =	25
"	10 Cent-piece, . .	=	0	$2\frac{2}{5}$	. =	10
Copper	5 Cent-piece, . .	=	0	$1\frac{1}{5}$	. =	5
"	2 do.				=	2
"	1 do.	= $\frac{24}{25}$ of a farthing,			=	1

THE DECIMALS OF A POUND—that is, florins and cents—may be written in two ways; thus—

£	Florins.	Cents.		£	Cents.
36	9	75	*or,*	36 · 975	

The first method may probably be used in merchants' books: the second, in which the 9 florins 75 cents are written as 975 cents, will be the most convenient in calculations; all that is necessary to distinguish the cents from the pounds, being the placing a decimal point between them.

In adding, subtracting, multiplying, dividing, &c. sums of money, the process will be exactly the same as in simple numbers, except that it will be necessary, in reading the figures, to attend to the placing of the decimal point in all cases before the *third* figure from the right when the given number is cents, and before the *first* figure on the right when the number is florins, to distinguish the florins and cents from the pounds.

It is to be observed that *three* figures are used to express the decimals of a pound—thus, ·346: the first decimal means florins, and the two last cents; or all the three figures may be read as cents.

In any number written as cents, the three last figures are read as cents (or florins and cents), and all the rest as pounds, the decimal point being placed before the third figure from the right: thus, 43624 cents is read as £43, 6 florins, 24 cents, or as £43, 624 cents.

Again, in any number of florins, the last figure is read as florins, and all the rest as pounds—thus, 7468 florins is read as £746, 8 florins.

The following examples, in the various rules of Arithmetic, will shew the method of calculating in decimal money:—

Examples.

1. Add together 43 pounds, 7 florins, 83 cents; 126 pounds, 2 florins, 74 cents; 342 pounds, 9 florins, 86 cents.

£	Florins.	Cents.		
43	7	83		£43 · 783
126	2	74	*or,*	126 · 274
342	9	86		342 · 986
513	0	43		£513 · 043 *Answer.*

2. Subtract from 4978 pounds, 9 florins, 87 cents; 3785 pounds, 8 florins, 98 cents.

£	Florins.	Cents.		
4978	9	87		£4978 · 987
3785	8	98	*or,*	3785 · 898
1193	0	89		£1193 · 089 *Answer.*

3. Multiply 2354 pounds, 6 florins, 49 cents, by 5.

$$\begin{array}{r} £2354 \cdot 649 \\ 5 \\ \hline £11773 \cdot 245 \end{array}$$ *Answer.*

4. Divide 8796 pounds, 8 florins, 96 cents, by 8.

$$\begin{array}{r} 8\,)\,£8796 \cdot 896 \\ \hline £1099 \cdot 612 \end{array}$$ *Answer.*

5. What is the price of 35 yards of cloth, at 7 florins 25 cents per yard (= 14*s.* 6*d.*)?

$$\begin{array}{r} \cdot725 \\ 35 \\ \hline 3625 \\ 2175 \\ \hline £25\cdot375 \end{array}$$ *Ans.*

Here the 7 florins 25 cents are written as 725 cents, and being multiplied by 35, amount to 25375 cents. The decimal point being placed before the third figure from the right, the three last figures express cents (or florins and cents), and all the rest pounds. The answer may be read either as £25·375 cents, or as £25, 3 florins, 75 cents.

6. What is the price of 42 quarters of wheat, at £2, 9 florins, 50 cents, per quarter (= £2, 19*s.*)?

$$\begin{array}{r} £2\cdot950 \\ 42 \\ \hline 5900 \\ 11800 \\ \hline £123\cdot900 \end{array}$$ *Ans.*

Here the three last figures are cents, and all the rest pounds, the decimal point being placed between them. The answer may be read either as £123, 900 cents; or as £123, 9 florins.

7. What is the interest for 1 year on £95, 8 florins, 45 cents, at 4 per cent.?

$$\begin{array}{r} £95\cdot845 \\ 4 \\ \hline 383\cdot380 \end{array}$$

Ans. $£3\cdot833\frac{80}{100}$

To divide ordinary numbers by 100, the two last figures are struck off. In decimals, the division is accomplished merely by removing the decimal point two places to the *left*, and then writing the two last figures as a fraction with the divisor, 100, placed below.

Exercises.

1. Add £2, 5 florins, 33 cents; £4, 9 florins, 86 cents; £7, 3 cents; £8, 6 florins, 75 cents, ... Ans. 23·197
2. Subtract £136, 9 florins, 29 cents, from £254, 3 florins, 18 cents, Ans. 117·389
3. Multiply £25, 9 florins, 27 cents, by 3, 5, 6, 8, Ans. £77·781; £129·635; £155·562; £207·416
4. Divide £75, 2 florins, 36 cents, by 2, 4, 7, 8, Ans. £37·618; £18·809; £10·748; £9·404½

What is the price of the following:—

5.	37 yards of cloth, at £1·075 per yard, . . .	Ans. £	39·775
6.	132 " " £·904 " . . .	"	119·328
7.	435 lbs. of sugar, at 17 cents per lb. . . .	"	7·395
8.	122½ yards of satin, at £1·326 per yard, .	"	162·435
9.	620 " of cotton, at 76 cents, " . . .	"	47·120
10.	336 lbs. of tea, at 2 florins, 35 cents per lb. .	"	78·960
11.	538 " of sugar, at 13 cents " .	"	6·994
12.	18 quarters of wheat, at £2·436 per quarter,	"	43·848

What is the interest for 1 year on:—

13.	£378,	4 florins,	37 cents,	at 2	per cent. . .	Ans. £	$7\cdot568\frac{74}{100}$
14.	296,	5 "	24 "	2½	" . . .	"	$7\cdot413\frac{10}{100}$
15.	425,	6 "	42 "	3	" . . .	"	$12\cdot769\frac{26}{100}$
16.	369,	4 "	82 "	4	" . . .	"	$14\cdot779\frac{28}{100}$
17.	654,	9 "	76 "	5	" . . .	"	$32\cdot748\frac{80}{100}$

SHILLINGS, PENCE, AND FARTHINGS may be converted into decimal money; and DECIMAL MONEY into shillings, &c. by the following rules: they give the answer within a farthing or a cent of the exact value.

I. TO CONVERT SHILLINGS, PENCE, AND FARTHINGS INTO DECIMAL MONEY.

RULE.—Reckon *half* the number of shillings as so many florins; and if the shillings are an odd number, reckon the 1*s.* over as 50 cents; convert the pence into farthings, which, with any farthings in the given sum, reckon as so many cents—adding one more for every 24; then add the whole for the answer.

Example.—Convert 7*s.* 8¼*d.* into decimal money.

	Florins.	Cents.
7*s.*	= 3	50
8¼*d.* = 33 farthings, and adding 1 =		84
	3	84 *Ans.*

Here 1 is added to the 33 farthings, there being *once* 24 in 33.

Exercises.—Convert into decimal money:—

					Florins.	Cents.
1.	£0	6	5	. Ans.	3	20
2.	0	7	8½	. „	3	85
3.	0	13	3	. „	6	62
4.	0	11	7¾	. „	5	82
5.	£0	17	6½	. Ans.	8	77
6.	0	12	9¼	. „	6	38
7.	0	16	10	. „	8	41
8.	0	19	4	. „	9	66

II. TO CONVERT DECIMAL MONEY INTO SHILLINGS, PENCE, AND FARTHINGS.

RULE.—Reckon *twice* the number of florins as so many shillings; and reckon the cents as so many farthings—less 1 for every 25; then add the whole for the answer.

Example.—Convert 5 florins, 42 cents, into shillings, &c.

5 florins, ... = 10*s.*
42 cents, less 1 = 41 farthings, = 10¼*d.*
10*s.* 10¼*d.* *Ans.*

Here 1 is deducted from 42 cents, there being once 25 in 42.

Exercises.—Convert into shillings, &c.:—

	Florins.	Cents.				
1.	0	62	. Ans. £0	1	3	
2.	1	43	. „	0	2	10½
3.	5	06	. „	0	10	1½
4.	7	92	. Ans. 0	15	10¼	
5.	8	87	. „	0	17	9
6.	9	65	. „	0	19	3¾

THE END.

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